Civinnovate

Discover, Learn, and Innovate in Civil Engineering

COURSE COVERAGE

Course Outcome 1

Course Outcome 2

Course Outcome 3

Course Outcome 4

Influence Lines 72

- Influence Lines for Beams and Frames by Equilibrium Method
- Influence Lines for Trusses
- Application of Influence Lines
- Response at a Particular Location due to a single moving Concentrated Load
- Response at a Particular Location due to a Uniformly Distributed Load
- Response at a Particular Location due to a Series of Moving Concentrated Loads
- Absolute Maximum Response

COURSE EVALUATION

Suggested Assessment Tools for Lecture Component	Weight
1. Quizzes	50%
2. Others	
• Attendance	
• Seatwork/Recitation (10%)	
• Homework (10%)	30%
• Case Study / Research Project (10%)	
3. Final Examination	20%
TOTAL	100%

Passing Grade is 70%

Course Outcome 1

Introduction to Structural Analysis

Role of Structural Analysis in Structural Engineering Projects

- To predict performance of a given structure under stipulated loads and/or other external effects such as support movements and temperature changes.

- Structural Analysis is a highly paid profession.

- Stipulated loads comprise:
  - Axial forces
  - Shear forces
  - Bending moments
  - Deflection
  - Support reaction

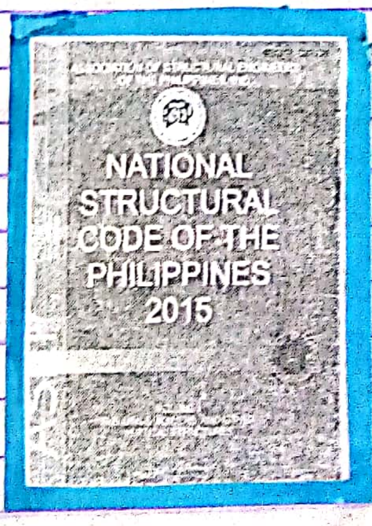
GOLDEN RULE :
Analysis of structure involves determination of the above-mentioned stipulated loads which are applied on a given structure.

No. of UNKNOWN = No. of EQUATION

* As a licensed Civil Engineer, you are bound to follow the codes and local specifications through structural analysis.

* This NSCP 7th edition is referred from the following:

a. Uniform Building Code UBC-1997
b. International Building Code IBC - 2009
c. American Society of Civil Engineers ASCE / SEI 7-10
d. American Concrete Institute ACI 318 - 14M
e. American Institute for Steel Construction AISC-05 with Supplementary Seismic Provisions
f. American Iron and Steel Institute AISI S100-2007
g. Reinforce Masonry Engineering Handbook of America
h. Concrete Masonry Handbook, 6th Edition
i. American National Standard Institute ANSI EIA/TIA-222-G-I-2007
j. American Society for Testing and Materials (ASTM) Standards



- Basic Objective:
  To produce a structure capable of

resisting all applied loads without failure and excessive deformations during its anticipated life.

- An engineering design must provide:
  - description of what is to be built or manufactured
  - materials
  - construction techniques
  - specifications
  - dimensions
  - explanation of the design proposal
  - detailed analytical models

Classification of Structures

- NSCP 7th Edition Section 103 : Nature of Occupancy

- Occupancy Category:

1. Essential Facility
   - surgery/emergency treatment areas
   - fire and police stations
   - shelters for emergency vehicles and aircraft
   - emergency preparedness centers
   - aviation control towers
   - communication centers and other facilities required for emergency response.
   - facilities for standby power-generating equipment for category I structures
   - structures containing water or other fire-suppression material, or equipment required for the protection of category I, II, III, IV, and V structures
   - public school buildings, hospitals
   - designated centers, power, and communication transmission lines

2. Hazardous Facility
   - structures housing toxic or explosive chemicals or substances
   - non-building structures storing same substances

3. Special occupancy structures
   - buildings with assembly room with 1,000 or more occupant capacity
   - educational buildings (museums, libraries, auditorium) with 300 or more occupants
   - institutional buildings with 50 or more incapacitated patients (not included in category I)
   - mental hospitals, sanitariums, jails, prisons, and other buildings where personal liberties of inmates are restrained.
   - churches, mosques, other religious facilities
   - structures with 5000 or more occupancy
   - structures and equipment in power-generating stations, and other public utility facilities not included in Category I or II and required for conditioned operation

4. Standard Occupancy Structures
- All structures housing occupancies or having functions not listed in category I, II, III, and IV

5. Miscellaneous Structures
- Private garages, carports, sheds, and fences over 1.5m high.

- Broad Categorization of Structures based on Member Types:
  - cable structure
  - skeletal structure
  - spatial structure
  - solid structure
  - A combination of the categories

Loads on Structures (Design Loads)

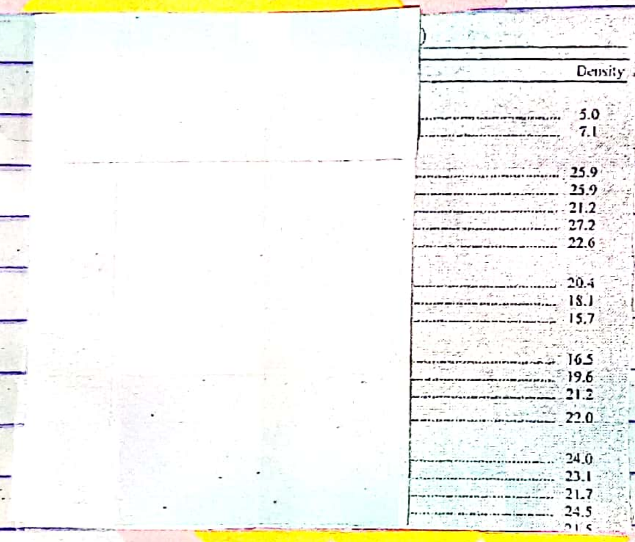
Dead Loads

- consists of the weight of all materials of construction incorporated into the building or other structure, and their similarly incorporated architectural and structural items, and fixed service equipment, including the width of cranes.

- actual weights of materials and constructions shall be used in determining dead loads for purposes of design.

- floors in office buildings and other buildings where partition locations are subject to change shall be designed to support, in addition to all other loads, a uniformly distributed dead load equal to 1.0 kPa.

- Taken from Section 204 : Dead Loads of NSCP 7th Ed.



Density
5.0
7.1
25.9
25.9
21.2
27.2
22.6
20.4
18.1
15.7
16.5
19.6
21.2
22.0
24.0
23.1
21.7
24.5

Live Loads

- the maximum loads expected by the intended use or occupancy but in no case shall be less than the loads required from NSCP.
  - floor and roof live loads
  - uniform
  - concentrated
  - special types

- Section 205 : Live Loads of NSCP 2015

Table 205-1 Minimum Uniform and Concentrated Live Loads

Use or Occupancy: Category	Use or Occupancy: Description	Uniform Load kPa	Concentrated Load kN
1. Access floor systems	Office use	2.4	9.0
	Computer use	4.8	9.0
2. Armories	–	7.2	0
3. Theaters, assembly areas and auditoriums	Fixed seats	2.9	0
	Movable seats	4.8	0
	Lobbies and platforms	4.8	0
	Stage areas	7.2	0
4. Bowling alleys, poolrooms and similar recreational areas	–	3.6	0
5. Catwalk for maintenance access	–	1.9	1.3

Impact

- unusual vibration, impact forces, and motion from live loads

- provisions in structural design:
  - elevators at 100% impact
  - machinery at
    - elevator = 100%
    - light, shaft, or motor-driven = 20%
    - reciprocating or power driven = 50%
    - hangers for floors and balconies = 33%

- Section 206: Other Minimum Loads of NSCP 2015

Wind Loads

- buildings and other vertical structures shall be designed and constructed to resist wind loads as specified and presented in NSCP.

- antenna towers and antenna supporting structures shall be designed and constructed to resist wind loads as specified in "Structural Standards for Steel Antenna Towers and Antenna Supporting Structures"

- procedure of design is:
  - definition of scope
  - checking of permitted procedures
  - definition of main wind-force resisting frame

- Section 207: Wind Loads of NSCP 2015

Earthquake Loads

- earthquake provisions
  - to design seismic-resistant structures to safeguard against major structural damage that may lead to loss of life and property.
  - not intended to assure zero-damage nor maintain their functionality after a severe earthquake.
  - structures and portions, as minimum, be designed and constructed to resist the effects of seismic ground motions

- Section 208: Earthquake Loads of NSCP 7th Ed.

Table 208-1 Seismic Importance Factors

Occupancy Category [1]	Seismic Importance Factor, I	Seismic Importance [2] Factor, I_p
I. Essential Facilities [3]	1.50	1.50
II. Hazardous Facilities	1.25	1.50
III. Special Occupancy Structures	1.00	1.00
IV. Standard Occupancy Structures	1.00	1.00
V. Miscellaneous structures	1.00	1.00

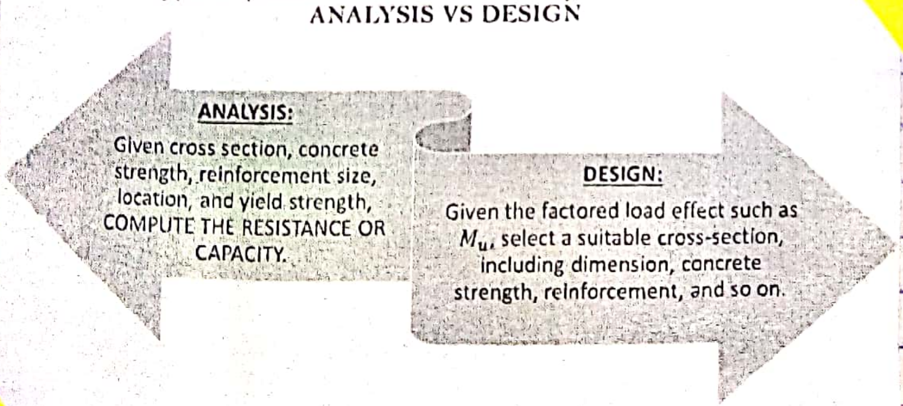
Load Combinations

- Buildings, towers, and other vertical structures and all portions thereof shall be designed to resist load combinations specified in NSCP.

- Working Stress Design (WSD)
  - traditional method of a structural design not only for reinforced concrete structures, but also for steel & timber.
  - method assumes that materials behave in a linear elastic manner and so no load combinations or factors are used.

- Ultimate Stress Design (USD)
  - due to shortcomings of WSD, it serves as an alternative method and commonly used in design problems and practice of CE profession

There are two types of problems that arise in the study of reinforced concrete.
ANALYSIS VS DESIGN



This is for DESIGN COURSES

- Section 203: Combination of Loads

Load Combination Equations:

$1.4\,(D+F)$

$1.2\,(D+F+T) + 1.6\,(L+H) + 0.5\,(L_r \text{ or } R)$

$1.2\,D + 1.6\,(L_r \text{ or } R) + (f_1 L \text{ or } 0.5W)$

$1.2\,D + 1.0\,W + f_1 L + 0.5\,(L_r \text{ or } R)$

$1.2D + 1.0E + f_1 L$

$0.9D + 1.0W + 1.6H$

$0.9D + 1.0E + 1.6H$

where:

f_1 = 1.0 for floors in places of public assembly, for live loads in excess of 4.8 KPa, and for garage live load
= 0.5 for other live loads

D = dead load

E = earthquake load set forth in Section 208.6.1

E_m = estimated max. earthquake force that can be developed in the structure as set forth in Sec. 208.6.1

F = load due to fluids with well-defined pressures and maximum heights

H = load due to lateral pressure of soil and water in soil

L = live load, except roof live load, including any permitted live load reduction

L_r = roof live load, including any permitted live load reduction

P = ponding load

R = rain load on the undeflected roof

T = self-straining force and effects arising from contraction or expansion resulting from temperature change, shrinkage, moisture change, creep in component materials, movement due to differential settlement, or combinations thereof

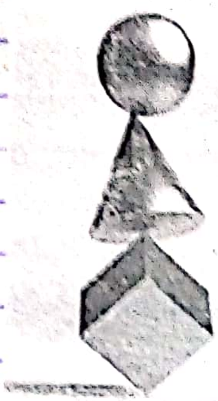
W = load due to wind pressure

$$1.2(D+\cancel{F+T}) + 1.6(L+\cancel{H}) + \cancel{0.5}(L_r \text{ or } R)$$

$$\left.\begin{array}{l} 1.2D + 1.6L : 2010 \\ \text{focus sa earthquake loads} \\ 1.4D + 1.7L : 2001 \end{array}\right\} \text{NSCP}$$

Equilibrium and Support Reactions

Equilibrium of Structures



e·qui·lib·ri·um (ē′kwə lib′rē əm, ek′wə-), n., pl. -ri·ums, -ri·a (-rē ə). 1. a state of rest or balance due to the equal action of opposing forces. 2. equal balance between any powers, influences, etc.; equality of effect. 3. mental or emotional balance; equanimity: The pressures of the situation caused her to lose her equilibrium. 4. Chem. the condition existing when a chemical reaction and its reverse reaction proceed at equal rates. [1600–10; < L aequilībrium, equiv. to aequi- EQUI- + lībr(a) balance + -ium -IUM] —e·qui·li·bra·to·ry (i kwil′ə brə tôr′ē, -tōr′ē, ē′kwə lib′rə-, ek′wə-), adj.
—Syn. 1. equipoise, steadiness, stability.

- A structure is in equilibrium if, initially at rest, it remains at rest when subjected to a system of forces and couples.

- If a structure is in equilibrium, then all its members and parts are also in equilibrium.

- Equilibrium equations provide both the necessary and enough conditions for equilibrium.

- "A structure is in equilibrium when all forces or moments acting upon it are balanced."

- This means that **each force acting upon a body**, or part of the body, is **resisted by** either another **equal and opposite force** or set of forces whose **net result** is **zero**.

- "There is a net result of zero in all directions for all of the applied loads and reactions."

- There are conditions of equilibrium that must be satisfied for both types.

Sum of All Vertical Forces $\Sigma F_y = 0$

Sum of All Horizontal Forces $\Sigma F_x = 0$

Sum of All Moments $\Sigma M_z = 0$

Sum of All Forces $\Sigma F_z = 0$

Sum of All Moments $\Sigma M_y = 0$

Sum of All Moments $\Sigma M_x = 0$

- The first three are the most common equations and will be utilized in all the problems. In this course.

- The other three are only necessary when considering three-dimensional force systems.

- Two Types of Equilibrium

  - External Equilibrium
    encompasses the loads upon, and reactions of, a structural system.

  - Internal Equilibrium
    describes the various forces that are acting within every member of the system.

External and Internal Forces

- External Forces
  - actions of other bodies on the structure under consideration.
  - classified as applied forces and reaction forces
    - Applied forces
      - referred to as loads (e.g. live loads and winds loads)
      - tend to move the structure
      - known in the analysis
    - Reaction forces / reactions
      - forces exerted by supports on the structure
      - tend to prevents motion and keep it in equilibrium
      - unknowns to be determined

- the state of equilibrium or motion of the structure is governed solely by the external forces acting on it.

- Internal Forces
  - the forces and couples exerted on a member or portion of the structure by the rest of the structure
  - forces develop within the structure and hold the various portions of it together
  - always occur in equal but opposite pairs, because each member or portion exerts back on the rest of the structure the same forces acting upon it but in opposite directions, according to Newton's third law.
  - cancels each other
  - do not appear in equilibrium equations of entire structure
  - appear as unknown in equilibrium equations of individual members or portions of the structure

- Force - the amount of pushing or pulling required to move an object

- When **external forces are applied to a structure**, **internal stresses** (internal forces) develop **resistance** to the outside forces.

- The opposition of external and internal forces is what holds the structure together.

- Once engineers know the loads acting on a structure, they calculate the resulting internal stresses, and design each piece of the structure so it is strong enough to carry the loads without breaking.

External Forces

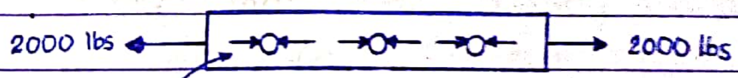
which resists the

when applied results to

Internal Stresses

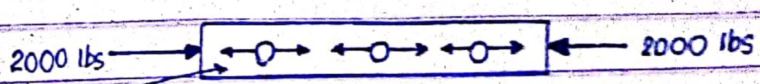
- Five types of loads that can act on a structure :

1. Tension
   - two pulling (opposing) forces that stretch an object trying to pull it apart.



inside the molecules are pulling back trying to stay together and keep from being ripped apart

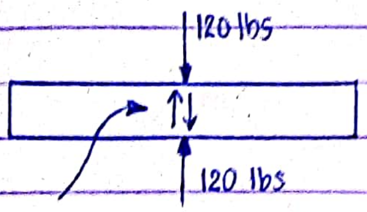
2. Compression
   - two pushing (opposing) forces that squeeze an object trying to compress it



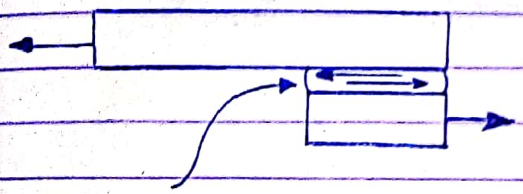
inside the molecules are pushing back trying to stay apart and not get crushed

3. Shear

- two pushing or pulling adjacent forces, acting close together but not directly opposing each other.
- A shearing load cuts or rips an object by sliding its molecules apart sideways.

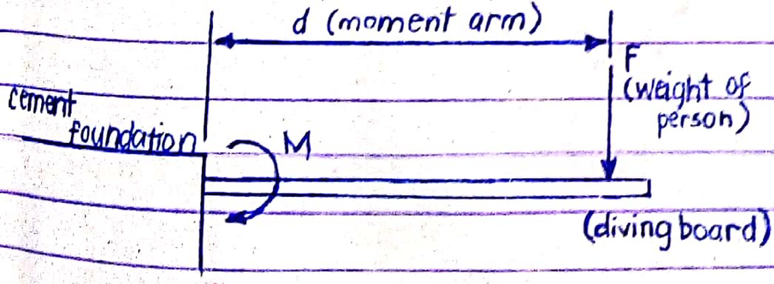


inside the molecules hold onto each other to resist being slid apart



inside the glue joint, the molecules are trying to hold onto one another to resist being ripped apart

- A moment is a "turning force" caused by a force acting on an object at some distance from a fixed point.

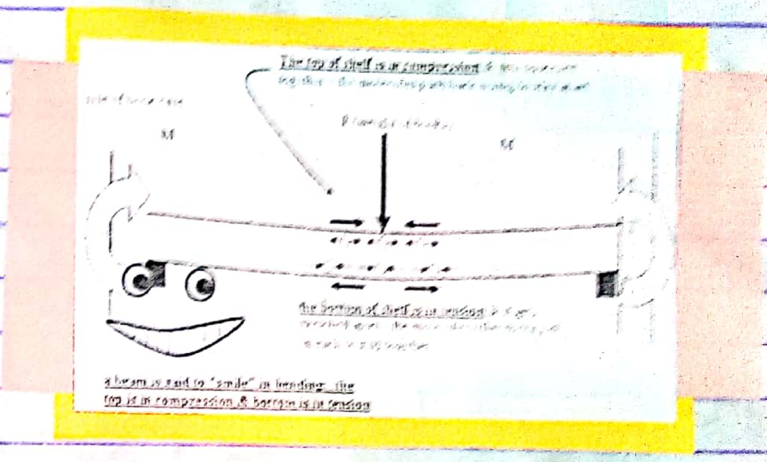


$$M = F \times d$$

4. Bending

- When a moment or "turning force" is applied to a structural member that is fixed on both ends, such as a pole beam, making it deflect or bend
- A moment that causes bending is called a bending moment
- produces tension and compression inside a beam or a pole, causing it to "smile."

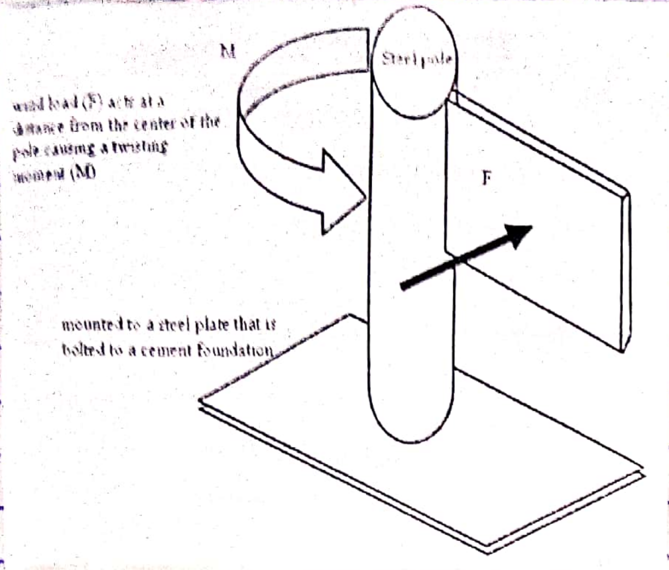
Flexural Stress: $f_b = \frac{Mc}{I_{NA}}$



5. Torsion (Twisting)

- created when a moment or "turning force" is applied to a structural member (or piece of material) making it deflect at an angle (twist).
- moment that causes twisting is called a twisting or torsional moment
- produces shear stresses inside the material.
- A beam in torsion will fail in shear; the twisting action causes the molecules to be slid apart sideways.

TORSION (TWISTING):

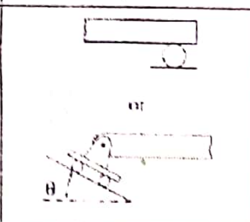
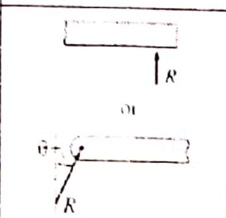
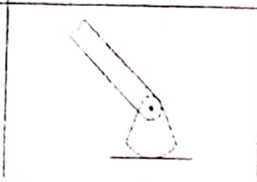
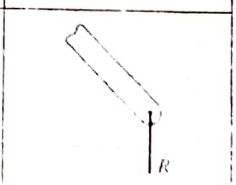


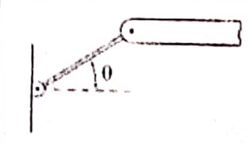
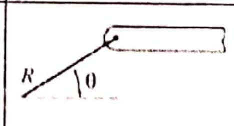
Static Determinacy, Indeterminacy, and Instability

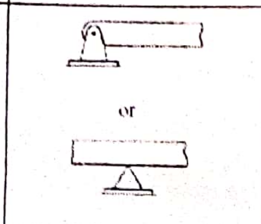
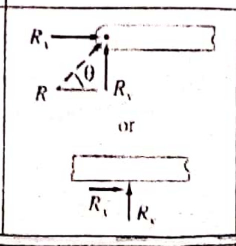
Internal Stability

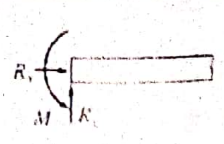
- A structure is internally stable, or rigid, if it maintains its shape and remains a rigid body when detached from the supports.

Types of Supports for Plane Structures

Category	Type of support	Symbolic representation	Reactions	Number of unknowns
I	Roller			1 The reaction force R acts perpendicular to the supporting surface and may be directed either into or away from the structure. The magnitude of R is the unknown.
	Rocker			

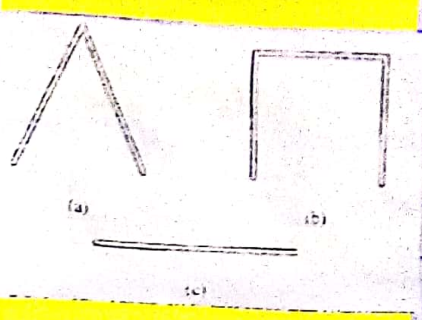
Category	Type of support	Symbolic representation	Reactions	Number of unknowns
	Link			1 The reaction force R acts in the direction of the link and may be directed either into or away from the structure. The magnitude of R is the unknown.

Category	Type of support	Symbolic representation	Reactions	Number of unknowns
II	Hinge			2 The reaction force R may act in any direction. It is usually convenient to represent R by its rectangular components, R_x and R_y. The magnitudes of R_x and R_y are the two unknowns

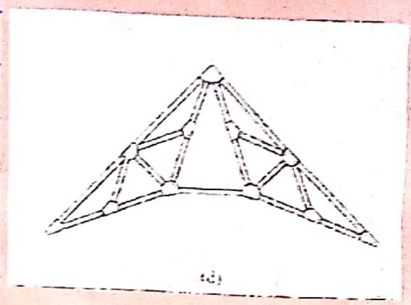
Category	Type of support	Symbolic representation	Reactions	Number of unknowns
III	Fixed			3 The reactions consist of two force components R_x and R_y and a couple of moment M. The magnitudes of R_x, R_y, and M are the three unknowns.

- These supports are grouped into three categories, depending on the number of reactions (1, 2, or 3) they exert on the structures.

- Conversely, a structure is termed internally unstable (or nonrigid) if it cannot maintain its shape and may undergo large displacements under small disturbances when not supported externally.



- It should be realized that all physical bodies deform when subjected to loads; the deformations in most engineering structures under service conditions are so small that their effect on the equilibrium state of structure can be neglected.



- rigid structure
  - the structure offers significant resistance to its change of shape

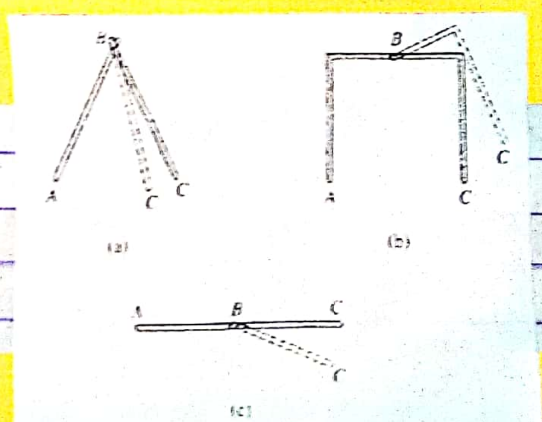
- nonrigid structure
  - offers negligible resistance to its change of shape when detached from the supports and would often collapse under its own weight when not supported externally.

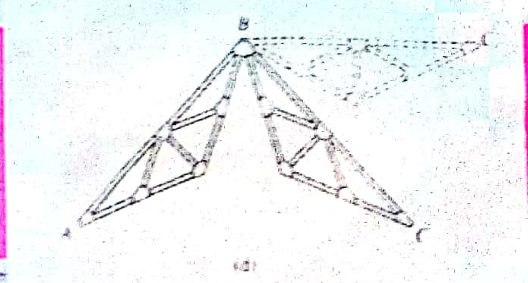
Internally Stable Structures

- Each of the structures shown forms a rigid body, and each can maintain its shape under loads.

Internally Unstable Structures

- Each structure is composed of two rigid parts, AB and BC, connected by a hinged joint B, which cannot prevent the rotation of one part with respect to the other.





Static Determinacy of Internally Stable Structures

- An internally stable structure is statically determinate externally if all its support reactions can be determined by solving the equations of equilibrium.

- If a structure is supported by more than three reactions, then all the reactions cannot be determined from the three equations of equilibrium.

- Such structures are termed statically indeterminate externally.

- The reactions in excess of those necessary for equilibrium are called external redundant, and the number of external redundant is referred to as the degree of external indeterminacy.

- The degree of external indeterminacy can be written as

External Indeterminacy = # of reaction − 3

$$i_e = r - 3$$

Conditions:

$r < 3$ the structure is statically unstable externally

$r = 3$ the structure is statically determinate externally

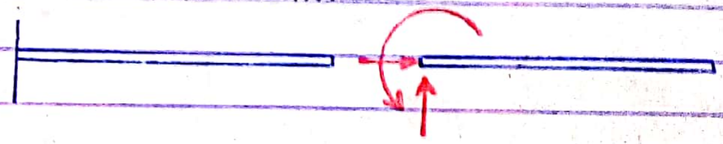
$r > 3$ the structure is statically indeterminate externally

- If a structure is supported by fewer than three support reactions, the reactions are not enough to prevent all possible movements of the structure in its plane.

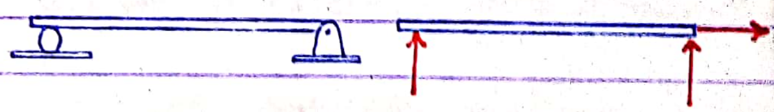
- Such a structure cannot remain in equilibrium under a general system of loads and is, therefore, referred to as statically unstable externally.

Externally Statically Determinate Plane Structures

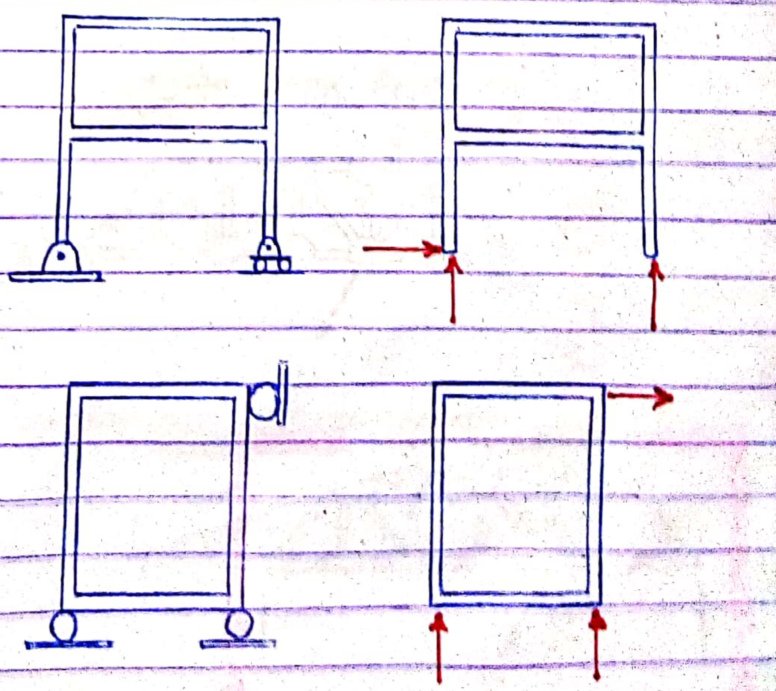
- Cantilever Beam



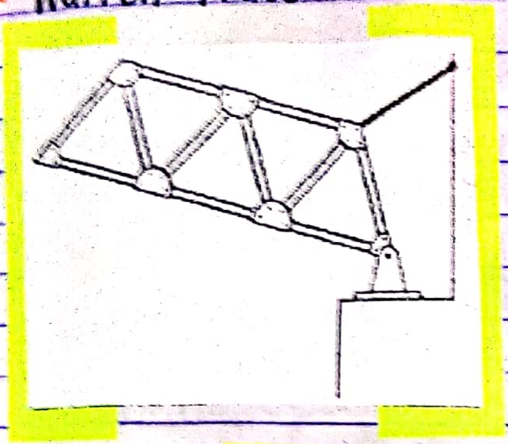
- Simple Beam

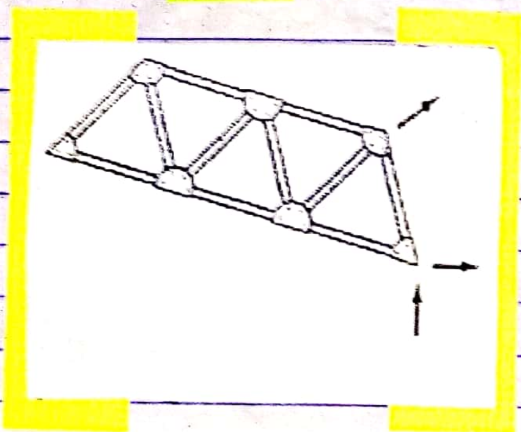


- Frame



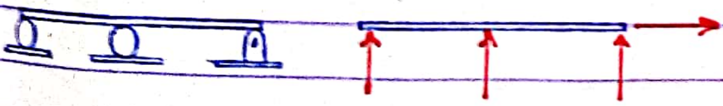
- Warren Truss





Externally Statically Indeterminate Plane Structures

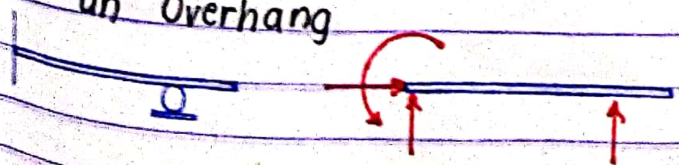
1. Indeterminate Simple Beam



$i_e = r - 3 = 4 - 3 = 1$

$\therefore$ Degree of External Indeterminacy 1.

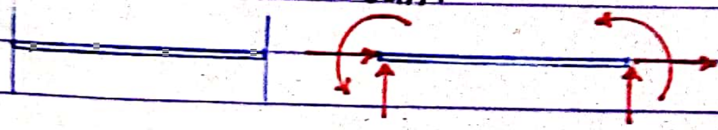
2. Indeterminate Propped Beam with an Overhang



$i_e = r - 3 = 4 - 3 = 1$

$\therefore$ Degree of External Indeterminacy 1

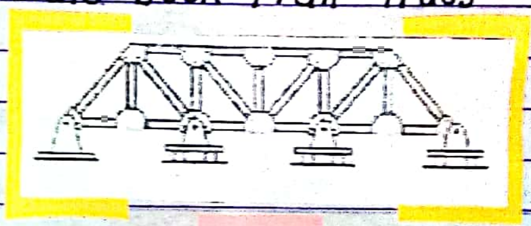
3. Indeterminate Fully Restrained / Fixed Beam

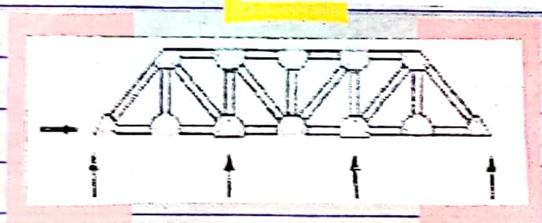


$i_e = r - 3 = 6 - 3 = 3$

$\therefore$ Degree of External Indeterminacy 3

4. Indeterminate Deck Pratt Truss



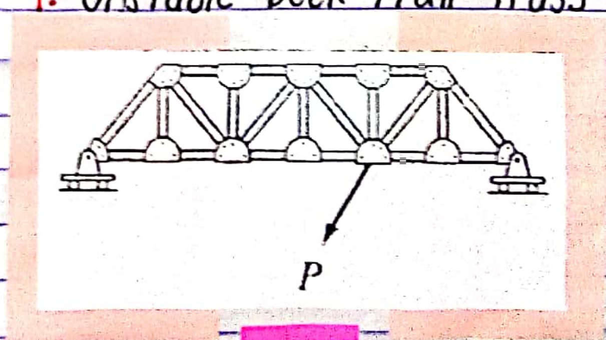


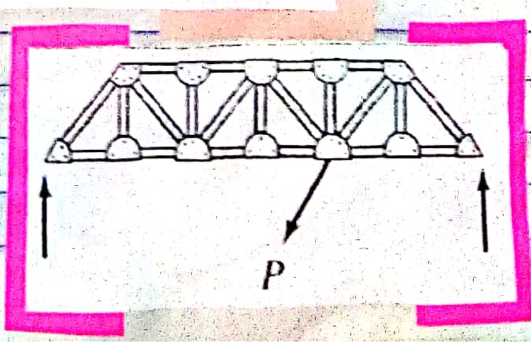
$i_e = r - 3 = 5 - 3 = 2$

$\therefore$ Degree of External Indeterminacy 2

Externally Statically Unstable Plane Structure

1. Unstable Deck Pratt Truss

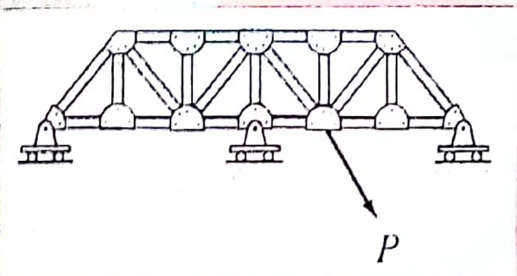


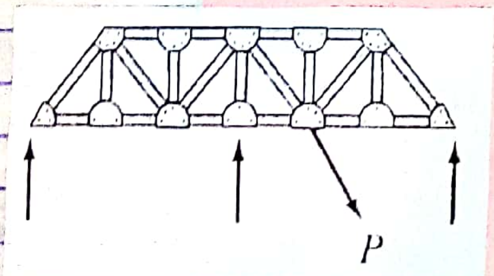


$i_e = r - 3 = 2 - 3 = -1$

- A structure may be supported by an enough reactions $r \geq 3$ but may still be unstable due to improper arrangement of supports
- Such structures are referred to as geometrically unstable externally.

Reaction Arrangements causing External Geometric Instability in Plane Structures



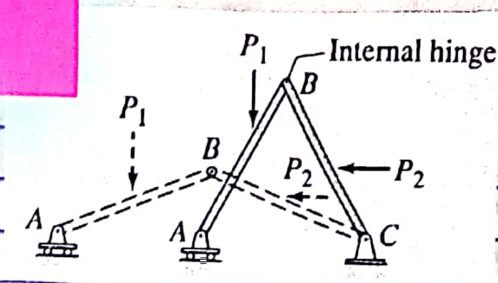


$i_e = r - 3 = 3 - 3 = 0$

∴ Determinate but geometrically unstable because of the supports. It only uses rollers which will result to swaying.

Static Determinacy of Internally Unstable Structures - Equations of Condition

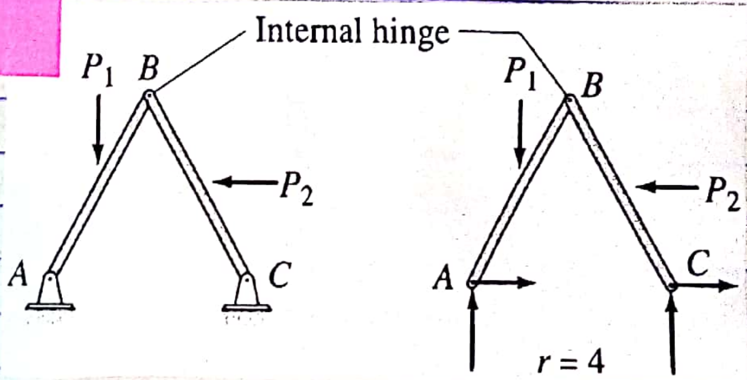
- Example of internally unstable structure are shown next.



$i_e = r - 3 = 4 - 3 = 1$

∴ Indeterminate and Internally Unstable Structure

∴ However, solvable due to hinge at B.



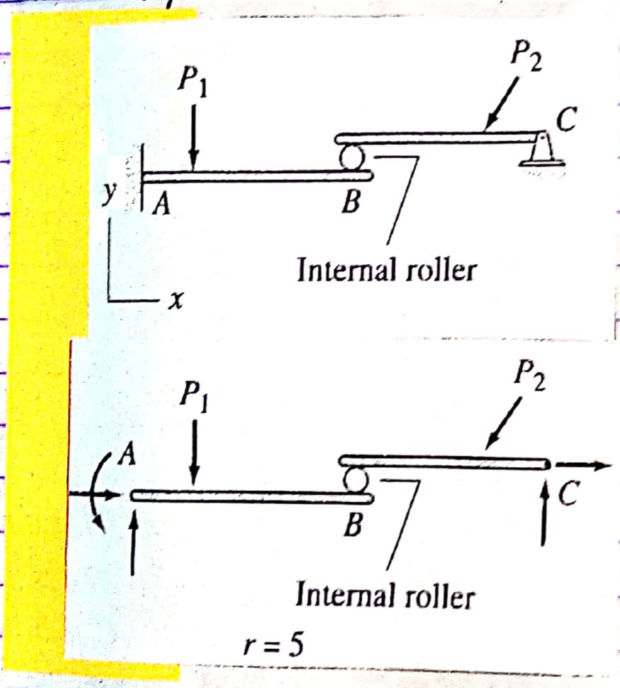
One equation of condition:

$$\Sigma M_B^{AB} = 0 \quad \text{or} \quad \Sigma M_B^{BC} = 0$$

- Occasionally, connections are used in structures that permit not only relative rotations of the member ends but also relative translations in certain directions of the ends of the connected members.

- Such connections are modeled as internal roller joints for the purposes of analysis.

- Indeterminate structure at first glance but statically determinate externally



Two equations of condition:

$\sum F_x^{AB} = 0$ or $\sum F_x^{BC} = 0$

$\sum M_B^{AB} = 0$ or $\sum M_B^{BC} = 0$

- The degree of external indeterminacy can be written as: (with internal hinge / roller)

$$i_e = r - 3 - e_c$$

where: e_c = equation of condition

- internal hinge: $e_c = 1$
- internal roller: $e_c = 2$

Conditions:

$r < 3 + e_c$ the structure is statically unstable externally

$r = 3 + e_c$ the structure is statically determinate externally

$r > 3 + e_c$ the structure is statically indeterminate externally

Alternative Approach:

1. Count the total number of support reactions, r.

2. Count the total number of internal forces, f_i, that can be transmitted through the internal hinges and the internal rollers of the structure.

* Recall that an internal hinge can transmit two force components, and an internal roller can transmit one force component.

3. Determine the total number of unknowns

$$r + f_i$$

4. Count the number of rigid members or portions, n_r, contained in the structure.

5. Because each of the individual rigid portions or members of the structure must be in equilibrium under the action of applied loads, reactions, and/or internal forces, each member must satisfy the three equations of equilibrium. ($\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$)

Thus, the total number of equations available for the entire structure is $3n_r$.

6. Determine whether the structure is statically unstable, determinate, or indeterminate by comparing the total number of unknowns, $r + f_i$, to the total number of equations.

For indeterminate structures, the degree of external indeterminacy is given by: (with internal hinge/roller)

$$i_e = (r + f_i) - 3n_r$$

Conditions:

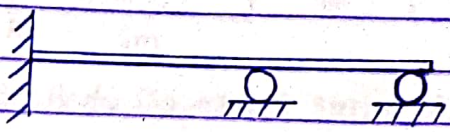
$r + f_i < 3n_r$ the structure is statically unstable externally

$r + f_i = 3n_r$ the structure is statically determinate externally

$r + f_i > 3n_r$ the structure is statically indeterminate externally

Sample Problems

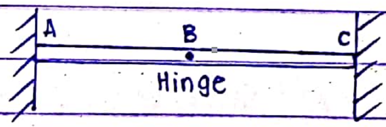
1. Direction: Classify each of the structures shown as externally unstable, statically determinate, or statically indeterminate. If the structure is statically indeterminate externally, then determine the degree of external indeterminacy.

a. 

The beam is internally stable.

$i_e = r - 3 = 5 - 3 = 2$

∴ The beam is statically indeterminate externally with the degree of external indeterminacy of 2.

b. 

The beam is internally unstable.

➤ It is composed of two rigid members AB and BC connected by an internal hinge at B.

$i_e = r - (3 + e_c) = 6 - (3 + 1) = 2$

∴ The beam is statically indeterminate externally with the degree of external indeterminacy of 2.

Using Alternate Method:

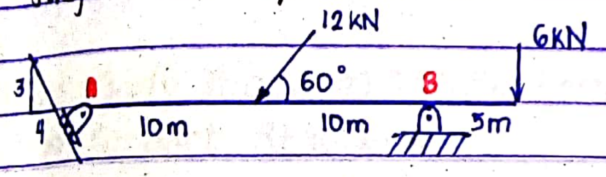
$f_i = 2, \; n_r = 2, \; r = 6$

$i_e = (r + f_i) - 3n_r = (6 + 2) - 3(2)$

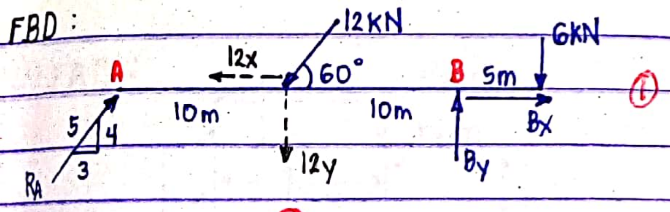
$i_e = 2$

As:
$r + f_i > 3n_r$ the structure is statically indeterminate externally
$\therefore$ The beam is statically indeterminate externally with the degree of external indeterminacy of 2.

2. Determinate the reactions of plane statically determinate structures subjected to coplanar loads as shown.

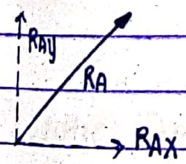


FBD :



Determinacy: 2
$i_e = r - 3 = 3 - 3 = 0$

Support Reactions : 3



$\frac{R_{Ay}}{R_A} = \frac{4}{5}$; $\frac{R_{Ax}}{R_A} = \frac{3}{5}$

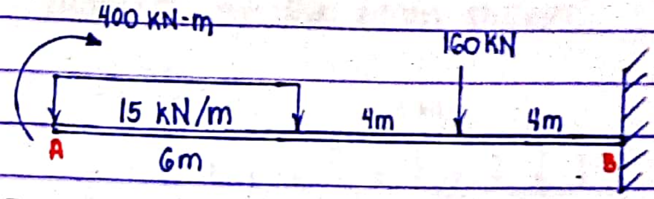
$R_{Ay} = \frac{4R_A}{5}$ $\quad R_{Ax} = \frac{3R_A}{5}$

$\Sigma M_B = 0$
$R_{Ay}(20) - 12\sin 60^\circ(10) + 6(5) = 0$
$R_{Ay} = 4/5\, R_A$
$R_A = 4.62$ kN

$\Sigma F_x = 0 = R_{Ax} + R_{x12} + B_x$
$3/5\, R_A - 12\cos 60 + B_x = 0$
$B_x = 3.23$ kN

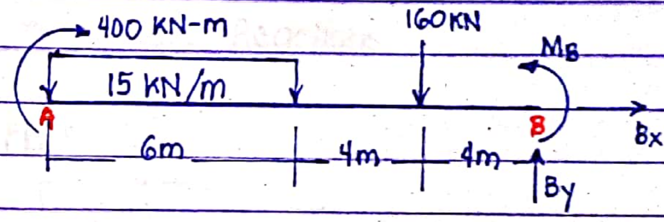
$\Sigma F_y = 0 = R_{Ay} + R_{12y} + B_y + 6$
$4/5\, R_A - 12\sin 60 + B_y - 6 = 0$
$B_y = 12.70$ KN

3. Determine the reactions at the supports for the beam shown.



Free-Body Diagram > Static Determinacy
>> Support Reactions

1 FBD :



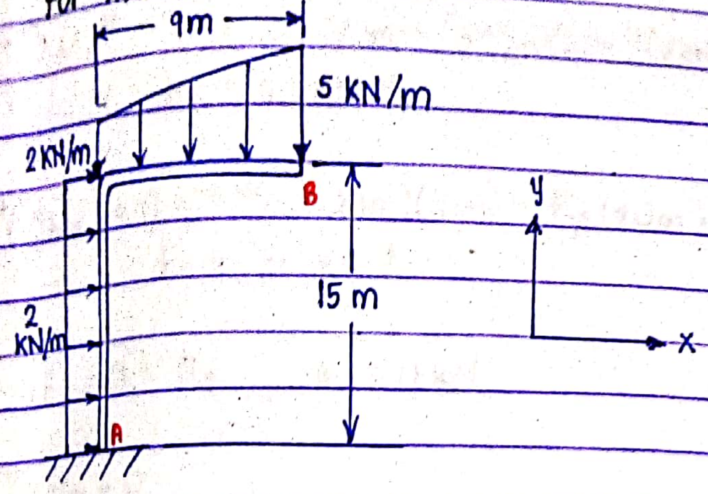
2 Static Determinacy:
$i_e = r - 3 = 3 - 3 = 0$.
$\therefore$ determinate externally

3 Support Reactions

$\Sigma F_y = 0 = -15(6) - 160 + B_y$
$B_y = 250$ kN

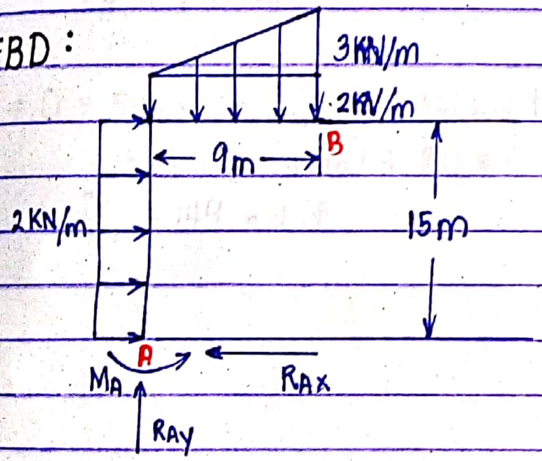
$\Sigma M_B = 0$
$400 = 15(6)(3+8) + 160(4) + M_B$
$M_B = -1230$ kN-m
$M_B = 1230$ kN-m

4. Determine the reactions at the support for the frame shown.



Free-Body Diagram > Static Determinacy
>> Support Reactions

① FBD:



② Static Determinacy:

$i_e = r - 3 = 3 - 3 = 0$

∴ determinate externally

③ Support Reactions

$$\Sigma F_x = 0 = (2)(15m) - A_x$$

$$A_x = 30 \text{ KN} \leftarrow$$

$$\Sigma F_y = 0 = A_y - 2(9m) - \tfrac{1}{2}\left(3\tfrac{KN}{m}\right)(9m)$$

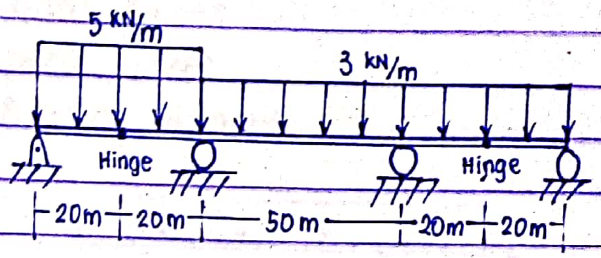
$$A_y = 31.5 \text{ KN} \uparrow$$

$$\Sigma M_A = 0$$

$$-M_A + (2\ KN/m)(15m)(7.5m) + \left(2\tfrac{KN}{m}\right)(9m)(4.5m) + \tfrac{1}{2}(3\ KN/m)(9m)\left(\tfrac{2}{3}\cdot 9m\right) = 0$$

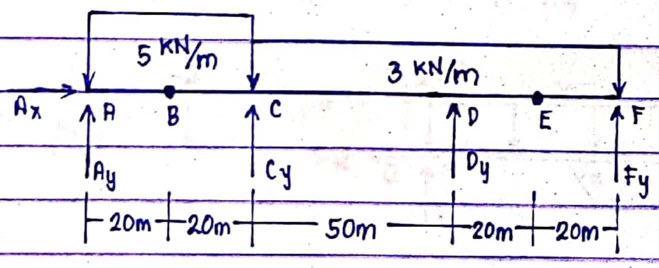
$$M_A = 387 \text{ KN-m} \circlearrowleft$$

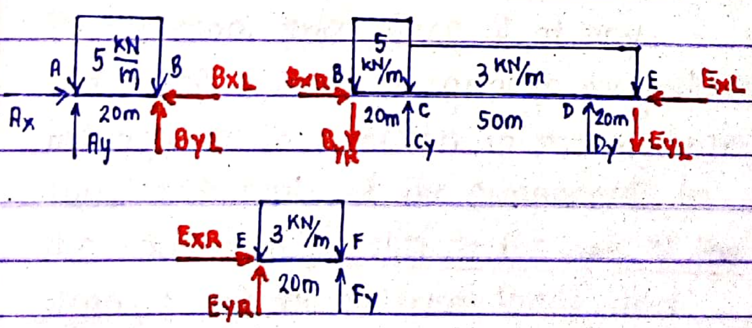
5. Determine the reactions at the supports for the beam shown.



Free-Body Diagram > Static Determinacy
>> Support Reactions

① FBD:





② Static Determinacy

$i_e = r - 3 - e_c = 5 - 3 - 2 = 0$

∴ determinate externally

③ Support Reactions

$$+\circlearrowleft \Sigma M_B^{AB} = 0 = A_y(20m) + (5\,kN/m)(20m)(10m)$$
$$A_y = 50\,kN \uparrow$$

$$+\circlearrowleft \Sigma M_E^{EF} = 0 = (3\,kN/m)(20m)(10m) - F_y(20m)$$
$$F_y = 30\,kN \uparrow$$

$$\Sigma F_x = 0 = A_x \quad ; \quad A_x = 0\,kN$$

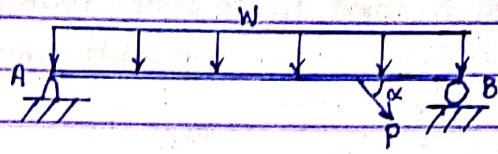
$$+\circlearrowleft \Sigma M_D = 0$$
$$50kN(90m) - 5\,kN/m\,(40m)(70m) + C_y(50m)$$
$$- 3\,kN/m\,(90m)(5m) - 30\,kN(40m) = 0$$
$$C_y = 241\,kN \uparrow$$

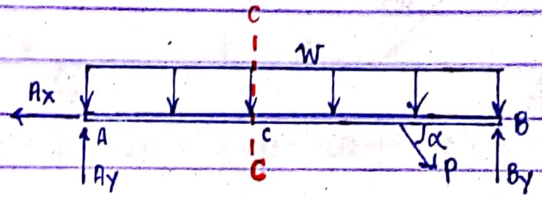
$$\Sigma F_y = 0 = 50kN - 5\tfrac{kN}{m}(40m) + 241\,kN + D_y$$
$$- 3\,kN/m\,(90m) + 30\,kN$$
$$D_y = 149\,kN \uparrow$$

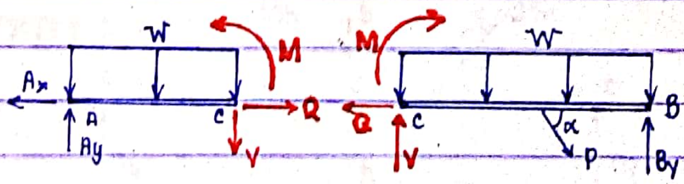
Beams and Frames: Shear and Bending Moment

Axial Force, Shear, and Bending Moment

Three Types of Stress Resultants:
1. Axial forces
2. shear forces
3. Bending Moments





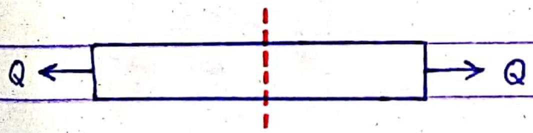


- The internal axial force Q at any section of a beam is equal in magnitude but opposite in direction to the algebraic sum (resultant) of the components in the direction parallel to the axis of the beam of all the external loads and support reactions acting on either side of the section under consideration

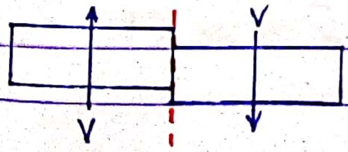
- The shear V at any section of a beam is equal in magnitude but opposite in direction to the algebraic sum (resultant) of the components in the direction perpendicular to the axis of the beam of all the external loads and support reactions acting on either side of the section under consideration.

- The bending moment M at any section of a beam is equal in magnitude but opposite in direction to the algebraic sum of the moments about (the centroid of the cross section of the beam at) the section under consideration of all the external loads and support reactions acting on either side of the section.

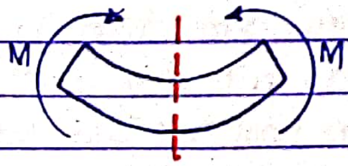
- The internal axial force Q is positive when the external forces acting on the member produce tension or have the tendency to pull the member apart at the section.



- The shear V is positive when the external forces tend to push the portion of the member on the left of the section upward with respect to the portion on the right of the section.

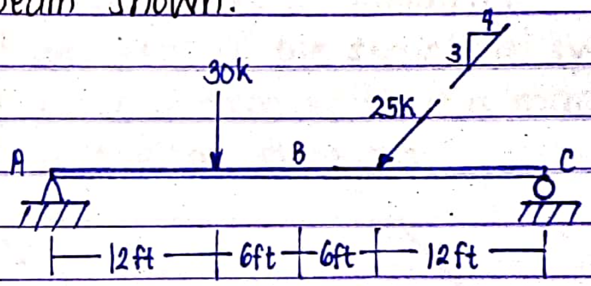


- The bending moment M is positive when the external forces and couples end to bend the beam concave upward, causing compression in the upper fibers and tension in the lower fibers of the beam at the section.

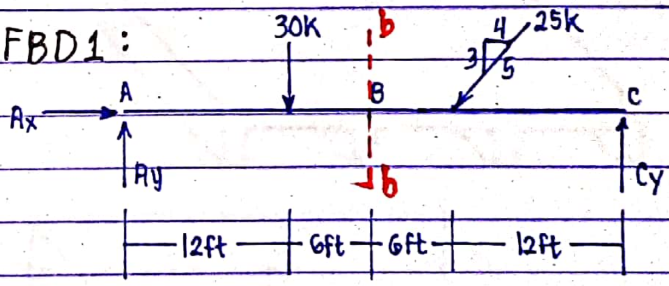


Sample Problems

6. Determine the axial force, shear, and bending moment at point B of the beam shown.



FBD1 :



$$\Sigma F_x = 0 = A_x - (4/5)(25K)$$

$$A_x = 20\ K \rightarrow$$

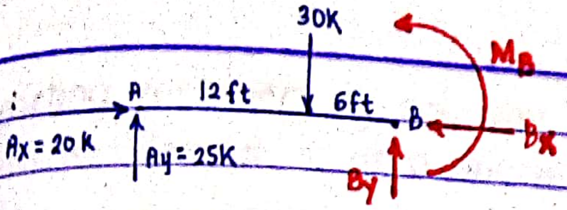
$$\overset{+}{\circlearrowleft} \Sigma M_c = 0 = A_y(36\,ft) - 30k\,(24\,ft) - 3/5\,(25k)(12\,ft)$$

$$A_y = 25\ K \uparrow$$

$$\Sigma F_y = 0 = 25k - 30K - (3/5)(25k) + C_y$$

$$C_y = 20\,k \uparrow$$

FBD 2:



$\Sigma F_x = 0 = 20K - B_x$; $B_x = 20K \leftarrow$

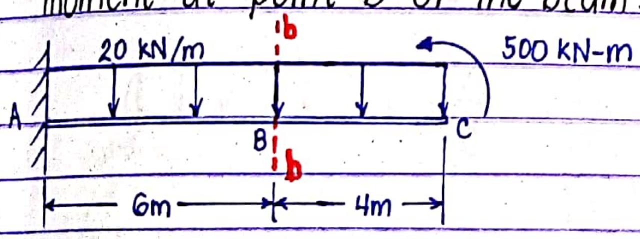
$\Sigma F_y = 0$; $B_y + 25K = 30K$

$B_y = 5K \uparrow$

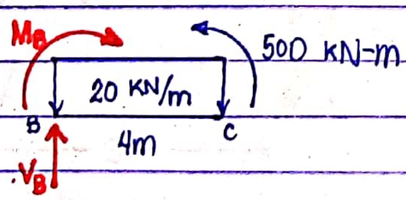
$\Sigma M_B = 0 = 25K(18ft) - 30K(6ft) - M_B$

$M_B = 270$ K-ft

7. Determine the shear and bending moment at point B of the beam shown.



FBD:



Solution: $\Sigma F_V = 0 = V_B - 20^{kN}/_m\,(4m)$

$V_B = 80\ kN \uparrow$

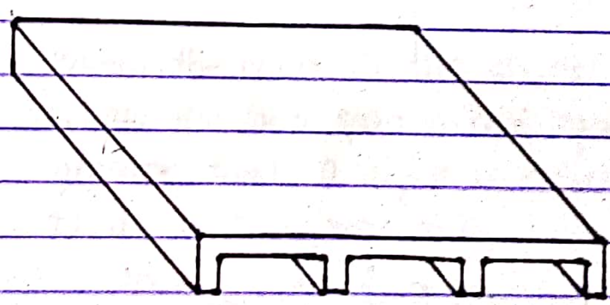
$\Sigma M_B = 0 = -500\ kN\cdot m + (20^{kN}/_m)(4m)(2m) + M_B$

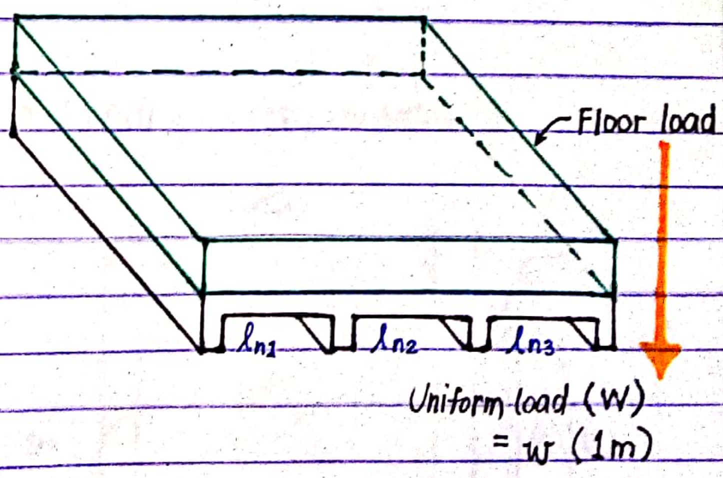
$M_B = 340\ kN\text{-}m$

Tributary Loadings

- When flat surfaces such as walls, floors, or roofs are supported by a structural frame, it is necessary to determine how the load on these surfaces is transmitted to the various structural elements used for their support.

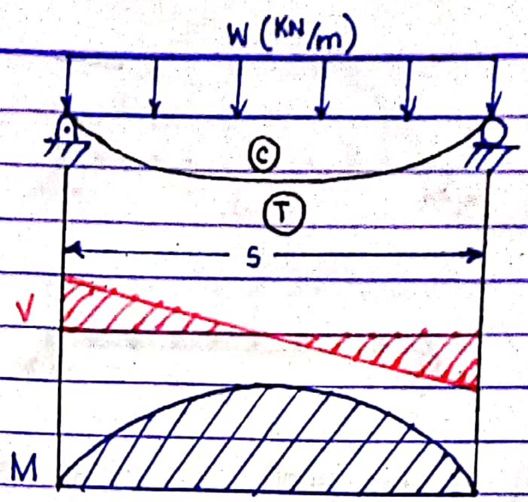
- There are generally two ways in which this can be done, including:
  - geometry of the structural system
  - material from which it is made
  - construction method/s





- One-Way System
  - A slab or deck that is supported such that it delivers its load to the

supporting members by one-way action is referred to as a one-way slab.



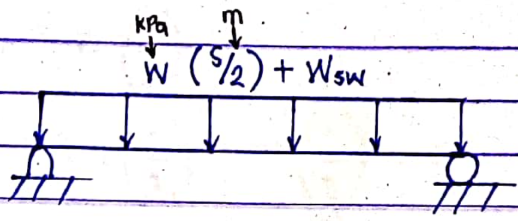
- The ratio of the shorter span versus the long span is less than 0.50.

$m = \frac{s}{l}$; $\frac{s}{l} < 0.5$

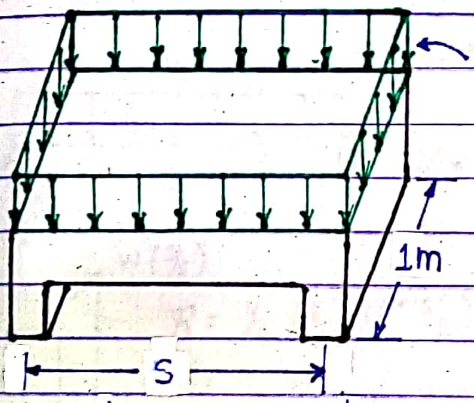
where :

s = shorter span

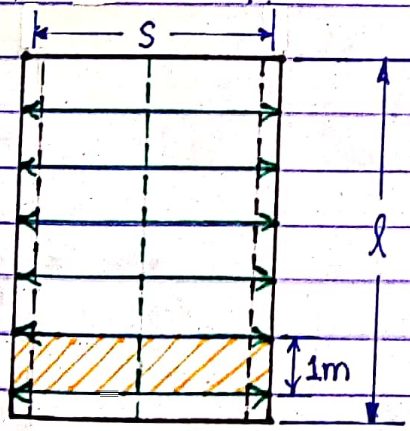
l = longer span

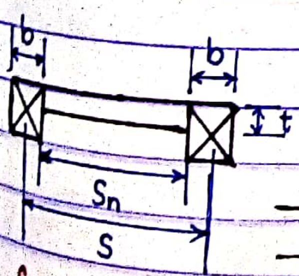


- short directional bending

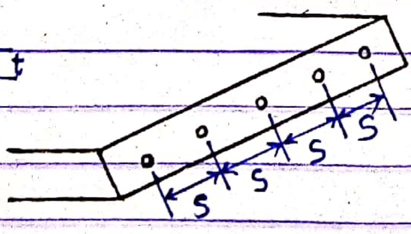


$\frac{KN}{m^2} \cdot (1m) = \frac{KN}{m}$





$S_n = S - b$

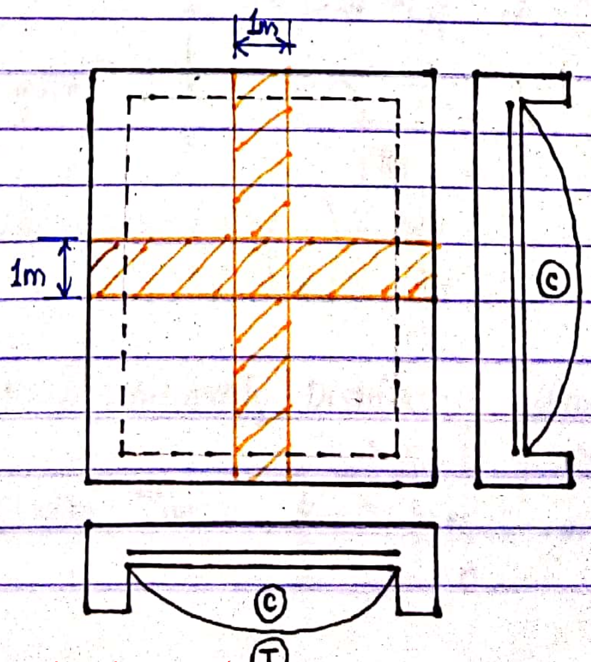


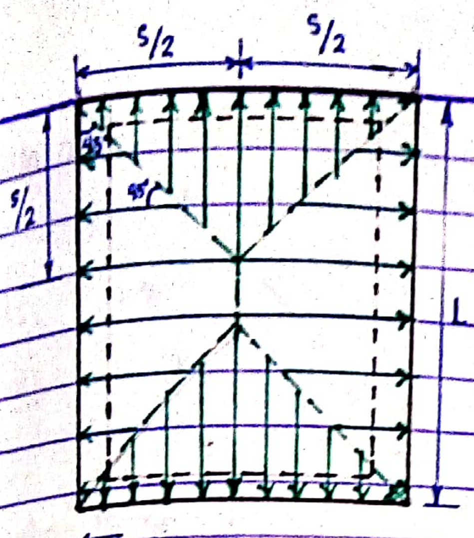
- Two - Way System

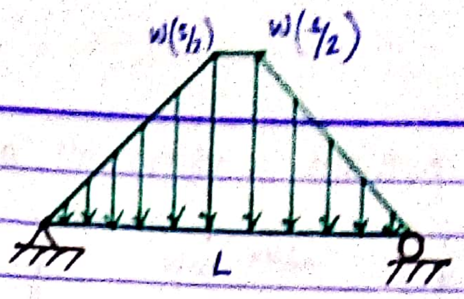
- When the ratio of the shorter span versus the long span is now equal or greater than 0.50, it is referred to as a two - way slab.

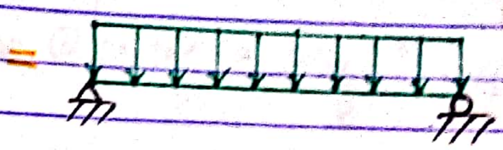
$m = \frac{s}{l}$; $\frac{s}{l} \geq 0.5$

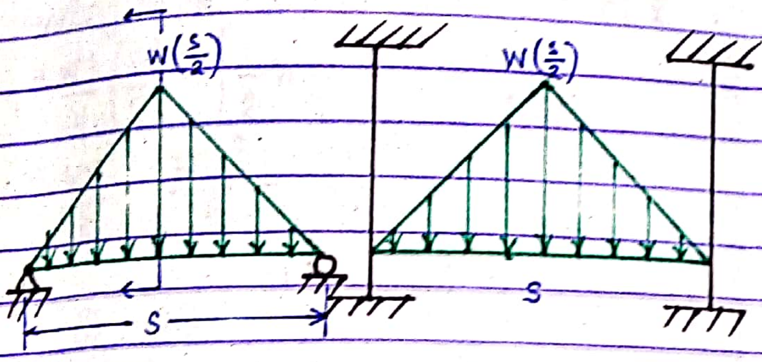
- bending both directions

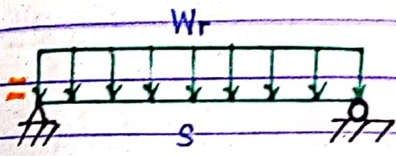






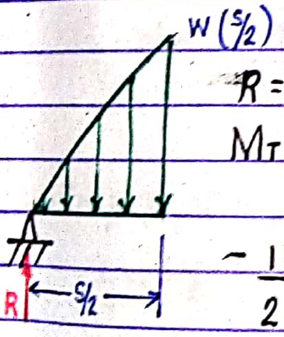






$$W_r = \frac{ws}{3}$$

$$M_R = \frac{ws^2}{8}$$



$$R = \tfrac{1}{2}(s/2)(ws/2) = ws^2/8$$

$$M_T = \frac{ws^2}{8}\left(\frac{s}{2}\right) - \frac{1}{2}\left(\frac{s}{2}\right)\left(\frac{ws}{2}\right)\left(\frac{1}{3}\right)\left(\frac{s}{2}\right) = \frac{ws^3}{24}$$

$$M_T = M_R$$

$$\frac{W_T s^3}{24} = \frac{W_R s^2}{8}$$

$$W_R = \frac{W_T S}{3}$$

- Let w be the floor load or pressure in kPa. Thus

$$W_T = w\left(\frac{s}{2}\right)$$

$$W_r = W_{tri}\left(\frac{s}{3}\right) = w\left(\frac{s}{3}\right)$$

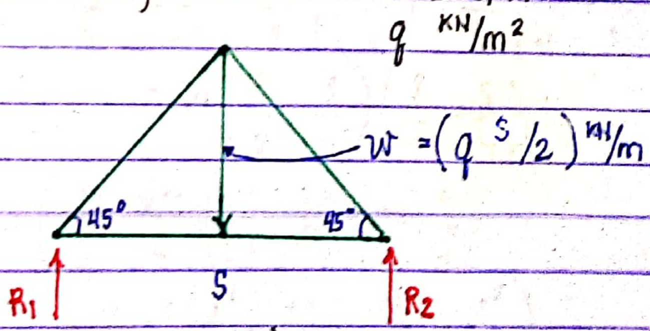
$$W_r = \frac{W_{tra} \cdot s}{3}\left(\frac{3-m^2}{2}\right) = \frac{ws}{3}\left(\frac{3-m^2}{2}\right)$$

* This is to convert from kPa to kN/m.

$$W_R = \frac{ws}{3}\left(\frac{3-m^2}{2}\right)$$

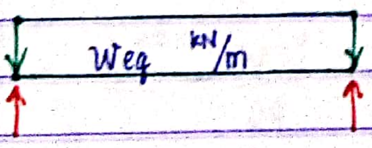
Two-Way Slab Load Derivation:

1. Triangular Load on Beam



$$R_1 = R_2 = w\left(\frac{s}{2}\right)\left(\frac{1}{2}\right) = \frac{ws}{4}$$

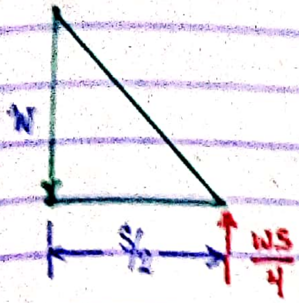
* In a Uniformly Distributed Load



B.M @ center

$$= \frac{W_{eq}\, s^2}{8} \rightarrow (1)$$

* In a Triangular Load



Bending Moment @ center = $FL - FL\bar{x}$

$= \frac{ws}{4}\left(\frac{s}{2}\right) - \left[\frac{1}{2}w\left(\frac{s}{2}\right)\right]\left[\frac{1}{3}\left(\frac{s}{2}\right)\right]$

$= \frac{ws^2}{8} - \frac{ws^2}{24} = \frac{ws^2}{12} \rightarrow (2)$

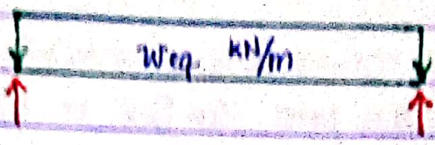
* (1) = (2)

$\frac{W_{eq}\, s^2}{8} = \frac{w s^2}{12}$

$W_{eq} = \frac{8}{12} w = \frac{8}{12}\left(q \frac{s}{2}\right)$

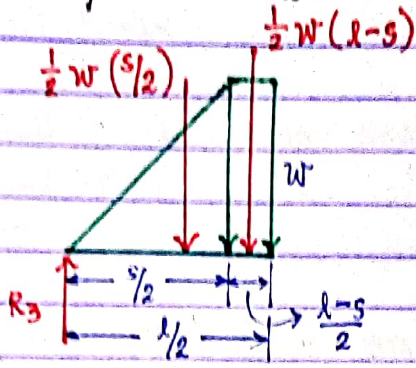
$W_{eq} = \frac{qs}{3}$

* In a Uniformly Distributed Load



BM @ center $= \frac{W_{eq}\, s^2}{8} \rightarrow (1)$

* In a Trapezoidal Load



B.M. @ center $= FL - F_1 L_1 \bar{x}_1 - F_2 L_2 \bar{x}_2$

$= \left[\frac{ws}{4} + \frac{w(l-s)}{2}\right]\left(\frac{l}{2}\right) - w\left(\frac{l-s}{2}\right)\left(\frac{l-s}{4}\right)$

$\quad - \frac{ws}{4}\left[\left(\frac{1}{3}\cdot\frac{s}{2}\right) + \frac{l-s}{2}\right]$

$= \left[\cancel{\frac{ws}{4}}^{\,0} + \frac{wl}{2} - \cancel{\frac{ws}{2}}^{\,-\frac{ws}{4}}\right]\left(\frac{l}{2}\right) - \frac{ws}{4}\left[\cancel{\frac{s}{6}}^{\,0} - \frac{l}{2} - \frac{s}{2}^{\,-s/3}\right]$

$\quad - w\left(\frac{l}{2} - \frac{s}{2}\right)\left(\frac{l}{4} - \frac{s}{4}\right)$

$= \left(\frac{wl}{2} - \frac{ws}{4}\right)\left(\frac{l}{2}\right) - \frac{ws}{4}\left(\frac{l}{2} - \frac{s}{3}\right)$

$\quad - w\left(\frac{l^2}{8} - \frac{sl}{8} - \frac{sl}{8} + \frac{s^2}{8}\right)$

$\frac{wl^2}{8}$

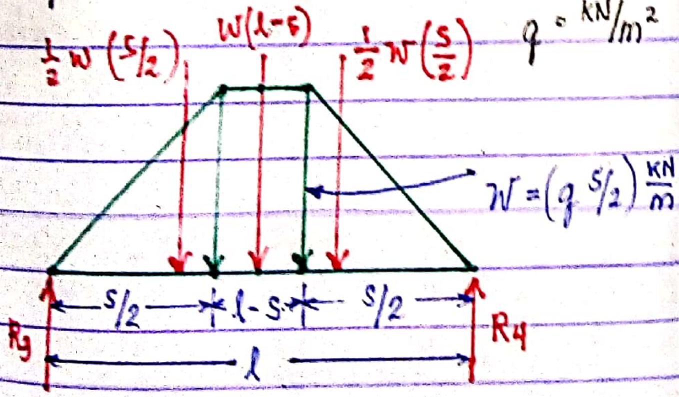
$= \cancel{\frac{wl^2}{4}} - \cancel{\frac{wsl}{8}}^{\,0} - \cancel{\frac{wsl}{8}}^{\,0} + \cancel{\frac{ws^2}{12}}^{\,20} - \cancel{\frac{wl^2}{8}}^{\,0}$

$\quad + \cancel{\frac{wsl}{4}}^{\,0} - \cancel{\frac{ws^2}{8}} - \frac{ws^2}{24}$

$= \frac{wl^2}{8} - \frac{ws^2}{24} = q\frac{s}{2}\left(\frac{l^2}{8} - \frac{s^2}{24}\right)$

2. Trapezoidal Load on Beam

$q = kN/m^2$



$R_3 = R_4 = \frac{ws}{4} + \frac{w(l-s)}{2}$

https://civinnovate.com/civil-engineering-notes/

$$= \frac{qsl^2}{16} - \frac{qs^3}{24} \rightarrow ②$$

① = ②

$$\frac{W_{eq}\,s^2}{8} = q\frac{s}{2}\left(\frac{l^2}{8} - \frac{s^2}{24}\right)$$

$$W_{eq} = q\frac{4}{s}\left(\frac{l^2}{8} - \frac{s^2}{24}\right)$$

$$W_{eq} = q\left(\frac{l^2}{2s} - \frac{s}{6}\right)$$

$$W_{eq} = q\left(1 - \frac{s}{③\,6(l^2/2s)}\right)$$

$$W_{eq} = q\left(1 - \frac{s^2}{3l^2}\right)$$

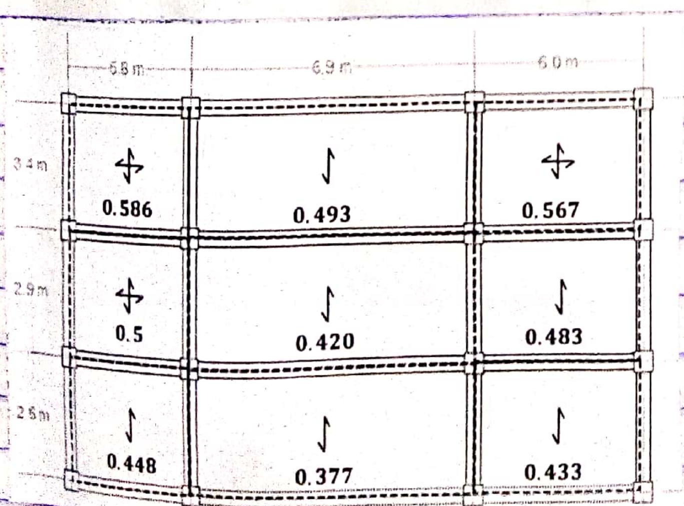
$$W_{eq} = q\left(1 - \frac{s^2}{3l^2}\right)$$

Practice

Determine / Specify if

↕ One-way slab ↔ Two-way slab

Solution : $m = \frac{s}{l}$



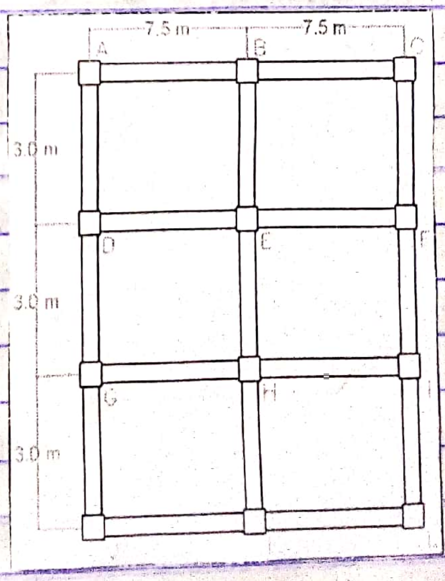
Sample Problem [Board Exam Problem]

8. Reinforced concrete beams having widths of 400 mm and overall depths of 600 mm are spaced 3 meters on centers as shown. These beams support a 100 mm thick slab. The superimposed loads on these beams are as follows :

Dead load (including floor finish, ceiling, etc.) . . . . 3.2 kPa
Live load . . . . 3.6 kPa

The columns E and H are omitted such that the girder BEHK supports the beams DEF at E and GHI at H. Assume EI = constant for all beams. $\gamma_c = 24\ kN/m^3$.

Determine the factored uniform load on beam GHI, in kN/m.

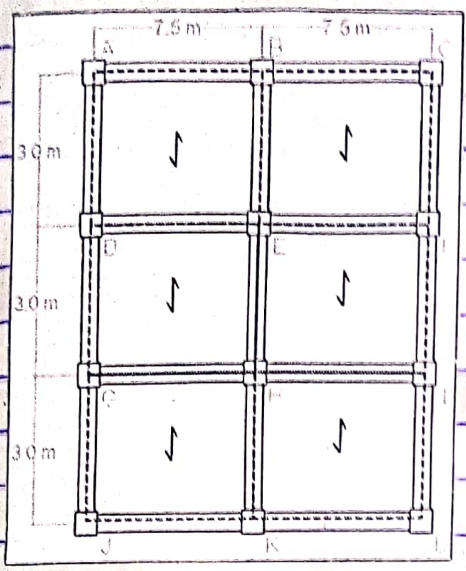


* At initial / original condition

Determine m.

$$m = \frac{s}{l} = \frac{3}{7.5} = 0.4$$

∴ one way slab

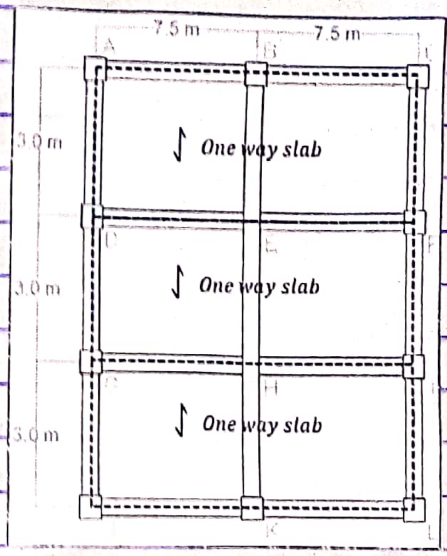


* New floor system.

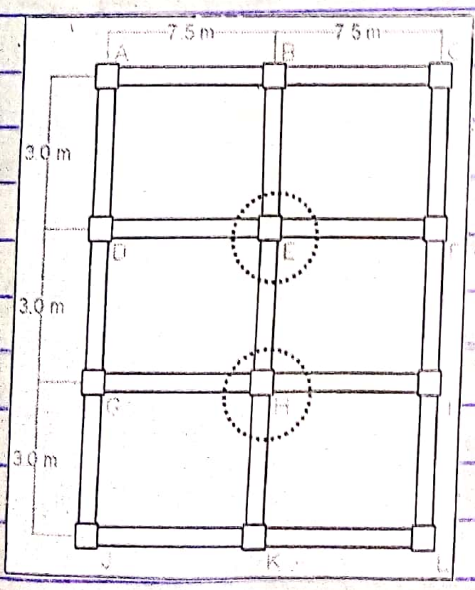
The new load path is then given, determine m.

$$m = \frac{s}{l} = \frac{3}{15} = 0.2$$

∴ one-way slab



* Eliminating columns E and H.



Given: $\gamma_c = 24 \text{ kN/m}^3$

DL = 3.2 kPa (given)

LL = 3.6 kPa (given)

slab thk. = 100 mm = 0.1 m

beam width = 400mm = 0.4 m

beam depth = 600mm = 0.6 m

Factored Load Formulas:

FL = 1.2 DL + 1.6 LL (2010 Edition)

FL = 1.4 DL + 1.7 LL (2001 Edition)

Superimposed → add weight of concrete in dead load

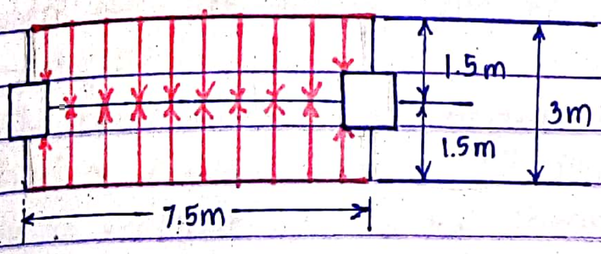
Dead Load:

$W_{slab} = \gamma_c\,(thk) = 24\ kN/m^3\ (0.1\ m)$
$= 2.4\ kN/m^2 = 2.4\ kPa$

$W_{beam} = \gamma_c\,(Area) = 24\ kN/m^3[(0.4m)(0.6m)]$
$= 5.76\ kN/m$

Factored Load : Pressure

$PFL = 1.2\,DL + 1.6\,LL$
$= 1.2\,(3.2\ kPa + 2.4\ kPa) + 1.6\,(3.6\ kPa)$
$= 12.48\ kPa$



Approach 1:

$W_r = 12.48\ kPa\ (1.5\ m + 1.5\ m)$
$= 37.44\ kN/m$

Total Factored Uniform Load:

$W_u = 1.2\,DL + W_r = UFL$
$UFL = 1.2\,(5.76\ kN/m) + 37.44\ kN/m$
$UFL = 44.352\ kN/m$

Approach 2:

$W_{DL} = (DL_{given} + W_{slab})\left[S_1/2 + S_2/2\right]$
$= (3.2\ kPa + 2.4\ kPa)(1.5m + 1.5m)$
$= 16.8\ kN/m$

$W_{LL} = (LL_{given})(S_1/2 + S_2/2)$
$= 3.6\ kPa\ (1.5m + 1.5m)$
$= 10.8\ kN/m$

Factored Load : Uniform Load

$UFL = 1.2\,DL + 1.6\,LL$
$UFL = 1.2(W_{DL} + W_{Beam}) + 1.6\,W_{LL}$
$UFL = 1.2\,(16.8\ kN/m + 5.76\ kN/m) + 1.6\,(10.8\ kN/m)$
$UFL = 44.352\ kN/m$

Review: Shear and Bending Moment Diagrams

Sign Convention

An easy way to remember the sign convention is to isolate a small segment of the member and note that:

- Positive normal force tends to elongate the segment.
- Positive shear force tends to rotate the segment clockwise.
- Positive bending moment tends to bend the segment concave upward.

Procedure of Analysis

1. Compute the reactions.
2. Compute the values of shear (V) at the change of load points using $V_2 = V_1 + A_{load\ diagram}$.
3. Sketch the shear diagram, drawing the correct shape and concavity of the shear diagram.
4. Locate the points of zero shear.
5. Compute the values of the moment

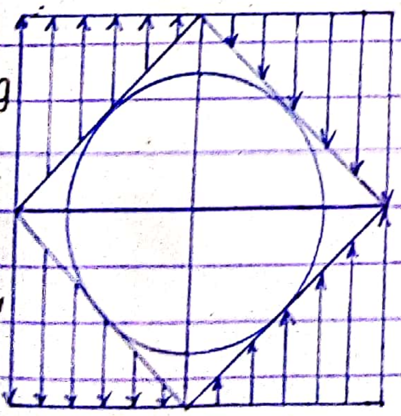
or bending moment (M) at the change of load points of zero shear using

$M_2 = M_1 + A_{shear\ diagram}$.

Properties of Shear Diagram

1. At every change of loading, shear shall be investigated.
2. For concentrated loads or reactions, the left and right portion of the point where they are acting must be investigated.
3. Whenever we have a concentrated load or reaction, there will always be a vertical line in the shear diagram.
4. The shear diagram in one degree higher than the load diagram.
5. Refer concavity to shear concavity diagram.

Shear Concavity Diagram



Properties of Moment Diagram

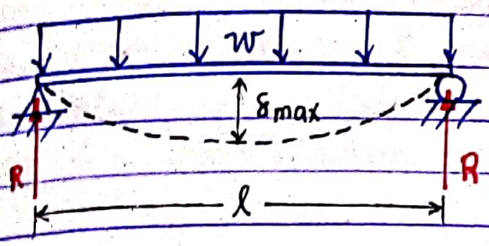
1. For every change in shear diagram, the moment must be investigated.
2. Consider only the moment at any point and not the left and the right portion of the point except when we have a moment load or reaction.
3. Analyze the point where the shear intersects the reference line (V=0) since when shear is zero, the moment is maximum or minimum.
4. Vertical line will only be observed in the moment diagram whenever we have a moment load or reaction.
5. The concavity of the moment diagram depends upon the load. If the load is downward, so is the moment diagram.
6. The moment diagram is one degree higher than the shear diagram.

Degree of Loading in V and M Diagram

Load	Initial Degree	V	M
None	–	0°	1°
Point / Concentrated	–	0°	1°
Uniform	0°	1°	2°
Triangular	1°	2°	3°
Parabolic	2°	3°	4°
n degree	N°	n+1	n+2

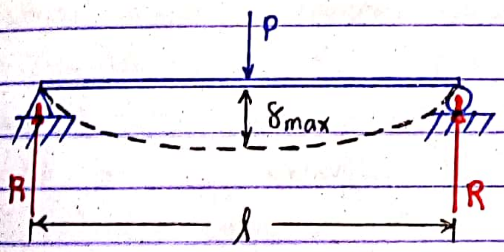
Common Loads on Beams

1. Simple Beam with Uniform Load on Entire Span



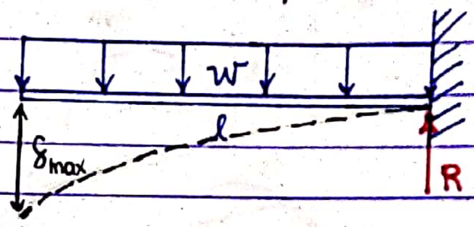
Vmax	Mmax	δmax or Δmax
$\frac{wl}{2}$	$\frac{wl^2}{8}$	$\frac{5wl^4}{384\,EI}$

2. Simple Beam with Point Load at Midspan



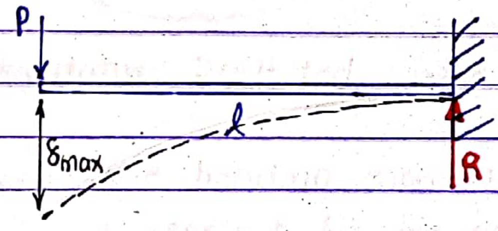
Vmax	Mmax	δmax or Δmax
$\frac{P}{2}$	$\frac{Pl}{4}$	$\frac{Pl^3}{48\,EI}$

3. Cantilever Beam with Uniform Load on Entire Span



Vmax	Mmax	δmax or Δmax
wl	$\frac{wl^2}{2}$	$\frac{wl^4}{8EI}$

4. Cantilever Beam with Point Load at Free End



Vmax	Mmax	δmax or Δmax
P	Pl	$\frac{Pl^3}{3EI}$

Qualitative Deflected Shape

- A **qualitative deflected shape** (elastic curve) of a structure is simply a **rough sketch** (usually exaggerated) **of the neutral surface of structure**, in the deformed position, under the action of a given loading condition.

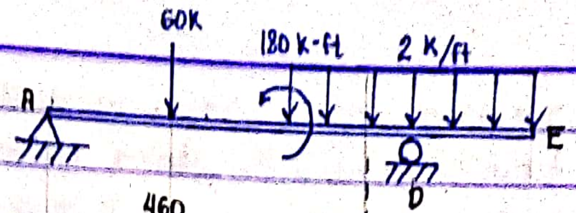
- Such sketches, which can be constructed without any knowledge of the numerical values of deflections, provide valuable insights into the behavior of structures and are often useful in computing the numerical values of deflections.

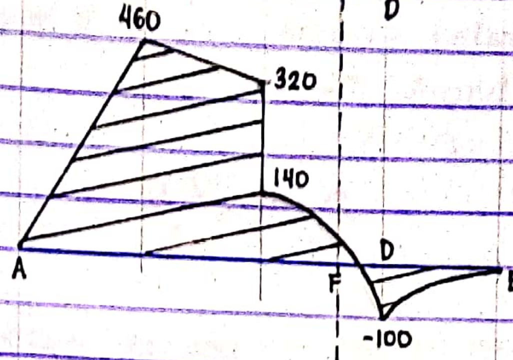
- A **positive bending** moment bends a beam **concave upward** (or toward the positive y direction), whereas a **negative bending** moment bends a beam **concave downward** (or toward the negative y direction).

- Thus, the sign (positive or negative) of the curvature at any point along the axis of a beam can be obtained from the bending moment diagram. Using the signs of curvatures, a qualitative deflected shape (elastic curve) of the beam, which is consistent with its support conditions, can be easily sketched.

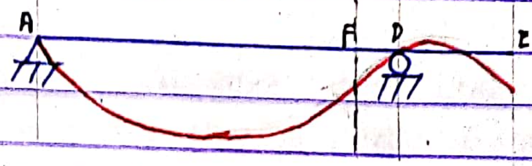
- A qualitative deflected shape of the beam is shown.

a. Beam





b. Bending Moment Diagram (K-ft)



c. Qualitative Deflected Shape

- Because the bending moment is positive in segment AF, the beam is bent concave upward in this region.

- Conversely, the bending moment is negative in segment FE; therefore, in this region, the beam is bent concave downward.

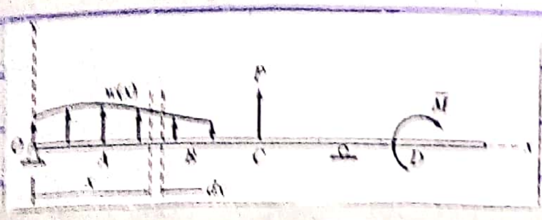
- Regarding the support conditions, note that at both supports A and D, the deflection of the beam is zero, but its slope (rotation) is not zero at these points.

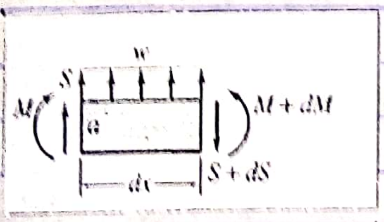
* It is important to realize that a qualitative deflected shape is approximate because it is based solely on the signs of curvatures; the numerical values

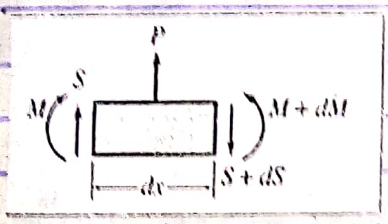
of deflections along the axis of the beam are not known (except at supports) and are just estimated.

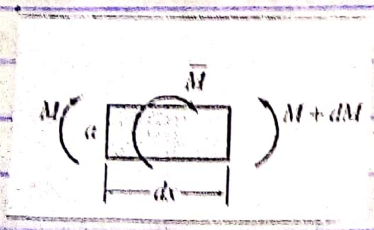
Relationship between Loads, Shear, and Bending Moment

Review : Shear and Bending Moment Diagrams









$$\frac{dS}{dx} = w$$

slope of shear diagram at a point = intensity of distributed load at that point

change in shear between points A and B = area under the distributed load diagram between points A and B

$$\frac{dM}{dx} = S$$

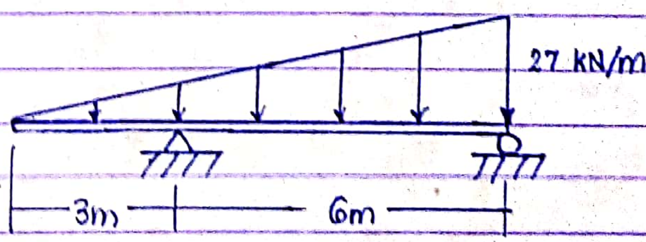
slope of bending moment diagram at a point = shear at that point

change in bending moment between points A and B = area under the shear diagram between points A and B

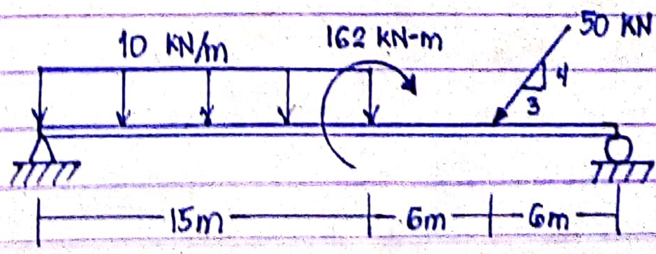
Sample Problems

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beams shown.

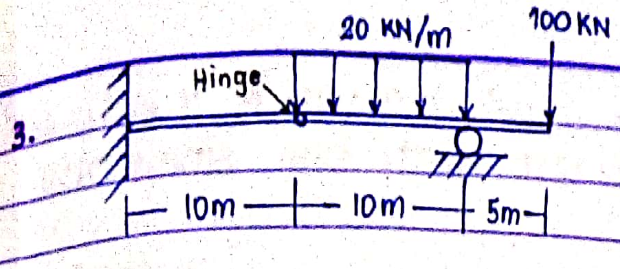
1.



2.



3.



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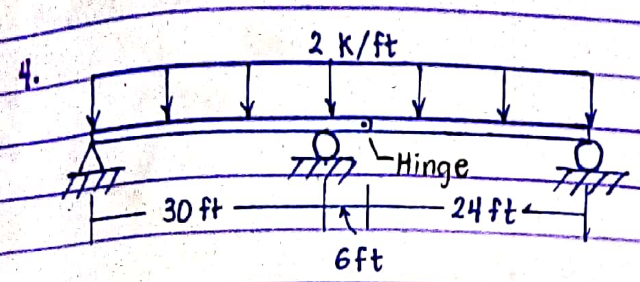


Static Determinacy, Indeterminacy, and Instability of Plane Frames

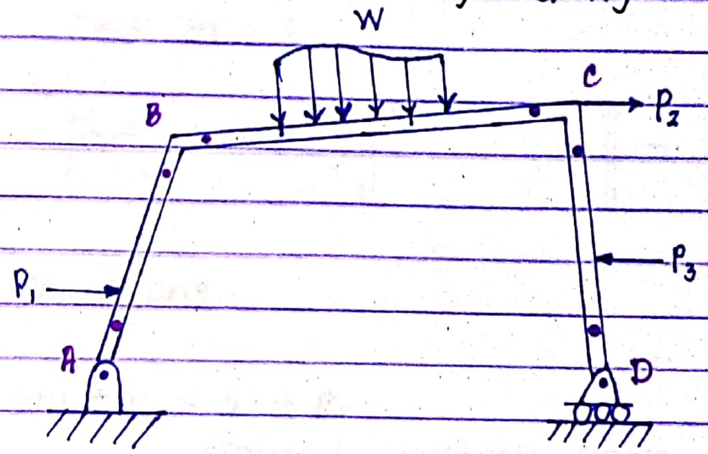
- Rigid frames, usually referred to simply as frames, are composed of straight members connected either by rigid (moment-resisting) connections or by hinged connections to form stable configurations.

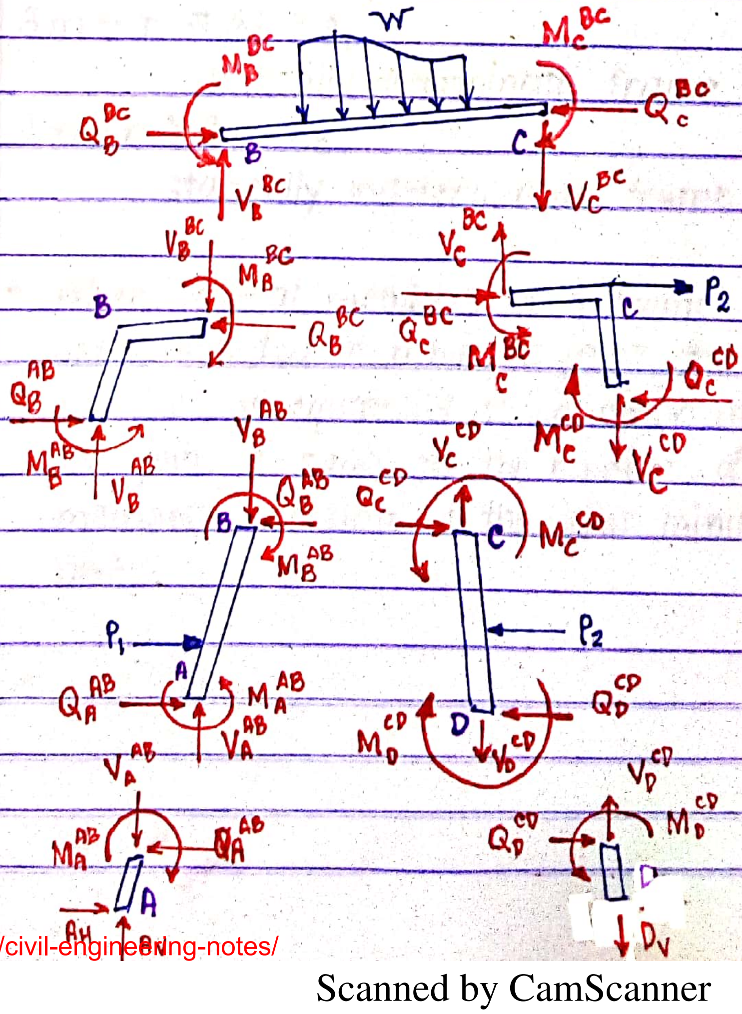
- The members of frames are usually connected by rigid joints, although hinged connections are sometimes used. A rigid joint prevents relative translations and rotations of the member ends connected to it, so the joint can transmit two rectangular force components and a couple between the connected members.

- A FRAME is statically determinate if the bending moments, shears, and axial forces in all its members, as well as all the external reactions, can be determined by using the equations of equilibrium and condition.

- Under the action of external loads, the members of a frame may be, in general, subjected to bending moment, shear, and axial tension or compression.

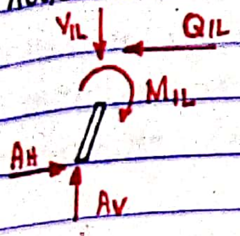
- To illustrate, consider a plane frame subjected to an arbitrary loading.





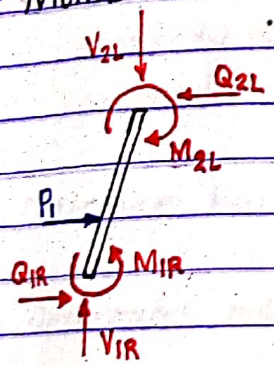
- **Equilibrium of internal forces in a determinate and stable frame**

- Reaction



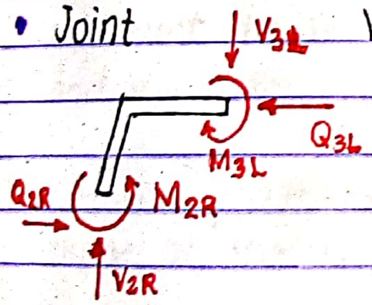
- Member

$$Q_{1R} + Q_{1L} = 0$$
$$V_{1R} + V_{1L} = 0$$
$$M_{1R} + M_{1R} = 0$$



- Joint

$$M_{2L} + M_{2R} = 0$$
$$V_{2L} + V_{2R} = 0$$
$$Q_{2L} + Q_{2R} = 0$$



- <u>Criteria for the Static Determinacy, Indeterminacy, and Instability of General Plane Frames</u> containing M members and J joints and supported by R number of external reactions

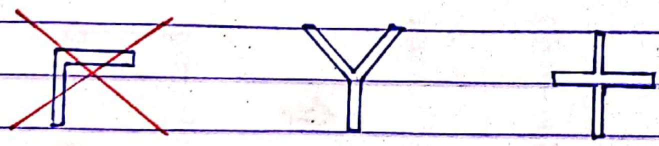
$$i = (3m + r) - (3j + e_c)$$

Where:

i = degree of static indeterminacy
m = number of members
r = # of external reactions
j = # of joints
e_c = equation of condition for plane frames

* Applicable only to members connected more than 2.

$$e_c = m - 1$$



Conditions:

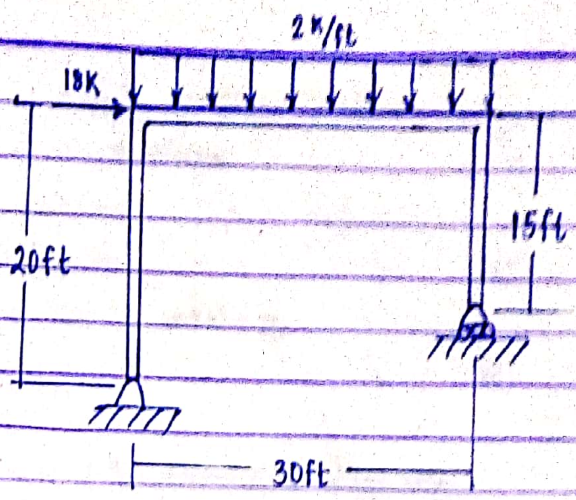
$3m + r < 3j + e_c$ statically unstable frame

$3m + r = 3j + e_c$ statically determinate frame

$3m + r > 3j + e_c$ statically indeterminate frame

* When several members of a frame are connected at a hinged joint, the number of equations of condition at the joint is equal to the number of members meeting at the joint minus one.

Analysis of Plane Frames

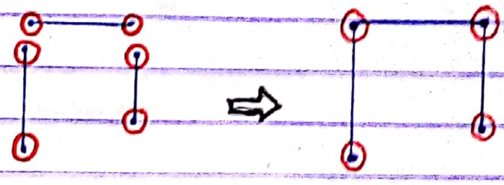


Procedure in determining the member end forces as well as the shears, bending moments, and axial forces in members of plane statically determinate frames

1. Check for static determinacy.
   - If the frame is found to be statically determinate and stable, proceed to step 2.

$m = 3 \qquad r = 3$ (roller & pin)

$e_c = 0 \qquad j = 6 - 2 = 4$



2. Determine the support reactions.

3. Determine member end forces.

- 6 = end points
- 2 = connected points in junctions

4. For each member of the frame, construct the shear, bending moment, and axial force diagrams.

a. $i = (3m + r) - (3j + e_c)$

$i = [3(3) + 3] - [3(4) + 0]$

$i = 0$

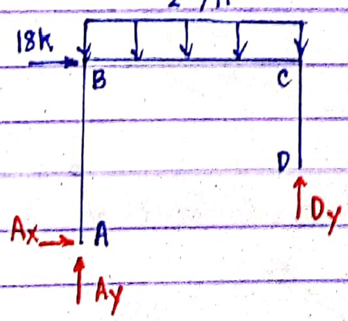
$\therefore$ statically determinate

5. Draw a qualitative deflected shape of the frame.

Sample Problem

9. Draw the shear, bending moment, and axial force diagrams and the qualitative deflected shape for the frame shown.

b.



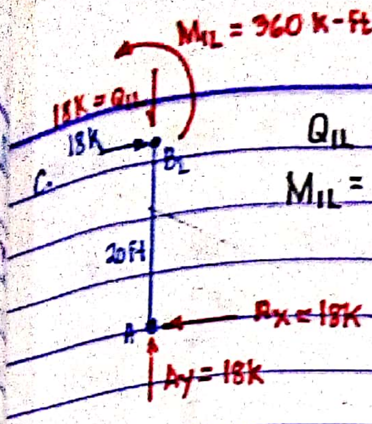
Reaction at supports : whole FBD

$\Sigma M_A = 0 ; \quad D_y = 42\,K$

$\Sigma F_V = 0 ; \quad A_y = 18\,K$

$\Sigma F_H = 0 ; \quad A_x = 18\,K$

c.



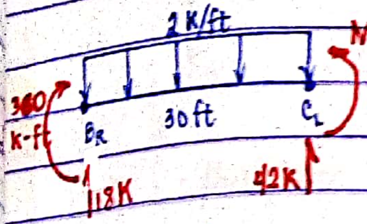
$Q_{1L} = A_y = 18K$

$M_{1L} = 18K(20ft) = 360 K\text{-}ft$

$\Sigma F_y = 0$

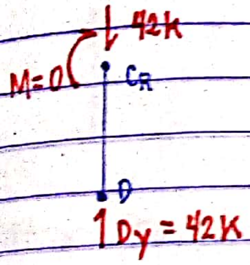
$0 = 18K - 2^{K}/_{ft}(30ft) + \uparrow$

$\uparrow = 42K$

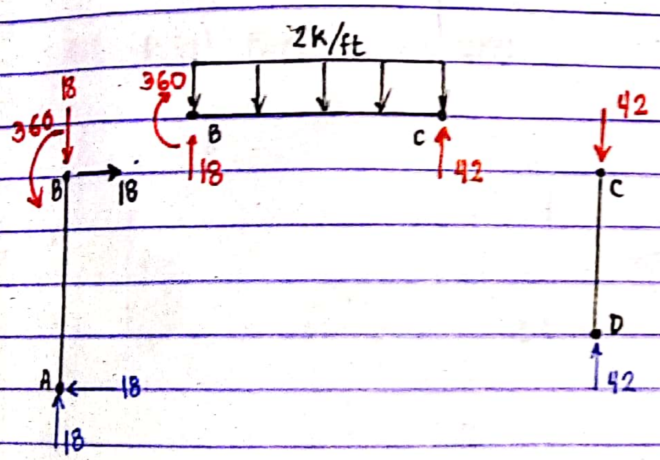


$Q_{1L} = \uparrow = 18K$

$M_{1L} = M_{1R} = 360 K\text{-}ft$



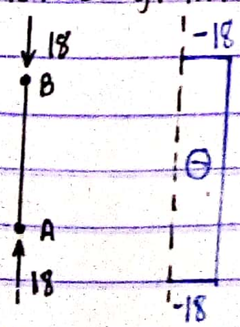
$\uparrow = \downarrow = 42K$



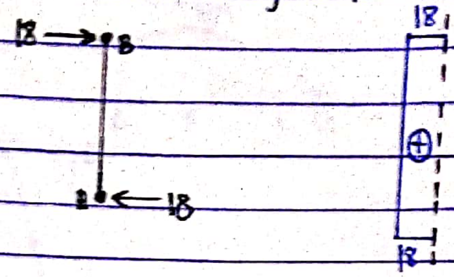
d. Diagrams

1. Member 1

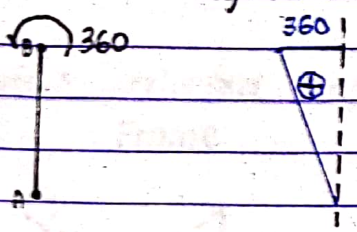
1.1. Axial Diagram



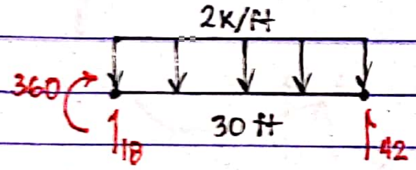
1.2. Shear Diagram



1.3. Moment Diagram

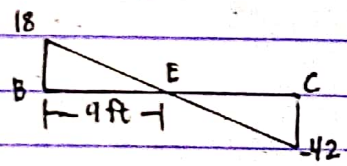


2. Member 2

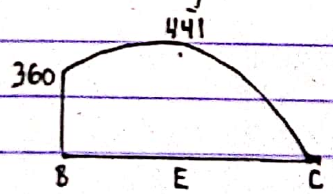


2.1. Axial Diagram

2.2. Shear Diagram

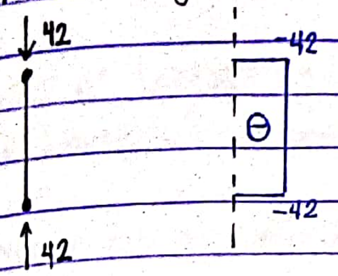


2.3. Moment Diagram

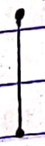


3. Member 3

3.1. Axial Diagram



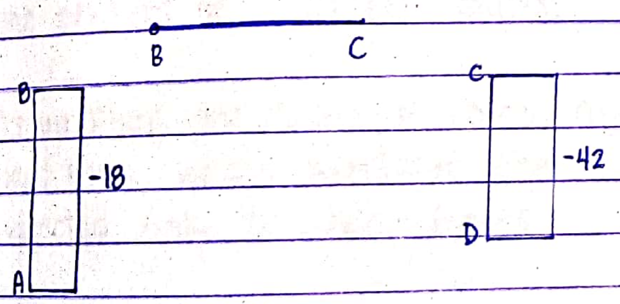
3.2. V-Diagram



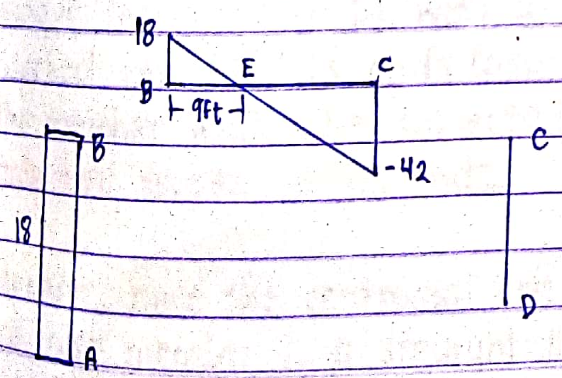
3.3. M-Diagram

4. Whole Frame

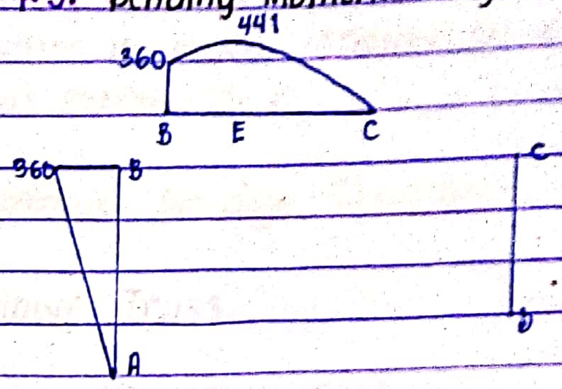
4.1 Axial Force Diagram



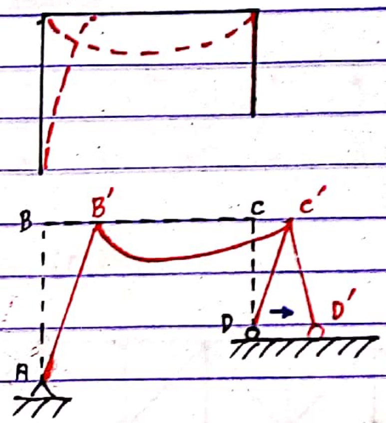
4.2. Shear Diagram



4.3. Bending Moment Diagram



e. Qualitative Deflected shape of the Frame



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Course outcome 2

Plane and Space Trusses

Assumptions for Analysis of Trusses

The analysis of trusses is usually based on the following simplifying assumptions:

1. All members are connected only at their ends by frictionless hinges in plane trusses and by frictionless ball-and-socket joints in space trusses.

2. All loads and support reactions are applied only at the joints.

3. The centroidal axis of each member coincides with the line connecting the centers of adjacent joints.

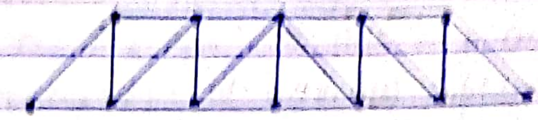
* Assumptions are made to obtain an ideal truss, whose members are subjected only to axial forces.

* Since each member of an ideal truss is connected at its ends by frictionless hinges (assumption 1) with no loads applied between its ends (assumption 2), the member would be subjected to only two forces at its ends.

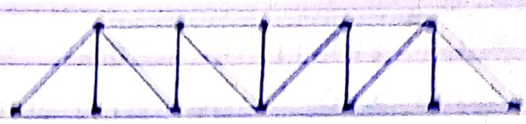
* Moreover, since the centroidal axis of each truss member is a straight-line coinciding with the line connecting the centers of the adjacent joints (assumption 3), the member is not subjected to any bending moment or shear force and is either in axial tension or in axial compression.

Common Bridge Trusses

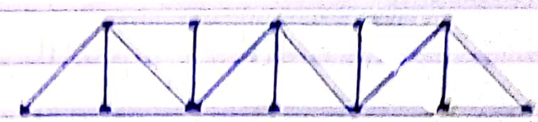
1. Howe Truss



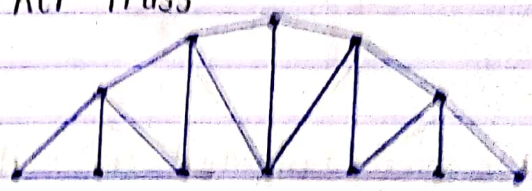
2. Pratt Truss



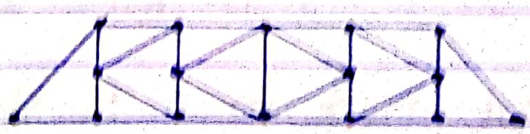
3. Warren Truss



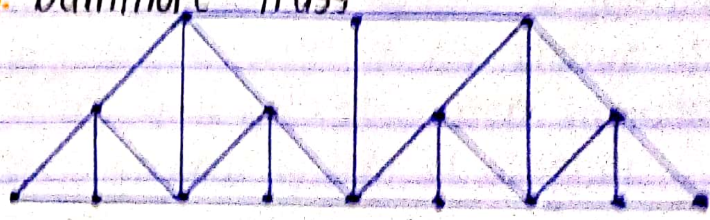
4. Parker Truss



5. K Truss

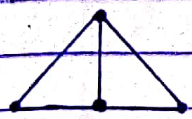


6. Baltimore Truss

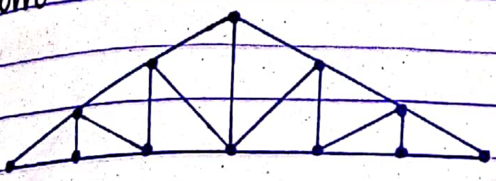


Common Roof Trusses

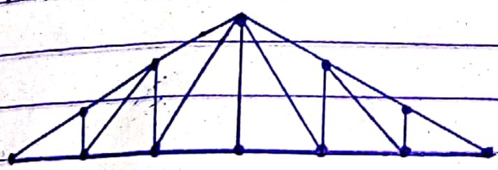
1. King Post Truss



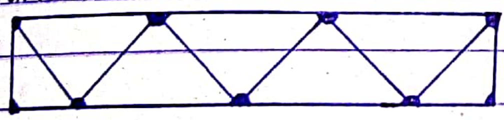
2. Howe Truss



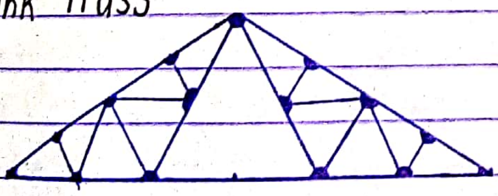
3. Pratt Truss



4. Warren Truss



5. Fink Truss



Arrangement of Members of Plane Trusses - Internal Stability

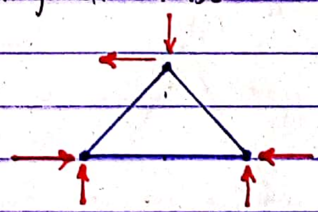
- Internal ► refer to the number and arrangement of members contained within the truss.

- External ► instability due to insufficient external supports or due to improper arrangement of external supports.

Basic Truss Element

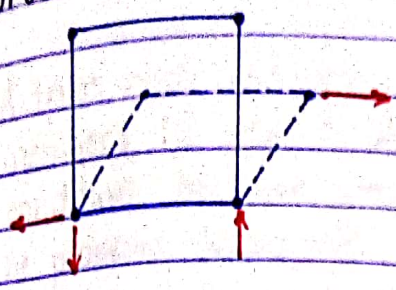
► The simplest internally stable (or rigid) plane truss can be formed by connecting three members at their ends by hinges to form a triangle.

► the triangular truss

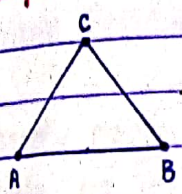


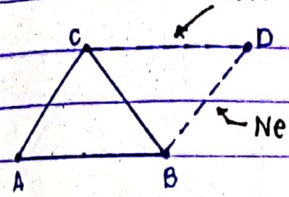
- NOTE : This triangular truss is internally stable, in a sense, that it is a rigid body that will not change its shape under loads.

- In contrast, a rectangular truss formed by connecting four members at their ends by hinges, is internally unstable because it will change its shape and collapse when subjected to a general system of coplanar forces.



Simple Trusses





- A simple truss is formed by enlarging the basic truss element, which contains three members and three joints, by adding two members for each joint, so the total number of members m in a simple truss is given by :

$$m = 2j - 3 \quad \text{(w/o hinge)}$$

where :
j = total # of joints (including those attached to the supports)
m = total # of members

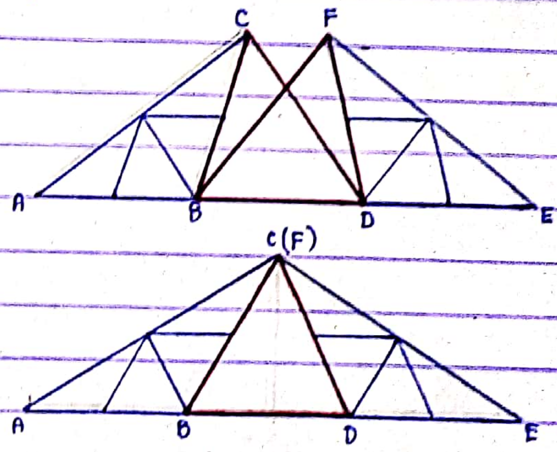
NOTE: Members are counted when in between the joints.

Compound Trusses

- These are constructed by connecting two or more simple trusses to form a single rigid body.

- To prevent any relative movement between the simple trusses, each truss must be connected to the other(s) by means of connections capable of transmitting at least three force components, all of which are neither parallel nor concurrent.

Example :



- Conditions for Plane Truss (w/o hinge)

$m < 2j - 3$ the truss is internally unstable

$m \geq 2j - 3$ the truss is internally stable

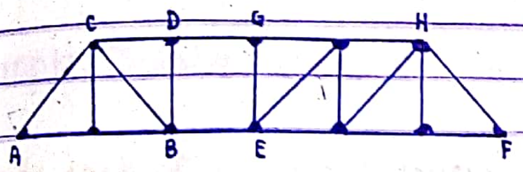
- It is very important to realize that although the foregoing criterion for internal stability is necessary, it is not enough to ensure internal stability.

- A truss must not only contain enough members to satisfy the $m \geq 2j-3$ condition, but the members must also be properly arranged to ensure rigidity of the entire truss.

Sample Problems :

Classify each of the plane trusses shown as internally stable or unstable.

1.

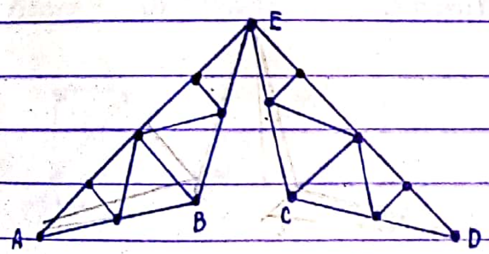


$m = 20 \qquad j = 12$

$m < 2j-3$

$\therefore$ internally unstable

2.

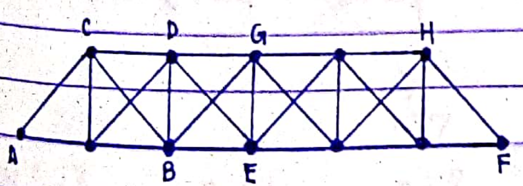


$m = 26 \qquad j = 15$

$m < 2j-3$

$\therefore$ internally unstable

3.



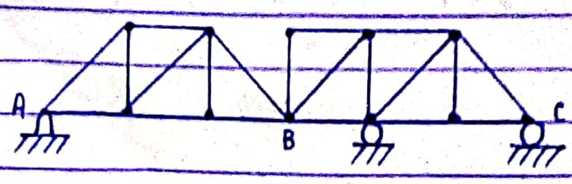
$m = 25 \qquad j = 12$

$m > 2j-3$

$\therefore$ internally stable

Equation of Condition for Plane Trusses

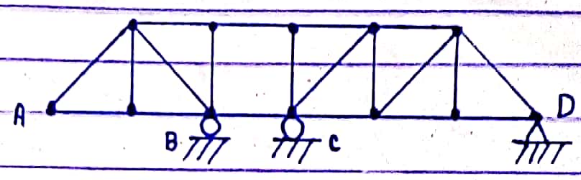
1. Single Hinge Connection



One equation of condition:

$$\sum M_B^{AB} = 0 \quad \text{or} \quad \sum M_B^{BC} = 0$$

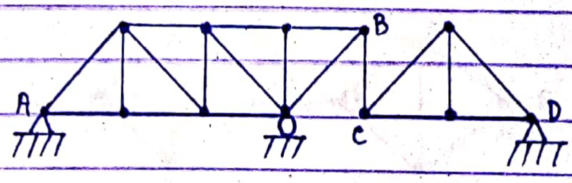
2. Parallel Members Connection



One equation of condition:

$$\sum F_y^{AB} = 0 \quad \text{or} \quad \sum F_y^{CD} = 0$$

3. Single Link connection



Two equations of condition:

$$\sum F_x^{AB} = 0 \quad \text{or} \quad \sum F_x^{CD} = 0$$

$$\sum M_B^{AB} = 0 \quad \text{or} \quad \sum M_C^{CD} = 0$$

Static Determinacy, Indeterminacy, and Instability of Plane Trusses

Conditions for Plane Trusses (with external reactions)

$m + r < 2j$ statically unstable truss

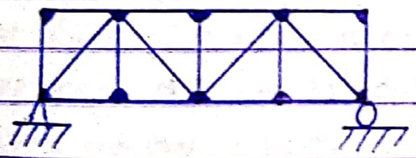
$m + r = 2j$ statically determinate truss

$m + r > 2j$ statically indeterminate truss

Sample Problems:

Classify each of the plane trusses shown as unstable, statically determinate, or statically indeterminate. If the truss is statically indeterminate, then determine the degree of static indeterminacy.

4.

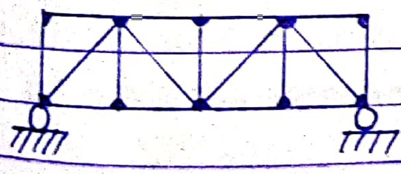


$m = 17$ $j = 10$ $r = 3$

$m + r = 2j$

$\therefore$ statically determinate

5.

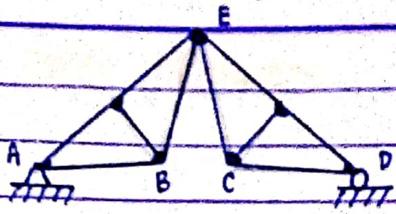


$m = 17$ $j = 10$ $r = 2$

$m + r < 2j$

$\therefore$ unstable

6.

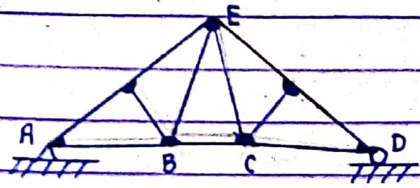


$m = 10$ $j = 7$ $r = 3$

$m + r < 2j$

$\therefore$ unstable

7.

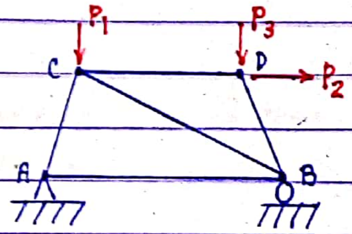


$m = 11$ $j = 7$ $r = 3$

$m + r = 2j$

$\therefore$ statically determinate

8.

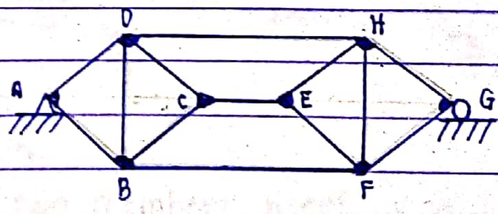


$m = 5$ $j = 4$ $r = 3$

$m + r = 2j$

$\therefore$ statically determinate

9.



$m = 13$ $j = 8$ $r = 3$

$m + r = 2j$

$\therefore$ statically determinate, unstable

* Unstable : contains 2 rigid bodies, ABCD and EFGH, connected by 3 parallel lines, BF, CE, and DH, which cannot prevent the relative displacement, in the vertical direction, of one rigid part of the truss

with respect to the other.

Analysis of Plane Trusses by the Method of Joints and Method of Sections

Method of Joints Analysis:

1. Draw the FBD.
2. Solve reactions.
3. Select joint with minimum number of unknowns (preferably only 2 unknowns)
4. Analyze magnitude of forces using equilibrium equation.
5. Proceed to other joints, concentrating with joints that has minimum number of unknowns.
6. Check member forces at unused joint/s.
7. Tabulate the value of member forces tension (+) and compression (-).

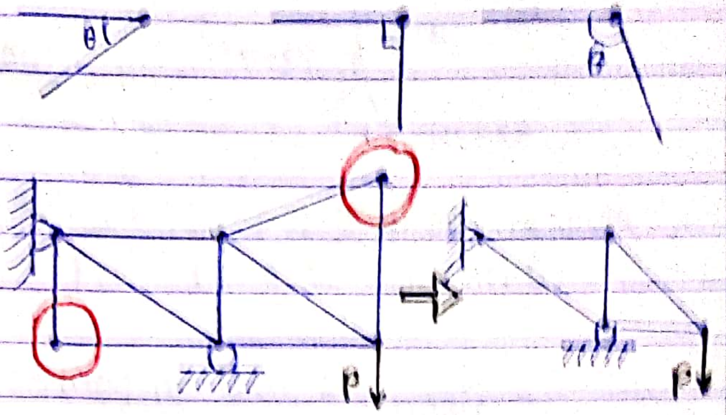
Method of Sections Analysis:

1. Pass a section through a maximum of three members, one of which is desired member (divide the truss into two completely separate parts.
2. For one part of the truss only, take moment about the point where the two members intersect.
3. Solve for the member force.
4. Solve the other two unknowns.

Zero - Force Members

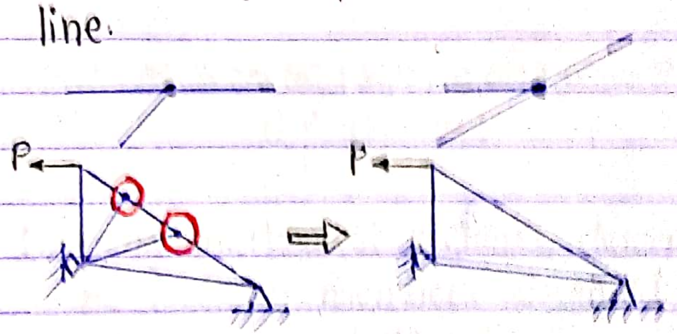
Case 1

When two members meet at an unloaded joint.



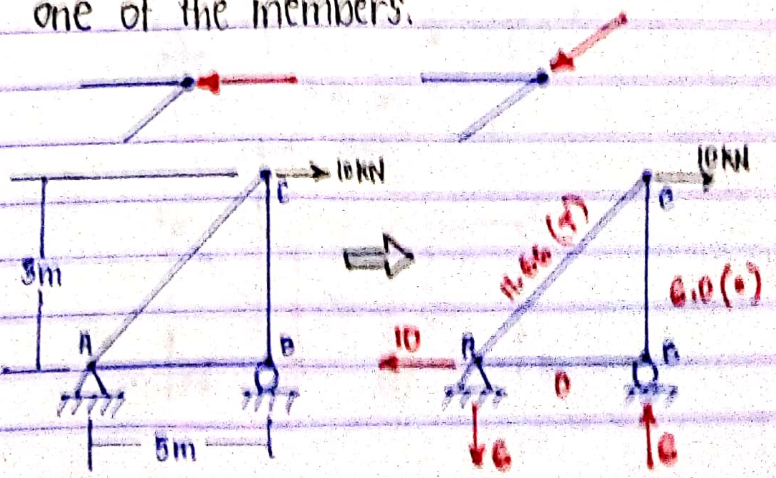
Case 2

When three members meet at an unloaded joint, where two are in line.



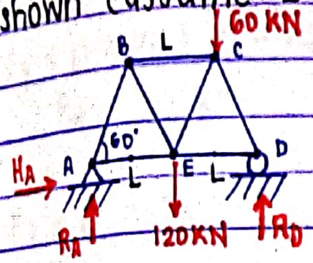
Case 3

When two members meet at a loaded joint, where the loading is in line with one of the members.



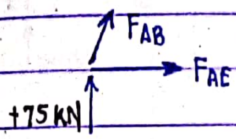
Sample Problems:

10. Using method of joints, determine the force in each member of the trusses shown (assume $L = 1m$).



Find reactions: $R_A = 75$ kN
$R_D = 105$ kN
$H_A = 0$

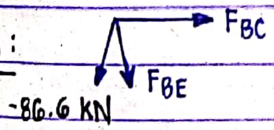
Joint A:



$\Sigma F_y = 0 = 75 + F_{AB} \sin 60$
$F_{AB} = -86.6$ KN $= 86.6$ KN (C)

$\Sigma F_x = 0 = F_{AE} + F_{AB} \cos 60$
$F_{AE} = +43.3$ KN $= 43.3$ KN (T)

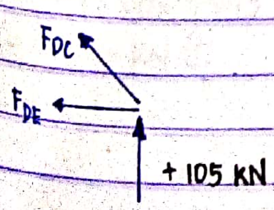
Joint B:



$\Sigma F_y = 0 = -F_{BE} \sin 60 - (-86.6) \sin 60$
$F_{BE} = +86.6$ KN $= 86.6$ KN (T)

$\Sigma F_x = 0 = F_{BC} - (-86.6) \cos 60 + 86.6 \cos 60$
$F_{BC} = -86.6$ KN $= 86.6$ KN (C)

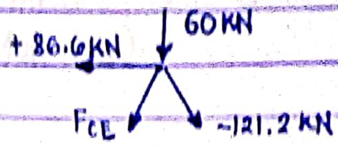
Joint D:



$\Sigma F_y = 0 = 105 + F_{DC} \sin 60$
$F_{DC} = -121.2$ KN $= 121.2$ KN (C)

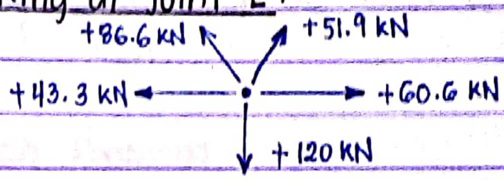
$\Sigma F_x = 0 = -F_{DE} - (-121.2 \text{ KN}) \cos 60$
$F_{DE} = +60.6$ KN $= 60.6$ KN (T)

Joint C:



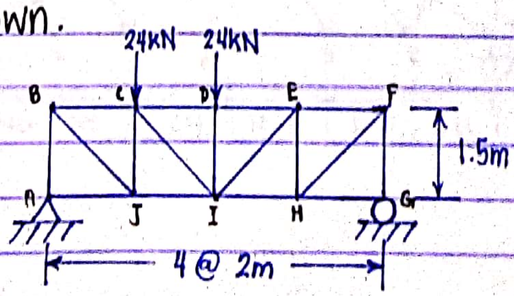
$\Sigma F_y = 0 = -60 - (-121.2) \sin 60 - F_{CE} \sin 60$
$F_{CE} = +51.9$ KN $= 51.9$ KN (T)

Checking at Joint E:

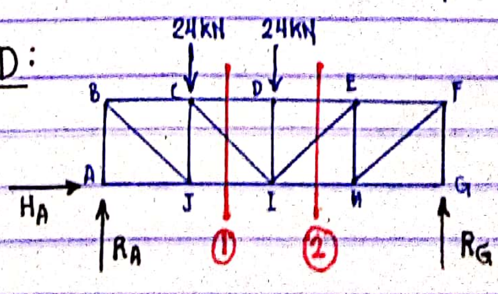


$\Sigma F_x = 0$
$-43.3 - 86.6 \cos 60 + 51.9 \cos 60 + 60.6 = 0$
$0 \stackrel{\checkmark}{=} 0$ OK!

11. Using method of sections, determine the force in members identified (CD, CI, EI, and HI) for the trusses shown.



FBD:



Find Reactions:

$\Sigma M_A = 0 = 24(2) + 24(4) - R_G(8)$

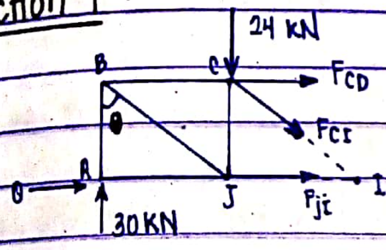
$R_G = 18 \text{ KN}$

$\Sigma F_y = 0 = R_A + R_G - 24 - 24$

$R_A = 30 \text{ KN}$

$\Sigma F_x = 0 = H_A$

Section 1:



$\tan\theta = 2/1.5$

$= 53.13°$

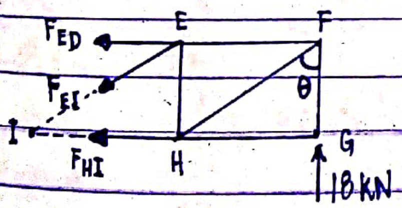
$\Sigma M_I = 0 = F_{CD}(1.5) - 24(2) + 30(4)$

$F_{CD} = -48 \text{ KN} = 48 \text{ KN}$ (C)

$\Sigma F_y = 0 = 30 - 24 - F_{CI}\cos 53.13$

$F_{CI} = 10 \text{ KN}$ (T)

Section 2:



$\Sigma M_E = 0 = F_{HI}(1.5) - 24(2)$

$F_{HI} = 24 \text{ KN}$ (T)

$\Sigma F_y = 0 = -F_{EI}\cos 53.13 + 18$

$F_{EI} = 30 \text{ KN}$ (T)

Compound Trusses

Analysis of Compound Trusses:

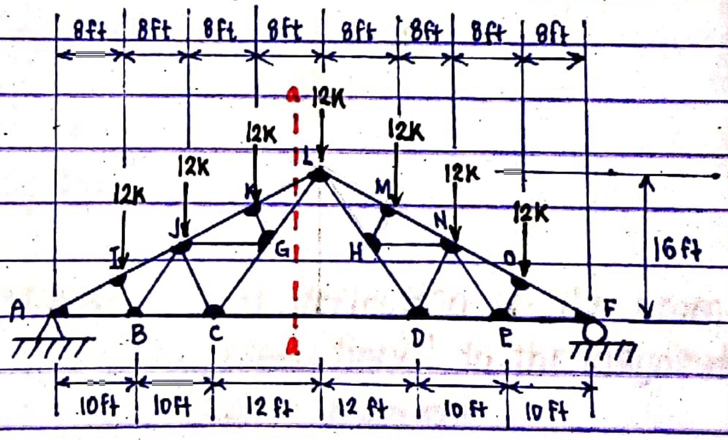
1. Determine static determinacy. If unstable or indeterminate, don't proceed with the analysis (for now).

2. Solve for reactions.

3. Calculate member forces using method of joints and/or method of sections. and indicate members under compression and tension.

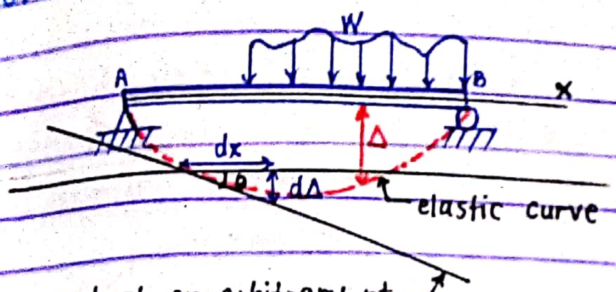
Sample Problem:

12. Determine the force in each member of the compound truss shown.



Deflection of Beams: Geometric Methods

Differential Equation for Beam Deflection



tangent at an arbitrary pt.

$$\tan\theta = \frac{d\Delta}{dx}$$

when $\theta = 10°$, $\tan\theta = 0.176$

$$\frac{\pi}{180} = 0.01745$$

$\therefore$ small $\theta \approx \tan\theta$

$$\theta = \frac{d\Delta}{dx}$$

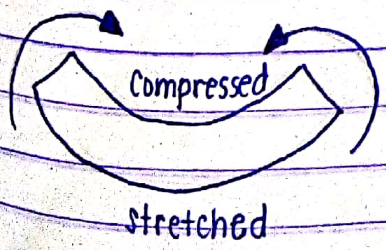
$$d\Delta = \theta\, dx$$

$$\int d\Delta = \int \theta\, dx$$

$$\Delta = \int \theta\, dx$$

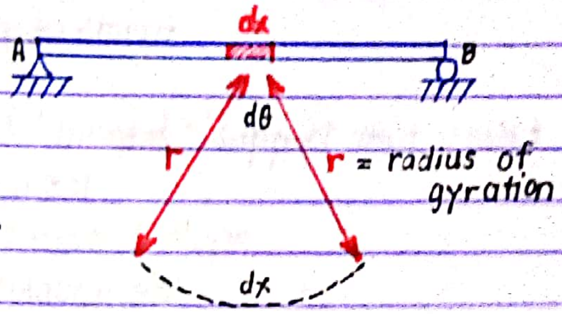
* Angle should be expressed or related in terms of x.

A bending beam:

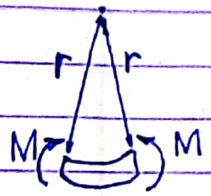


- Top fiber is compressed, outer fiber is stretched. The amount of compression and stretching varies from fiber to fiber.

- The neutral surface on the other is neither stretched or compressed. In 2D, it is the NEUTRAL AXIS.

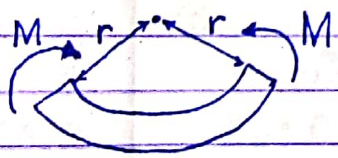


$M\uparrow$, $r\downarrow$
$M\downarrow$, $r\uparrow$



$$d\theta(r) = dx$$

$$d\theta = \frac{dx}{r}$$



$$\theta = \int \frac{1}{r}\, dx$$

Recall:

- The amount of deflection on the beam is directly proportional to the magnitude of the bending moment.

- The radius of curvature is also inversely proportional to the bending moment.

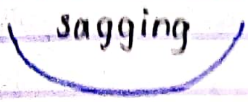
- The product of the bending moment and radius of curvature is constant.

$$M(r) = EI$$

Analysis of Beam Shape

- Hogging and sagging describe the shape of a beam or similar long object when loading is applied.
- **Hogging** described a beam which curves **upward** in the middle. (negative moment)

- **Sagging** describes a beam which curves **downwards** in the middle. (positive moment)



Direct Integration Method / Double Integration Method

Deflection ▸ the degree to which a structural element is displaced under a load.

Sources :

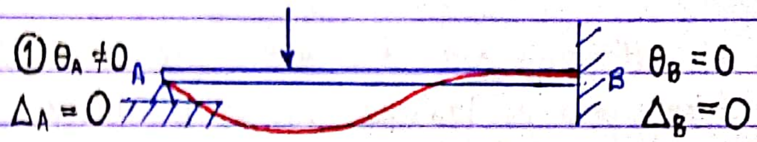
- Load
- Temperature
- Fabrication Error
- Settlement

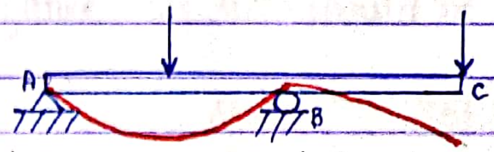
Elastic Curve

▸ the deflection diagram of the longitudinal axis that passes through the centerfold of each cross-sectional area of the beam.

▸ **Pin / Roller** ▸ support that resist :
- force
- displacement

▸ **Fixed / Hinged** ▸ support that resist :
- moment
- rotation / slope
- displacement

① 

② 

$\theta_A \neq 0$ $\quad$ $\theta_B \neq 0$ $\quad$ $\theta_C \neq 0$

$\Delta_A = 0$ $\quad$ $\Delta_B = 0$ $\quad$ $\Delta_C = 0$

Euler - Bernoulli Theory

▸ known as elastic-beam theory

▸ this theory form important differential equation that relate the internal moment in a beam to the displacement and slope of its elastic curve.

- this equation form the basis for the deflection methods.

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad \text{(Equation 1)}$$

Double Integration Method (DIM)

- Moment (M) is known expressible as a function of position (x), the successive integrations of Equation 1 will yield the beam's slope (θ).

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI}\,dx$$

- The equation of the elastic curve, y (displacement).

$$y = f(x) = \iint \frac{M}{EI}\,dx$$

- This method depend on the loading of the beam.
- All function for moment must be written each valid within the region between discontinuities.
- Using equation 1 and the function for M, will give the slope and deflection for each region of the beam for which they are valid.
- $\frac{dy}{dx}$ is also termed as θ or y'
- $\frac{d^2y}{dx^2}$ is also termed as Δ or δ or y.
- Slope Equation:

$$EIy' = Mx + C_1$$

- Deflection Equation:

$$EIy = \frac{Mx^2}{2} + C_1x + C_2$$

Considerations when performing DIM applying Macaulay's method:

1. Having a uniform dead load extended only on a part of the span.
2. If for any value of x, the quantity within a square bracket in any term is negative, the term should be neglected.

	Slope	Deflection
Fixed Support	Zero	Zero
Pin / Roller	Maximum	Zero (unless with settlement)
Free End	Varies	Varies

* Slope is zero at maximum deflection.

Macaulay's Method

- Enables us to write a single equation for bending moment for the full length of the beam

• Allows us to 'turn off' partial of moment function when the value inside a bracket in that function is zero or negative.

Moment Formulas:

1. Point Load (P.L.)

   $P.L. = FL$

2. Uniformly Distributed Load (U.L.)

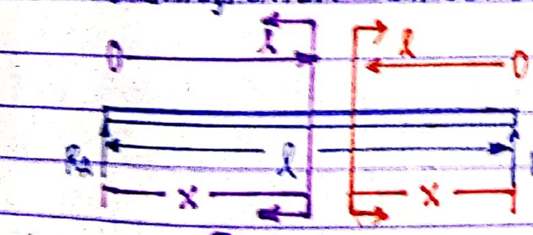
   $U.L. = FL\bar{x}$

3. Gradually Varying Load (V.L.)

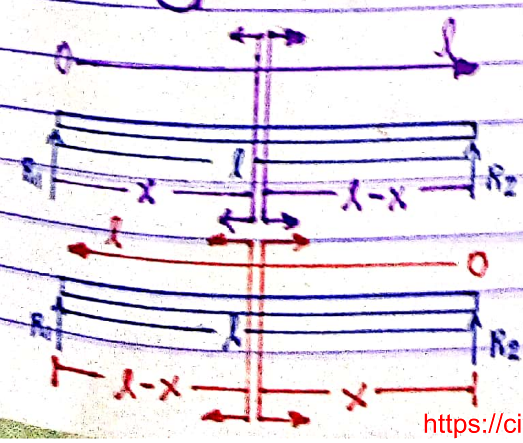
   $V.L. = \dfrac{FL\bar{x}}{n+1}$

where: F = force (N or N/m)
L = length (m)
$\bar{x}$ = centroid (m)
n = degree of slope (°)

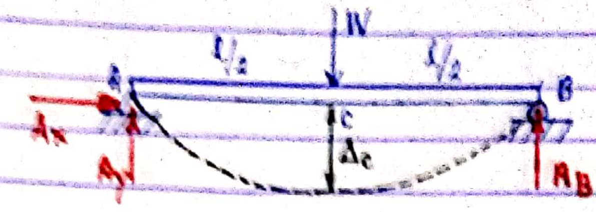
Double Integration Direction:



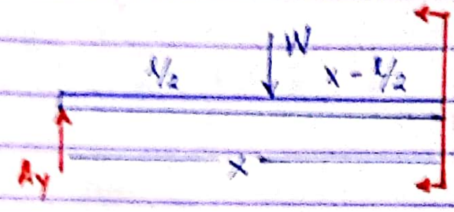
Left Side: (+) (−)   Right Side: (−) (+)



Case I: Simply Supported Beam with a Central Point Load



Reactions: $A_y = R_B = \dfrac{W}{2}$; $A_x = 0$



Moment Equation:

$$EIy'' = \frac{W}{2}x - W\left\langle x - \frac{l}{2}\right\rangle$$

Slope Equation:

$$EIy' = \frac{Wx^2}{4} - \frac{W\langle x - l/2\rangle^2}{2} + C_1$$

Deflection Equation:

$$EIy = \frac{Wx^3}{12} - \frac{W\langle x - l/2\rangle^3}{6} + C_1x + C_2$$

Constants of Integration:

@ $x = 0$, $y = 0$, $C_2 = 0$

@ $x = l/2$, $y' = 0$

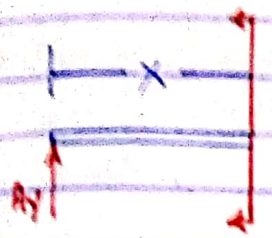
$$SE: 0 = \frac{W}{4}\left(\frac{l}{2}\right)^2 - \frac{W\langle l/2 - l/2\rangle^2}{2} + C_1$$

$$C_1 = -\frac{Wl^2}{16}$$

True Equations for the Whole Beam:

SE: $EIy' = \frac{Wx^2}{4} - \frac{W\langle x - l/2 \rangle^2}{2} - \frac{Wl^2}{16}$

DE: $EIy = \frac{Wx^3}{12} - \frac{W\langle x - l/2 \rangle^3}{6} - \frac{Wl^2}{16}x$



Moment Equation: $EIy'' = \frac{Wx}{2}$

Slope Equation: $EIy' = \frac{Wx^2}{4} + C_1$

Deflection Equation: $EIy = \frac{Wx^3}{12} + C_1x + C_2$

Constants of Integration:

@ $x = 0$, $y = 0$; $C_2 = 0$

@ $x = l/2$, $y' = 0$

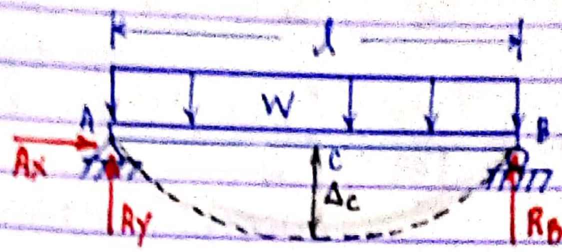
SE: $0 = \frac{W}{4}\left(\frac{l}{2}\right)^2 + C_1$; $C_1 = -\frac{Wl^2}{16}$

True Equations for a Part of Beam:

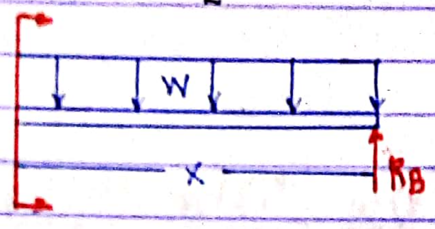
SE: $EIy' = \frac{Wx^2}{4} - \frac{Wl^2}{16}$

DE: $EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x$

Case II: Simply Supported Beam with a Uniformly Distributed Load



Reactions: $A_y = R_B = \frac{wl}{2}$; $A_x = 0$



Moment Equation:

$EIy'' = \frac{wl}{2}x - w\frac{x^2}{2}$

Slope Equation:

$EIy' = \frac{wl}{4}x^2 - \frac{wx^3}{6} + C_1$

Deflection Equation:

$EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$

Constants of Integration

@ $x = 0$, $y = 0$

DE: $EI(0) = \frac{wl(0)^3}{12} - \frac{w(0)^4}{24} + C_1(0) + C_2$

$C_2 = 0$

@ $y = 0$, $x = l$

DE: $EI(0) = \frac{wl(l)^3}{12} - \frac{w(l)^4}{24} + C_1(l) + 0$

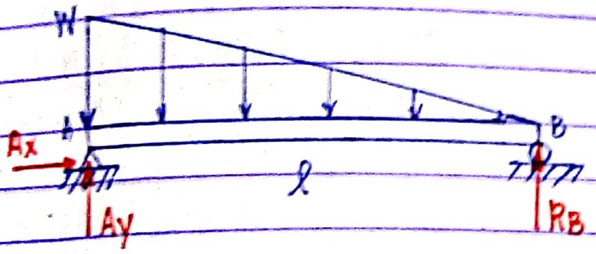
$C_1 = -\frac{wl^3}{24}$

True Equations for the Whole Beam

SE: $EIy' = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$

DE: $EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3}{24}x$

Case III: Simply Supported Beam with a Gradually Varying Load



$\Sigma M_A = 0 = \frac{1}{2} wl\left(\frac{1}{3}l\right) - R_B(l)$

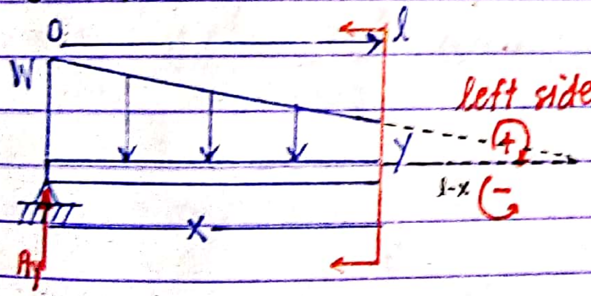
$R_B = \frac{1}{6} wl$

$\Sigma M_B = 0 = A_y(l) - \frac{1}{2} wl\left(\frac{2}{3}l\right)$

$A_y = \frac{1}{3} wl$

$\Sigma F_x = 0 = A_x$

a.



Ratio & Proportion:

$\frac{y}{l-x} = \frac{w}{l} \Rightarrow y = w - \frac{wx}{l}$

$M = \frac{wlx}{3} - \frac{yx^2}{2} - \frac{1}{2}(w-y)x\left(\frac{2}{3}x\right)$

$M = \frac{wlx}{3} - \left(w - \frac{wx}{l}\right)\frac{x^2}{2} - \frac{x^2}{3}\left[w - w + \frac{wx}{l}\right]$

$M = \frac{wlx}{3} - \frac{wx^2}{2} + \frac{wx^3}{6l}$

Moment Equation

$EIy'' = \frac{wlx}{3} - \frac{wx^2}{2} + \frac{wx^3}{6l}$

Slope Equation

$EIy' = \frac{wlx^2}{6} - \frac{wx^3}{6} + \frac{wx^4}{24l} + C_1$

Deflection Equation:

$EIy = \frac{wlx^3}{18} - \frac{wx^4}{24} + \frac{wx^5}{120l} + C_1x + C_2$

Constants of Integration:

@ $x = 0,\ y = 0\ ;\ C_2 = 0$

@ $x = l,\ y = 0$

$EI(0) = \frac{wl(l^3)}{18} - \frac{w(l)^4}{24} + \frac{w(l)^5}{120l} + C_1 l$

$C_1 = -\frac{wl^3}{45}$

True Equation for Whole Beam:

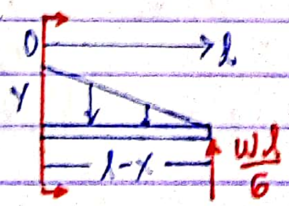
SE: $EIy' = \frac{wlx^2}{6} - \frac{wx^3}{6} + \frac{wx^4}{24l} - \frac{wl^3}{45}$

DE: $EIy = \frac{wlx^3}{18} - \frac{wx^4}{24} + \frac{wx^5}{120l} - \frac{wl^3x}{45}$

b. Right side

(-)

(+)



Recall: $y = w - \frac{wx}{l}$

$M = \frac{wl}{6}(l-x) - \frac{1}{2}y(l-x)\left[\frac{1}{3}(l-x)\right]$

$M = \frac{wl^2}{6} - \frac{wlx}{6} - \frac{y}{6}(l^2 - 2xl + x^2)$

$$M = \frac{wl^2}{6} - \frac{wlx}{6} - \frac{yl^2}{6} + \frac{yxl}{3} - \frac{yx^2}{6}$$

$$M = \frac{wl^2}{6} - \frac{wlx}{6} - \frac{(w - \frac{1}{l} wx)l^2}{6} + \frac{(w - {}^{wx}/_{l})lx}{3} - \frac{(w - {}^{wx}/_{l})x^2}{6}$$

$$M = \frac{wl^2}{6} - \frac{wlx}{6} - \frac{wl^2}{6} + \frac{wxl^2}{6l} + \frac{wlx}{3} - \frac{wlx^2}{3l} - \frac{wx^2}{6} + \frac{wx^3}{6l}$$

$$M = \frac{wxl}{3} - \frac{wx^2}{2} + \frac{wx^3}{6l}$$

Moment Equation:

$$EIy'' = \frac{wlx}{3} - \frac{wx^2}{2} + \frac{wx^3}{6l}$$

Slope Equation:

$$EIy' = \frac{wlx^2}{6} - \frac{wx^3}{6} + \frac{wx^4}{24l} + C_1$$

Deflection Equation:

$$EIy = \frac{wlx^3}{18} - \frac{wx^4}{24} + \frac{wx^5}{120l} + C_1x + C_2$$

Constants of Integration:

@ $x = 0$, $y = 0$; $C_2 = 0$

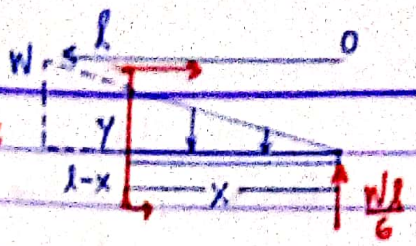
@ $x = l$, $y = 0$; $C_1 = -\frac{wl^3}{45}$

True Equations:

SE: $EIy' = \frac{wlx^2}{6} - \frac{wx^3}{6} + \frac{wx^4}{24l} - \frac{wl^3}{45}$

DE: $EIy = \frac{wlx^3}{18} - \frac{wx^4}{24} + \frac{wx^5}{120l} - \frac{wl^3}{45}x + 0.$

C. Right side (+)



Ratio & Proportion:

$$\frac{y}{x} = \frac{w}{l} \Rightarrow y = \frac{wx}{l}$$

$$M = \frac{wl}{6}x - \frac{1}{2}yx\left(\frac{1}{3}x\right)$$

$$M = \frac{wlx}{6} - \frac{wx^3}{6l}$$

Moment Equation:

$$EIy'' = \frac{wlx}{6} - \frac{wx^3}{6l}$$

Slope Equation:

$$EIy' = \frac{wlx^2}{12} - \frac{wx^4}{24l} + C_1$$

Deflection Equation:

$$EIy = \frac{wlx^3}{36} - \frac{wx^5}{120l} + C_1x + C_2$$

Constants of Integration:

@ $x = 0$, $y = 0$; $C_2 = 0$

@ $x = l$, $y = 0$

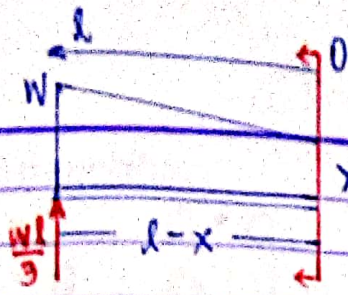
$$0 = \frac{wl(l)^3}{36} - \frac{w(l)^5}{120l} + C_1 l$$

$$C_1 = -\frac{7wl^3}{360}$$

True Equations for Whole Beam

SE: $EIy' = \frac{wlx^2}{12} - \frac{wx^4}{24l} - \frac{7wl^3}{360}$

DE: $EIy = \frac{wlx^3}{36} - \frac{wx^5}{120l} - \frac{7wl^3}{360}x$



d. left side
(+) ↺

Recall: $y = \frac{wx}{l}$

$$M = \frac{wl}{3}(l-x) - y\frac{(l-x)^2}{2} - \frac{(w-y)}{2}\cdot\frac{2(l-x)^2}{3}$$

$$M = \frac{wl^2}{3} - \frac{wxl}{3} - \frac{y}{2}(l^2 - 2xl + x^2) - \frac{1}{3}(w-y)(l^2 - 2xl + x^2)$$

$$M = \frac{wl^2}{3} - \frac{wlx}{3} - \frac{wx}{2l}(l^2 - 2xl + x^2) - \frac{1}{3}\left(w - \frac{wx}{l}\right)(l^2 - 2xl + x^2)$$

$$M = \frac{wl^2}{3} - \frac{wxl}{3} - \frac{wxl}{2} + wx^2 - \frac{wx^3}{2l} - \frac{1}{3}\left(wl^2 - 2wxl + wx^2 - wxl + 2wx^2 - \frac{wx^3}{l}\right)$$

$$M = \frac{wl^2}{3} - \frac{wxl}{3} - \frac{wxl}{2} + wx^2 - \frac{wx^3}{2l} - \frac{wl^2}{3} + \frac{2wxl}{3} - \frac{wx^2}{3} + \frac{wxl}{3} - \frac{2wx^2}{3} + \frac{wx^3}{3l}$$

$$M = \frac{wxl}{6} - \frac{wx^3}{6l}$$

Moment Equation:

$$EIy'' = \frac{wxl}{6} - \frac{wx^3}{6l}$$

Slope Equation:

$$EIy' = \frac{wlx^2}{12} - \frac{wx^4}{24l} + C_1$$

Deflection Equation:

$$EIy = \frac{wlx^3}{36} - \frac{wx^5}{120l} + C_1x + C_2$$

Constants of Integration:

@ $x = 0,\ y = 0$; $C_2 = 0$

@ $x = l,\ y = 0$

$$0 = \frac{wl(l)^3}{36} - \frac{w(l)^5}{120l} + C_1 l$$

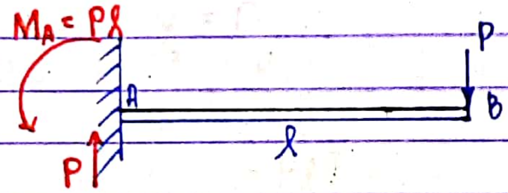
$$C_1 = -\frac{7wl^3}{360}$$

True Equations for Whole Beam

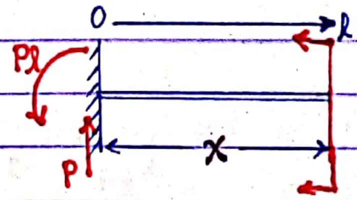
SE: $EIy' = \frac{wlx^2}{12} - \frac{wx^4}{24l} - \frac{7wl^3}{360}$

DE: $EIy = \frac{wlx^3}{36} - \frac{wx^5}{120l} - \frac{7wl^3}{360}x$

Case IV. Cantilever with a Point Load at its Free End



a. left side
(+) ↺



Moment Equation:

$$EIy'' = Px - Pl$$

Slope Equation:

$$EIy' = \frac{Px^2}{2} - Plx + C_1$$

Deflection Equation:

$$EIy = \frac{Px^3}{6} - \frac{Plx^2}{2} + C_1x + C_2$$

Constants of Integration:

@ $x = 0$, $y' = 0$; $C_1 = 0$

@ $x = 0$, $y = 0$; $C_2 = 0$

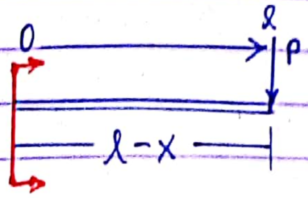
True Equations for the Whole Beam:

SE: $EIy' = \frac{Px^2}{2} - P\ell x$

DE: $EIy = \frac{Px^3}{6} - \frac{P\ell x^2}{2}$

b. right side

$\circlearrowleft$ (+)



$M = -P(\ell - x) = -P\ell + Px$

Moment Equation:

$EIy'' = Px - P\ell$

Slope Equation:

$EIy' = \frac{Px^2}{2} - P\ell x + C_1$

Deflection Equation:

$EIy = \frac{Px^3}{6} - \frac{P\ell x^2}{2} + C_1 x + C_2$

Constants of Integration:

@ $x = 0$, $y' = 0$; $C_1 = 0$

@ $x = 0$, $y = 0$; $C_2 = 0$

True Equations for the Whole Beam:

SE: $EIy' = \frac{Px^2}{2} - P\ell x$

DE: $EIy = \frac{Px^3}{6} - \frac{P\ell x^2}{2}$

c. right side

$\circlearrowright$ (+)

Moment Equation:

$EIy'' = -Px$

Slope Equation:

$EIy' = -\frac{Px^2}{2} + C_1$

Deflection Equation:

$EIy = -\frac{Px^3}{6} + C_1 x + C_2$

Constants of Integration:

@ $x = \ell$, $y' = 0$

$EI(0) = -\frac{P\ell^2}{2} + C_1$; $C_1 = \frac{P\ell^2}{2}$

@ $x = \ell$, $y = 0$

$EI(0) = -\frac{P\ell^3}{6} + \frac{P\ell^2(\ell)}{2} + C_2$

$C_2 = -\frac{P\ell^3}{3}$

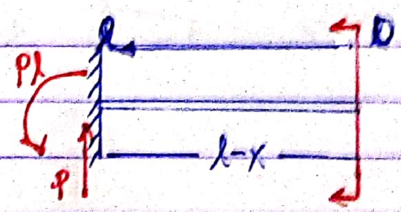
True Equations for the Whole Beam

SE: $EIy' = -\frac{Px^2}{2} + \frac{P\ell^2}{2}$

DE: $EIy = -\frac{Px^3}{6} + \frac{P\ell^2}{2}x - \frac{P\ell^3}{3}$

d. left side

$\circlearrowright$ (+) $\circlearrowleft$ (-)



$M = P(\ell - x) - P\ell = P\ell - Px - P\ell = -Px$

Moment Equation:

$EIy'' = -Px$

Slope Equation:

$$EIy' = -\frac{Px^2}{2} + C_1$$

Deflection Equation:

$$EIy = -\frac{Px^3}{6} + C_1x + C_2$$

Constants of Integration:

@ $x = l,\ y' = 0\ ;\ C_1 = \frac{Pl^2}{2}$

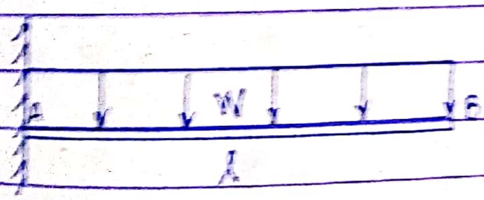
@ $x = l,\ y = 0\ ;\ C_2 = -\frac{Pl^3}{3}$

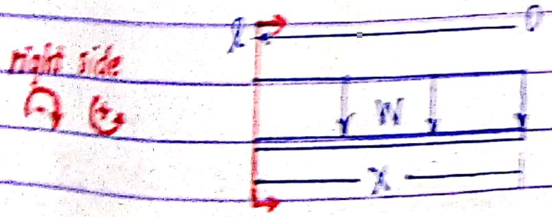
True Equations for the Whole Beam

SE: $EIy' = -\frac{Px^2}{2} + \frac{Pl^2}{2}$

DE: $EIy = -\frac{Px^3}{6} + \frac{Pl^2}{2}x - \frac{Pl^3}{3}$

Case V. Cantilever with a Uniformly Distributed Load





Moment Equation:

$$EIy'' = -\frac{Wx^2}{2}$$

Slope Equation:

$$EIy' = -\frac{Wx^3}{6} + C_1$$

Deflection Equation:

$$EIy = -\frac{Wx^4}{24} + C_1x + C_2$$

Constants of Integration:

@ $x = l,\ y' = 0$

SE: $EI(0) = -\frac{W(l)^3}{6} + C_1\ ;\ C_1 = \frac{Wl^3}{6}$

@ $x = l,\ y = 0$

DE: $EI(0) = -\frac{Wl^4}{24} + \frac{Wl^3}{6}(l) + C_2$

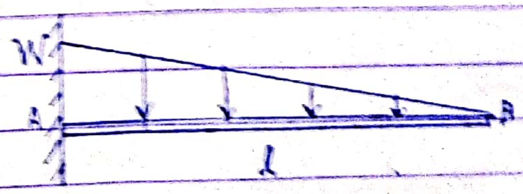
$$C_2 = -\frac{Wl^4}{8}$$

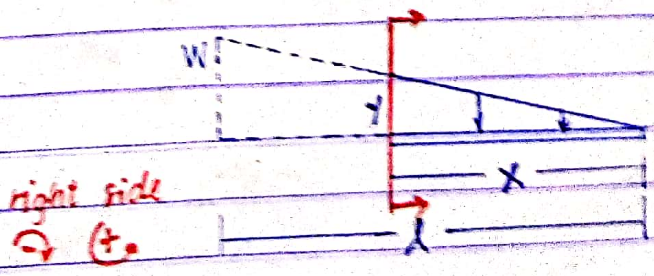
True Equations for the Whole Beam:

SE: $EIy' = -\frac{Wx^3}{6} + \frac{Wl^3}{6}$

DE: $EIy = -\frac{Wx^4}{24} + \frac{Wl^3}{6}x - \frac{Wl^4}{8}$

Case VI. Cantilever with a Gradually Varying Load





Ratio & Proportion:

$$\frac{y}{x} = \frac{W}{\ell} \Rightarrow y = \frac{Wx}{\ell}$$

$$M = -\frac{1}{2} y x \left(\frac{x}{3}\right) = -\frac{Wx^3}{6\ell}$$

Moment Equation:

$$EIy'' = -\frac{Wx^3}{6\ell}$$

Slope Equation:

$$EIy' = -\frac{Wx^4}{24\ell} + C_1$$

Deflection Equation:

$$EIy = -\frac{Wx^5}{120\ell} + C_1 x + C_2$$

Constants of Integration

@ $x = \ell$, $y' = 0$

$$EI(0) = -\frac{W(\ell)^4}{24\ell} + C_1 \quad ; \quad C_1 = \frac{W\ell^3}{24}$$

@ $x = \ell$, $y = 0$

$$EI(0) = -\frac{W(\ell)^5}{120\ell} + \frac{W\ell^3(\ell)}{24} + C_2$$

$$C_2 = -\frac{W\ell^4}{30}$$

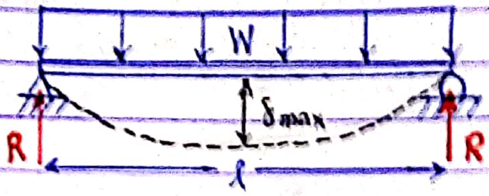
True Equations for the Whole Beam

SE: $$EIy' = -\frac{Wx^4}{24\ell} + \frac{W\ell^3}{24}$$

DE: $$EIy = -\frac{Wx^5}{120\ell} + \frac{W\ell^3 x}{24} - \frac{W\ell^4}{30}$$

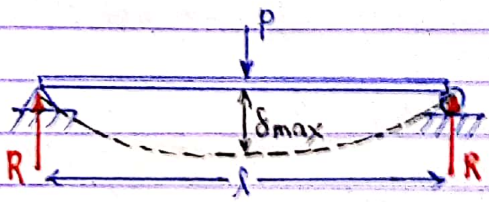
Summary

1. Simple Beam with Uniform Load on Entire Span



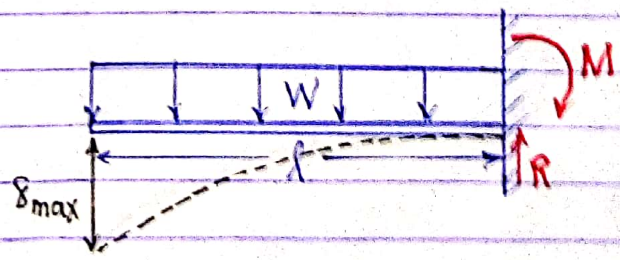
Vmax	Mmax	δ_{max} or Δ_{max}
$\frac{W\ell}{2}$	$\frac{W\ell^2}{8}$	$\frac{5W\ell^4}{384EI}$

2. Simple Beam with Point Load at Midspan



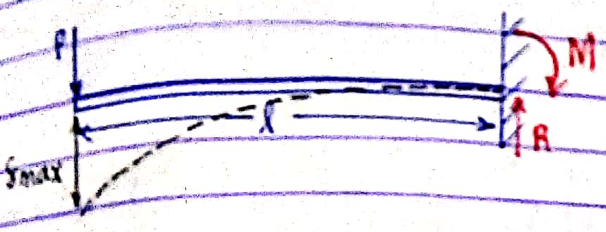
Vmax	Mmax	δ_{max} or Δ_{max}
$\frac{P}{2}$	$\frac{P\ell}{4}$	$\frac{P\ell^3}{48EI}$

3. Cantilever Beam with Uniform Load on Entire Span



Vmax	Mmax	δ_{max} or Δ_{max}
$W\ell$	$\frac{W\ell^2}{2}$	$\frac{W\ell^4}{8EI}$

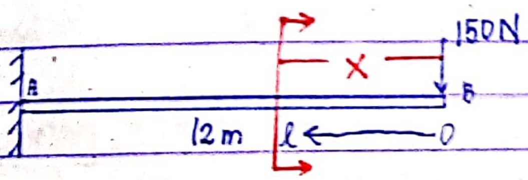
4. Cantilever Beam with Point Load at Free End



Vmax	Mmax	δmax or Δmax
P	Pℓ	$\frac{P\ell^3}{3EI}$

Sample Problem:

13. Determine the equation of the elastic curve, the deflection at midspan, and at free end, θ_{mid} and θ_{end}. Use DIM. EI is constant.



Required: equation for slope & deflection
Δ_{mid} & θ_{mid}
Δ_{end} & θ_{end}

$M = EIy'' = -150x$
$EIy' = -75x^2 + C_1$
$EIy = -25x^3 + C_1x + C_2$

@ $x = \ell$, $y' = 0$: $0 = -75\ell^2 + C_1$
$C_1 = 75\ell^2$

@ $x = \ell$, $y = 0$: $0 = -25\ell^3 + 75\ell^2(\ell) + C_2$
$C_2 = -50\ell^3$

∴ SE: $EIy' = -75x^2 + 75\ell^2$
DE: $EIy = -25x^3 + 75\ell^2x - 50\ell^3$

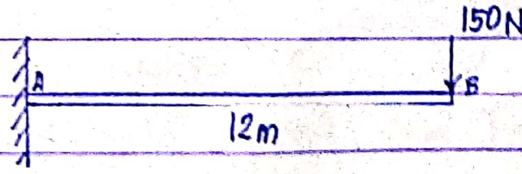
@ $x = 0$, $\theta_{end} = ?$
$EI\theta_{end} = -75(0)^2 + 75(12)^2$: SE
$\theta_{end} = \frac{10800}{EI}$

@ $x = 6$, $\Delta_{mid} = ?$
DE: $EI\Delta_{mid} = -25(6)^3 + 75(12)^2(6) - 50(12)^3$
$\Delta_{mid} = -\frac{27000}{EI}$

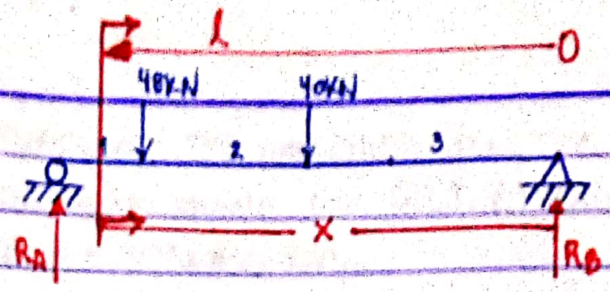
@ $x = 6$, $\theta_{mid} = ?$
SE: $EI\theta_{mid} = -75(6)^2 + 75(12)^2$
$\theta_{mid} = \frac{8100}{EI}$

@ $x = 0$, $\Delta_{end} = ?$
DE: $EI\Delta_{end} = -25(0)^3 + 75(12)^2(0) - 50(12)^3$
$\Delta_{end} = -\frac{86400}{EI}$

14. Determine the equation of the elastic curve, the Δ_{mid} and Δ_{end}, θ_{mid} and θ_{end}. Use DIM. E = 200 GPa. B x H : 200 x 300 mm.



from # 13 :
SE: $EIy' = -75x^2 + 75\ell^2$ $(N-m^2)$
DE: $EIy = -25x^3 + 75\ell^2x - 50\ell^3$ $(N-m^3)$



Given: $E = 200\,GPa = 200\times10^3\ N/mm^2$
$B\times H = 200\times 300\ mm$

$I = \frac{bh^3}{12} = \frac{200(300)^3}{12} = 450\times10^6\ mm^4$

$\theta_{end} = \frac{10800}{EI} = \frac{10800(1000)^2}{(200\times10^3)(450\times10^6)}\left[\frac{N\text{-}m^2\left(\frac{mm}{m}\right)^2}{N/mm^2\,(mm^4)}\right]$

$\theta_{end} = 1.2\times10^{-4}\ rad\ \left(^{180}/_{\pi}\right)$

$\theta_{end} = 6.88\times10^{-3}$ degree

$\theta_{mid} = \frac{8100}{EI} = \frac{8100(1000)^2}{(200\times10^3)(450\times10^6)}\left[\frac{N\text{-}m^2\left(\frac{mm}{m}\right)^2}{N/mm^2\,(mm^4)}\right]$

$\theta_{mid} = 9\times10^{-5}\ rad\ \left(^{180}/_{\pi}\right)$

$\theta_{mid} = 5.16\times10^{-3}$ degree

$\Delta_{end} = \frac{-86400}{EI} = \frac{-86400(1000)^3}{(200\times10^3)(450\times10^6)}\left[\frac{N\text{-}m^3\left(\frac{mm}{m}\right)^3}{N/mm^2\,(mm^4)}\right]$

$\Delta_{end} = -0.96\ mm$

$\Delta_{mid} = \frac{-27000}{EI} = \frac{-27000(1000)^3}{(200\times10^3)(450\times10^6)}\left[\frac{N\text{-}m^3\left(\frac{mm}{m}\right)^3}{N/mm^2\,(mm^4)}\right]$

$\Delta_{mid} = -0.3\ mm$

15. A beam of length 6m is simply supported at its ends and carries two-point loads of 48 kN and 40kN at 1m and 3m respectively from the left support. Use DIM to find:

a. deflection under each load
b. maximum deflection.
c. The point at which maximum deflection occurs. Use $E = 200$ GPa. $I = 85\times10^6\ mm^4$.

Required : Δ_{48kN} or Δ_{5m}
Δ_{40kN} or Δ_{3m}
Δ_{max} & location

$\Sigma M_A = 0;\ 48(1) + 40(3) = R_B(6)$
$R_B = 28\ kN$
$R_A = 60\ kN$

$M = R_B x - 48\langle x-5\rangle - 40\langle x-3\rangle$
$EIy'' = 28x - 48\langle x-5\rangle - 40\langle x-3\rangle$
$EIy' = 14x^2 - 24\langle x-5\rangle^2 - 20\langle x-3\rangle^2 + C_1 \rightarrow$ ②
$EIy = 4.67x^3 - 8\langle x-5\rangle^3 - 6.67\langle x-3\rangle^3 + C_1x + C_2 \rightarrow$ ①

@ $x = 0,\ y = 0;\ C_2 = 0$
@ $x = 6,\ y = 0$
$0 = 4.67(6)^3 - 8(6-5)^3 - 6.67(6-3)^3 + C_1(6)$
$C_1 = -136.77$

SE: $EIy' = 14x^2 - 24\langle x-5\rangle^2 - 20\langle x-3\rangle^2 - 136.77\ (kN\text{-}m^2)$
DE: $EIy = 4.67x^3 - 8\langle x-5\rangle^3 - 6.67\langle x-3\rangle^3 - 136.77x\ \ (kN\cdot m^3)$

@ $x = 5m,\ \Delta@48 = ?$
$EI\Delta_{48} = 4.67(5)^3 - 8(5-5)^3 - 6.67(5-3)^3 - 136.77(5)$

$\Delta_{48} = \frac{-153.467}{EI} = \frac{-153.467(1000)^4}{(200\times10^3)(85\times10^6)}\left[\frac{kN\text{-}m^3\left(\frac{N}{kN}\right)\left(\frac{mm}{m}\right)^3}{(N/mm^2)(mm^4)}\right]$

$\Delta_{48} = -9.03\ mm$

@ $x = 3m$, $\Delta_{40} = ?$

$$EI\Delta_{40} = 4.67(3)^3 - 8(3-5)^3 - 6.67(3-3)^3 - 136.77(3)$$

$$\Delta_{40} = -16.72 \text{ mm}$$

Δ_{max} occurs at $\theta = 0$;
set $dy/dx = y' = 0$:

$$\therefore EIy' = 14x^2 - 24\langle x-5\rangle^2 - 20\langle x-3\rangle^2 - 136.77$$

* Take note, max moment results to max deflection and so, zero rotation. Based on the loaded beam, zero shear would manifest between 48 KN and 40KN or at 3 m to 5m. This is where maximum deflection occurs.

If we set between 3m and 5m:

$$EIy' = 14x^2 - \cancel{24\langle x-5\rangle^2} - 20\langle x-3\rangle^2 - 136.77$$

$$0 = 14x^2 - 20\langle x-3\rangle^2 - 136.77$$

$$0 = 14x^2 - 20(x^2 - 6x + 9) - 136.77$$

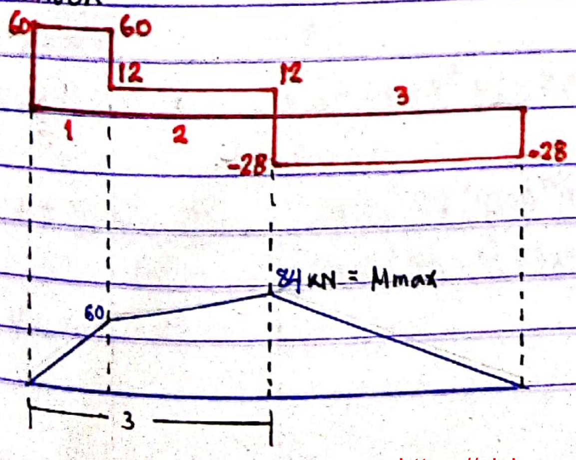
$$0 = 14x^2 - 20x^2 + 120x - 316.77$$

$x = 3.13$ from right support

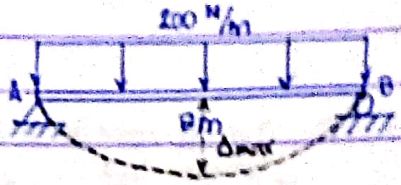
$$EI\Delta_{max} = 4.67(3.13)^3 - 8\langle 3.13-5\rangle^3 - 6.67\langle 3.13-3\rangle^3 - 136.77(3.13)$$

$$\Delta_{max} = -16.76 \text{ mm}$$

To check:



16. Determine the maximum deflection of the beam shown. Use DIM. $E = 200$ GPa. $B \times H = 250 \times 400$ mm.



Required: Δ_{max}

$$x = \frac{l}{2} = \frac{8m}{2} = 4m$$

from Case II: $EIy = \frac{wlx^3}{12} - \frac{wx^4}{12} - \frac{wl^3x}{24}$

where: w = uniform load = 200 N/m
x = location = 4m

$$EI\Delta_{max} = \frac{200(8)(4)^3}{12} - \frac{200(4)^4}{12} - \frac{200(8)^3(4)}{24}$$

$$EI\Delta_{max} = \frac{32000}{3EI} \quad (N\text{-}m^3)$$

$$\Delta_{max} = \frac{32000(1000)^3 \quad (N\text{-}m^3)\left(\frac{mm}{m}\right)^3}{3(200 \times 10^3 \text{ N/mm}^2)\left[\frac{1}{12}(250)(400)^3\right] mm^4}$$

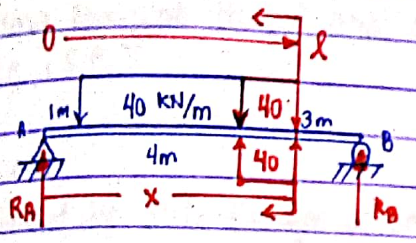
$$\Delta_{max} = -0.04 \text{ mm}$$

or just use:

$$y_{max} = \frac{5wl^4}{384EI} = \frac{5(200)(8)^4}{384EI}$$

y_{max} = 0.04mm downward

17. A beam of length 8m is simply supported at its ends. It carries a uniformly distributed load of 40 kN/m at shown. Determine the deflection of the beam at its midpoint and the position of maximum deflection and Δmax. Use DIM. Use $E = 200$ GPa, $I = 4.3 \times 10^8\ mm^4$.



Required: Δ_{4m}, Δ_{max}, & x_{max}

$\Sigma M_B = 0$; $R_A(8) = 40(4)(2+3)$

$R_A = 100$ kN

$R_B = 60$ kN

$EIy'' = M = 100(x) - 40\langle x-1\rangle + 40\langle x-5\rangle$ (kN-m)

$EIy' = 50x^2 - 20\langle x-1\rangle^2 + 20\langle x-5\rangle^2 + C_1$ (kN-m²)

$EIy = 16.67x^3 - 6.67\langle x-1\rangle^3 + 6.67\langle x-5\rangle^3 + C_1x + C_2$ (kN-m³)

@ $x=0$, $y=0$; $C_2 = 0$

@ $x=8$, $y=0$

$0 = 16.67(8)^3 - 6.67\langle 7\rangle^3 + 6.67\langle 3\rangle^3 + C_1(8)$

$C_1 = -803.415 = 2410/3$

@ $x=4$, $\Delta mid = ?$

$EIy_{mid} = 16.67(4)^3 - 6.67\langle 4-1\rangle^3 + 6.67\langle 4-5\rangle^3 - 803.415(4)$

$y_{mid} = \frac{-2326.87}{EI} = \frac{-2326.87(1000)^4(kNm^3)\left(\frac{N/mm}{kN/m}\right)^3}{(200\times10^3\ N/mm^2)(4.3\times10^8\ mm^4)}$

$y_{mid} = -27.06$ mm

* To solve for Δ_{max} and its location, set $dy/dx = 0$ and see where shear is zero or where Δmax deflection occurs.

* From the beam shown, $V=0$, Mmax between 1m to 5m.

$\therefore\ 1 < x < 5$

$EIy' = 50x^2 - 20\langle x-1\rangle^2 + \cancel{20\langle x-5\rangle^2} - 803.415$

$0 = 50x^2 - 20(x^2 - 2x + 1) - 803.415$

$0 = 50x^2 - 20x^2 + 40x - 20 - 803.415$

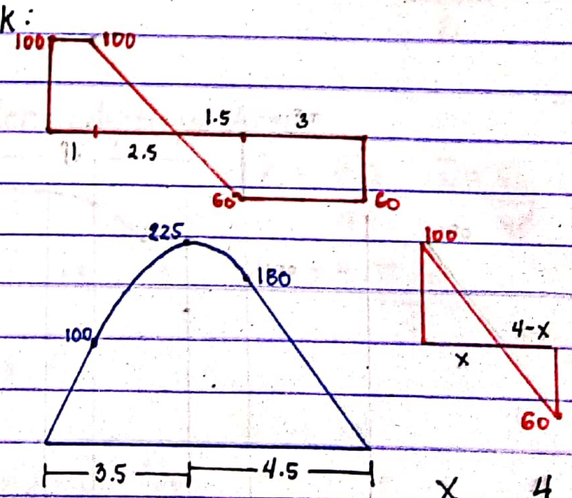
$x = 4.61$ m from left

$EIy_{max} = 16.67(4.61)^3 - 6.67(4.61-1)^3 + \cancel{6.67(4.61-5)^3} - 803.415(4.61)$

$y_{max} = \frac{-2384.35}{EI} = \frac{-2384.35(1000)^4(kN\text{-}m^3)\left(\frac{N}{kN}\right)\left(\frac{mm}{m}\right)^3}{(200\times10^3\ N/mm^2)(4.3\times10^8\ mm^4)}$

$y_{max} = -27.73$ mm

To check:



$\frac{x}{100} = \frac{4}{160}$

$x = 2.5$

Moment-Area or Area-Moment Method

* **Design Criteria** for the deflection of beams include:

1. Strength
   * It should be strong enough to resist the Bending Moment (B.M.) and Shear Force (S.F.)

2. Stiffness
   * It should be stiff enough to resist the deflection of beam.

* **Theorem I**: The change in slope between the tangents to the elastic curve at any two points is equal to the area under the M/EI diagram between the two points, provided that the elastic curve is continuous between the two points.

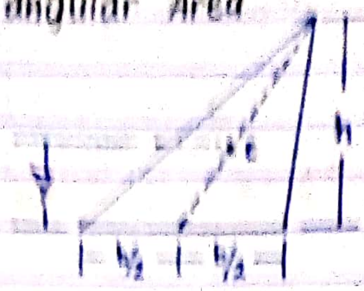
$$\theta = \frac{A}{EI}$$

* **Theorem II**: The tangential deviation in the direction perpendicular to the undeformed axis of the beam of a point on the elastic curve from the tangent to the elastic curve at another point is equal to the moment of the area under the M/EI diagram between the two points about the point at which the deviation is desired, provided that the elastic curve is continuous between the two points.

$$\Delta \text{ or } \delta = \frac{A\bar{x}}{EI}$$

Area and Centroid of Diagrams

1. Triangular Area

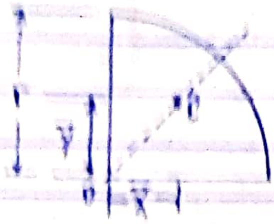


$\bar{x} = 0$

$\bar{y} = \frac{h}{3}$

$\text{Area} = \frac{bh}{3}$

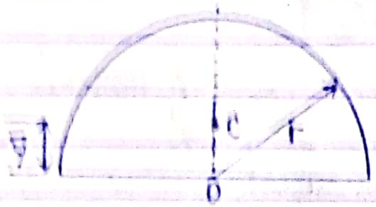
2. Quarter-Circular Area



$\bar{x} = \bar{y} = \frac{4r}{3\pi}$

$\text{Area} = \frac{\pi r^2}{4}$

3. Semicircular Area

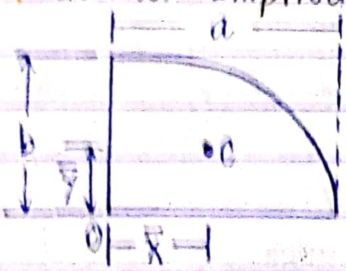


$\bar{x} = 0$

$\bar{y} = \frac{4r}{3\pi}$

$\text{Area} = \frac{\pi r^2}{2}$

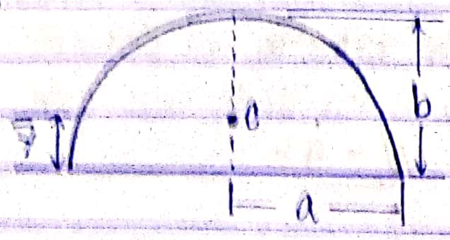
4. Quarter-Elliptical Area



$\bar{x} = \frac{4a}{3\pi}$ $\quad \bar{y} = \frac{4b}{3\pi}$

$\text{Area} = \frac{\pi ab}{4}$

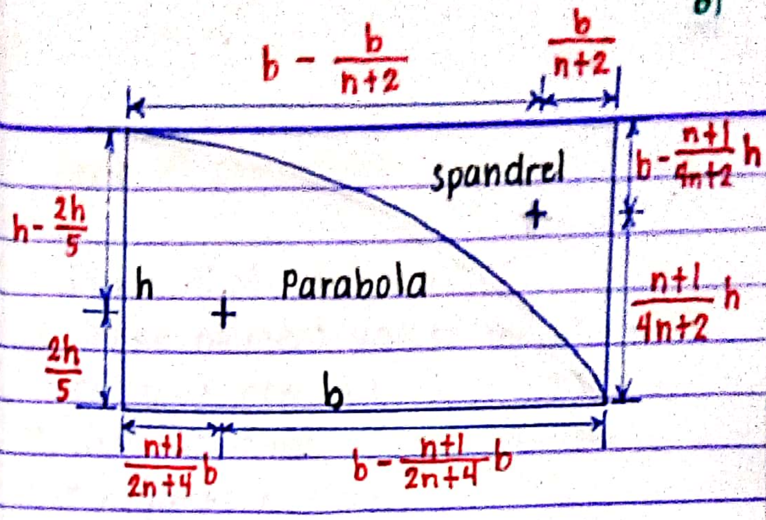
5. Semielliptical Area



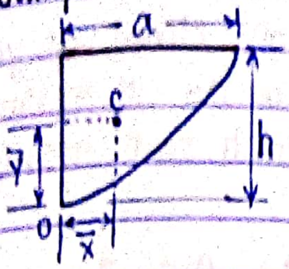
$\bar{x} = 0$

$\bar{y} = \frac{4b}{3\pi}$

$\text{Area} = \frac{\pi ab}{2}$



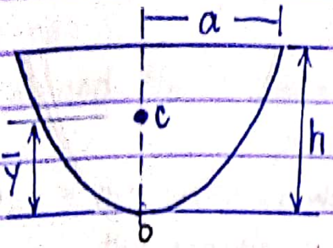
6. Semiparabolic Area



$\bar{x} = \dfrac{3a}{8}$ $\bar{y} = \dfrac{3h}{8}$

$\text{Area} = \dfrac{2ah}{3}$

7. Parabolic Area

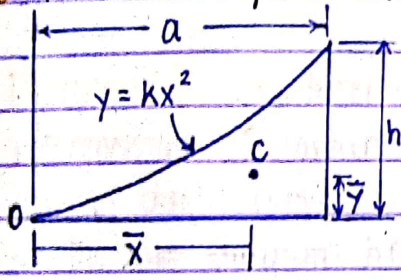


$\bar{x} = 0$

$\bar{y} = \dfrac{3h}{5}$

$\text{Area} = \dfrac{4ah}{3}$

8. Parabolic Spandrel



$\bar{x} = \dfrac{3a}{4}$

$\bar{y} = \dfrac{3h}{10}$

$\text{Area} = \dfrac{ah}{3}$

Spandrel & Parabola

Area of Spandrel:

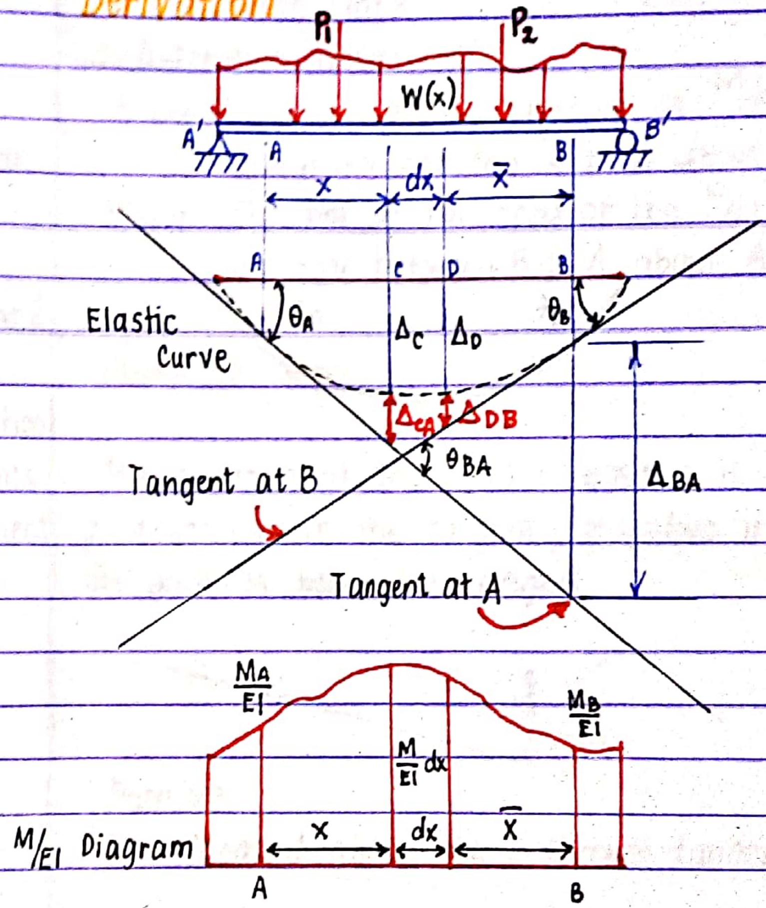
$$A_{spandrel} = \frac{1}{n+1} bh$$

Area of Parabola:

$$A_{parabola} = \frac{n}{n+1} bh$$

where: n - degree of the curve

Derivation



- To derive the moment-area theorems, consider a beam subjected to an arbitrary loading as shown.

- Focusing our attention on dx of the beam, we recall that dθ is given by:

$$d\theta = \frac{M}{EI} dx$$

Theorem I: The change on slope between any two points, on an elastic curve is equal to the net area of B.M. diagram between these points divided by EI.

$$\theta_{BA} = \theta_B - \theta_A = \int_A^B \frac{M}{EI}\,dx$$

where:

θ_A and θ_B = slopes of curve at points A and B

θ_{BA} = the angle between the tangents to the elastic curve at A and B

$\int_A^B \frac{M}{EI}\,dx$ = the area under the M/EI diagram between points A and B.

Theorem II: The intercept taken on a vertical reference line of tangents at any two points on an elastic curve, is equal to the moment of the B.M. diagram between these points about the reference line divided by EI.

$$\Delta_{BA} = \int_A^B \frac{M}{EI}\,\bar{x}\,dx$$

where:

Δ_{BA} = tangential deviation of B from the tangent at A; deflection of point B

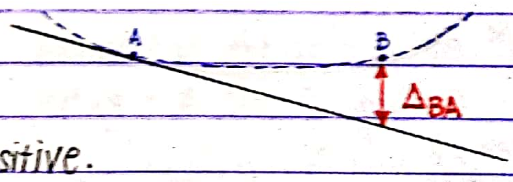
$\int_A^B \frac{M}{EI}\bar{x}\,dx$ = moment of the area under the M/EI diagram between points A and B about point B.

General Procedure:

1. Draw B.M. diagram
2. Divide moment values by EI and construct new B.M.D. if needed.
3. Construct Elastic Curve Diagram
   - negative moment:
   - positive moment:
4. Determine slope
5. Determine deflection
   - Δ_{BA}: moment of the area of the M/EI diagram between A to B about B
   - Δ_{AB}: moment of the area of the M/EI diagram between B to A about A.

Rules of Sign

- The deviation at any point is positive if the point lies above the tangent, negative if the point is below the tangent.

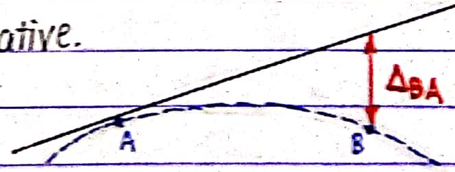


Positive.

B is located above the reference tangent

B is located below the reference tangent

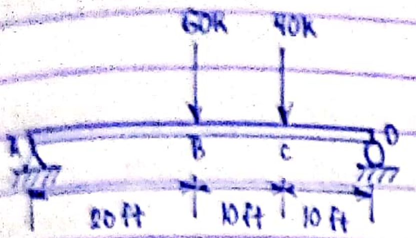
Negative.



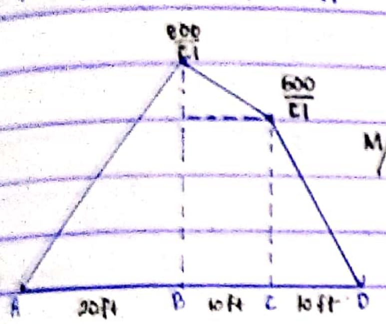
- Δ_{BA} or $t_{B/A}$ tangential deviation of B from the tangent at A.

Sample Problem:

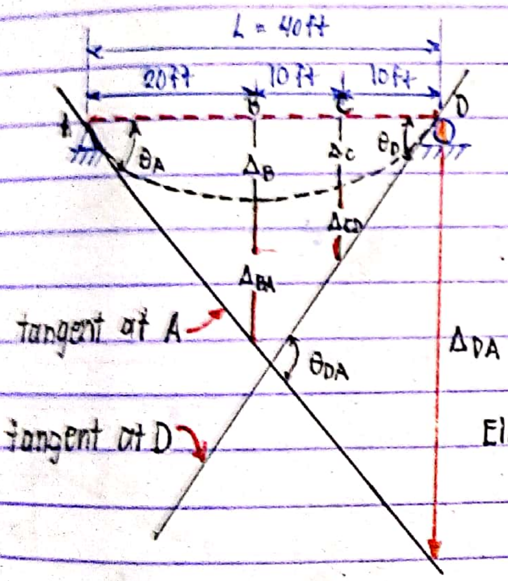
Use the moment-area or area-moment method to find Δ_B, Δ_C, θ_A, and θ_D.



EI = constant
E = 1800 ksi
I = 46,000 in^4



M/EI Diagram
(k-ft/EI)



Elastic Curve

θ_A = too small ; $\therefore \tan\theta_A = \theta_A$

$$\theta_A = \frac{\Delta_{DA}}{L}$$

Δ_{DA} = moment of the area of the M/EI diagram between A and D about D

$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(800)(20)\left(\frac{20}{3}+20\right)+\frac{1}{2}(200)(10)\left(\frac{20}{3}+10\right) + 600(10)(15) + \frac{1}{2}(600)(10)\left(\frac{20}{3}\right)\right]$$

$$\Delta_{DA} = \frac{340{,}000 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{L} = \frac{340{,}000/EI}{40} = \frac{8{,}500 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{8{,}500\,(12)^2}{(1800)(46{,}000)} = \underline{0.015 \text{ rad}}$$

θ_{DA} = area of the M/EI diagram between A and D

$$\theta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(800)(20)+\frac{1}{2}(200)(10)+600(10)+\frac{1}{2}(600)(10)\right]$$

$$\theta_{DA} = \frac{18{,}000 \text{ k-ft}^3}{EI}$$

$$\theta_D = \theta_{DA} - \theta_A$$

$$\theta_D = \frac{18{,}000}{EI} - \frac{8{,}500}{EI} = \frac{9{,}500 \text{ k-ft}^3}{EI}$$

$$\theta_D = \frac{9500\,(12)^2}{(1800)(46{,}000)} = \underline{0.017 \text{ rad}}$$

Δ_{BA} = moment of the area of the M/EI diagram between A and B about B

$$\Delta_{BA} = \frac{1}{EI}\left[\frac{1}{2}(800)(20)\left(\frac{20}{3}\right)\right] = \frac{53{,}333.33 \text{ k-ft}^3}{EI}$$

$$\Delta_B = 20\,\theta_A - \Delta_{BA}$$

$$\Delta_B = 20\left(\frac{8{,}500}{EI}\right) - \frac{53{,}333.33}{EI} = \frac{116{,}666.67 \text{ k-ft}^3}{EI}$$

$$\Delta_B = \frac{116{,}666.67\,(12)^3}{(1{,}800)(46{,}000)} = \underline{2.43 \text{ in} \downarrow}$$

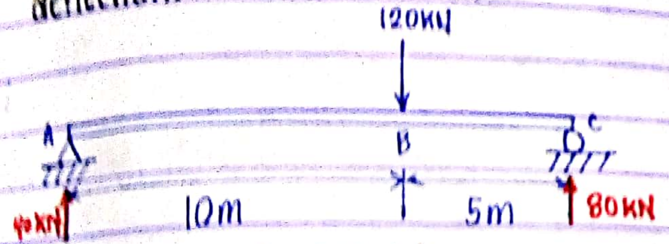
$$\Delta_{CD} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{10}{3}\right)\right] = \frac{10{,}000 \text{ k-ft}^3}{EI}$$

$\Delta_c = 10\,\theta_D - \Delta_{cD}$

$$\Delta_c = 10\left(\frac{9{,}500}{EI}\right) - \frac{10{,}000}{EI} = \frac{85{,}000 \text{ k-ft}^3}{EI}$$

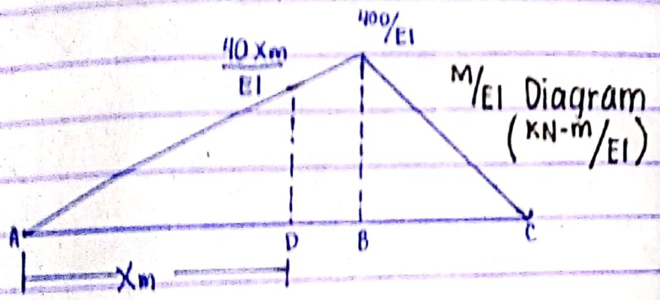
$$\Delta_c = \frac{85{,}000\,(12)^3}{(1{,}000)(46{,}000)} = 1.77 \text{ in} \downarrow$$

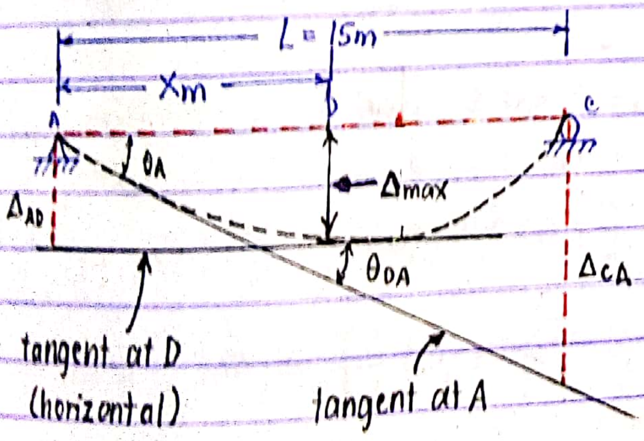
19. Use Moment-Area or Area-Moment method to determine the maximum deflection.



EI = constant : E = 200 GPa

$I = 700 \times 10^6 \text{ mm}^4$





Δ_{CA} = moment of the area of the M/EI diagram between A and C about C

$$\Delta_{CA} = \frac{1}{EI}\left[\frac{1}{2}(400)(10)\left(\frac{10}{3}+5\right) + \frac{1}{2}(400)(5)\left(\frac{10}{3}\right)\right]$$

$$\Delta_{CA} = \frac{20{,}000 \text{ KN-m}^3}{EI}$$

$$\theta_A = \frac{\Delta_{CA}}{L} = \frac{20{,}000/EI}{15} = \frac{1{,}333.33 \text{ kN-m}^2}{EI}$$

$$\theta_{DA} = \theta_A = \frac{1{,}333.33 \text{ kN-m}^2}{EI}$$

θ_{DA} = area of the M/EI diagram between A and D

$$\theta_{DA} = \frac{1}{2}\left(\frac{40X_m}{EI}\right)X_m = \frac{1{,}333.33 \text{ kN-m}^2}{EI}$$

$X_m = 8.16 \text{ m}$

Δ_{AD} = moment of the area of the M/EI diagram between A and D about A

$$\Delta_{AD} = \frac{1}{2}\frac{(40)(8.16)}{EI}(8.16)\left(\frac{2}{3}\right)(8.16)$$

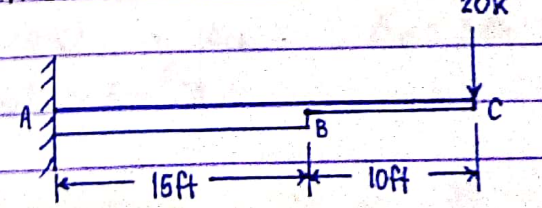
$$\Delta_{AD} = \frac{7{,}244.51 \text{ kN-m}^3}{EI}$$

$$\Delta_{max} = \Delta_{AD} = \frac{7{,}244.51 \text{ kN-m}^3}{EI}$$

$$\Delta_{max} = \frac{7{,}244.51 \text{ kN-m}^3}{(200\times10^6 \text{ kN/m}^2)(700\times10^{-6} \text{ m}^4)}$$

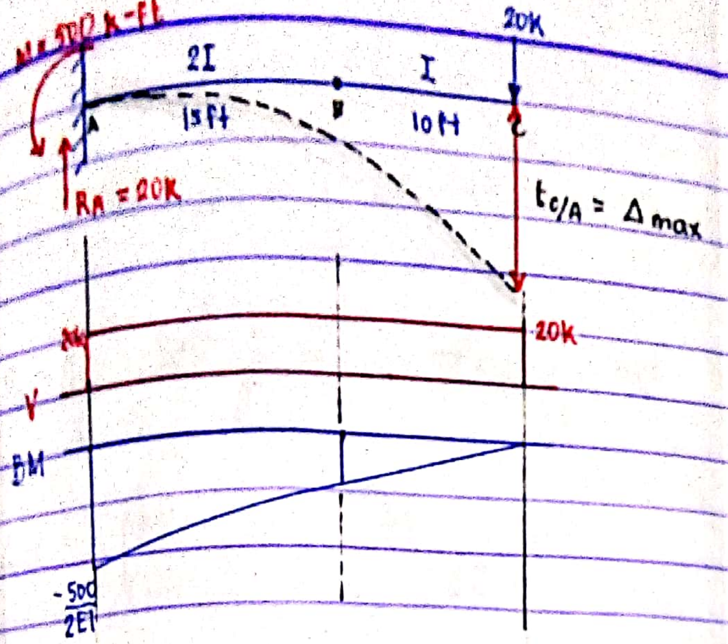
$\Delta_{max} = 0.0517 \text{ m} = 51.7 \text{ m} \downarrow$

20. Use moment-area or area-moment method to determine the maximum deflection.

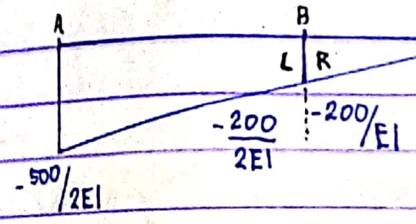


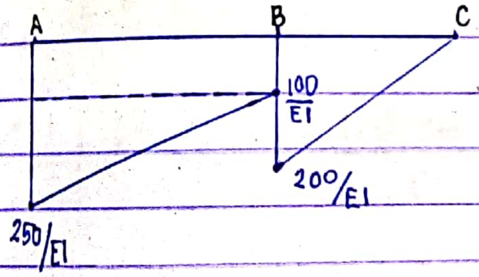
$I = 6{,}000 \text{ in}^4 \quad I = 3{,}000 \text{ in}^4$

$E = 29{,}000 \text{ Ksi}$



Corrected Bending Moment:





Δ_{CA} = moment of the area of the M/EI diagram between A and C about C

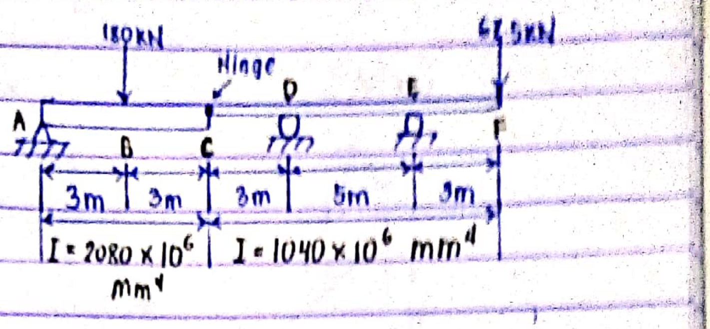
$$t_{C/A} = \frac{1}{EI}\left[\frac{1}{2}(200)(10)\left(\frac{2}{3}\right)(10) + (100)(15)\left(\frac{15}{2}+10\right) + \frac{1}{2}(150)(15)\left(\frac{2}{3}(15)+10\right)\right]$$

$$t_{C/A} = \Delta_{CA} = \frac{166250}{3EI} \quad (k\text{-}ft^3)$$

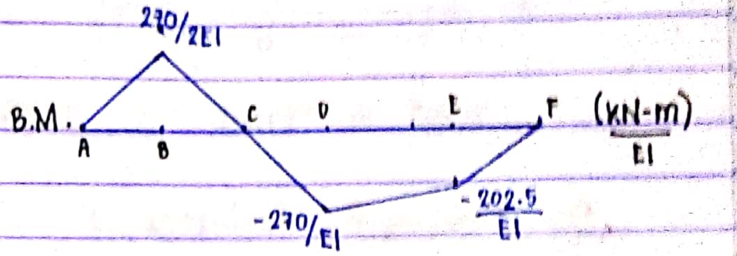
$$\Delta_{max} = \frac{166250\,(12)^3\,(k\text{-}ft^3)\left(\frac{in}{ft}\right)^3}{3\,(3000\,in^4)(29{,}000\,ksi)}$$

$$\Delta_{max} = 1.10 \text{ in.}$$

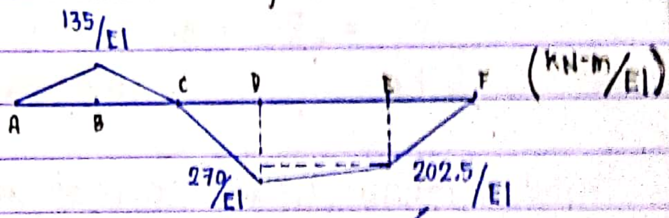
21. Use the moment-area method to determine the slope at point A and the deflection at point C of the beam shown.

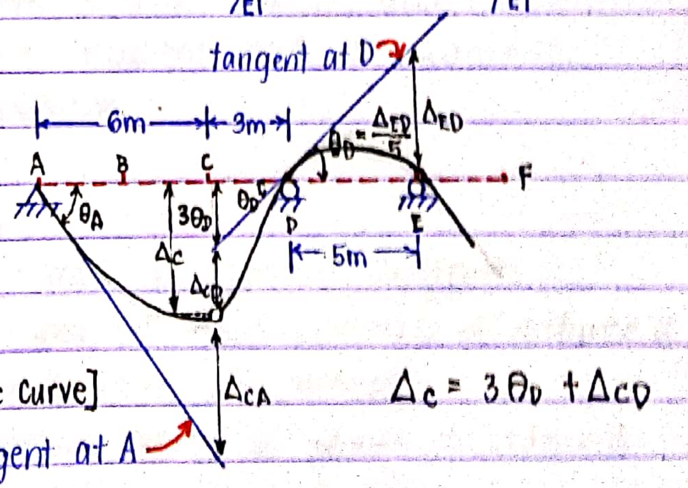


E = 200 GPa



Corrected M/EI Diagram:





[Elastic Curve]

$$\Delta_C = 3\theta_D + \Delta_{CD}$$

θ_D = too small ; $\tan\theta_D \approx \theta_D$

$$\theta_D = \frac{\Delta_{ED}}{5m}$$

$$\Delta_{ED} = \frac{1}{EI}\left[202.5(5)(2.5) + \frac{1}{2}(67.5)(5)\left(\frac{10}{3}\right)\right]$$

$$\Delta_{ED} = \frac{3093.75\ kN\text{-}m^3}{EI}$$

$$\theta_D = \frac{\Delta_{ED}}{5} = \frac{3093.75/EI}{5} = \frac{618.75\ kN\text{-}m^2}{EI}$$

$$\Delta_{CD} = \frac{1}{2}\left(\frac{270}{EI}\right)(3)(2) = \frac{810\ kN\text{-}m^3}{EI}$$

$$\Delta_c = 3\theta_D + \Delta_{CD}$$

$$\Delta_c = 3\left(\frac{618.75}{EI}\right) + \frac{810}{EI} = \frac{2666.25\ kN\text{-}m^3}{EI}$$

$$\Delta_c = \frac{2666.25\ kN\text{-}m^3}{(200\times10^6\ kN/m^2)(1040\times10^{-6}\ m^4)}$$

$$\Delta_c = 12.8\ mm \downarrow$$

$$\Delta_{CA} = \frac{1}{2}\left(\frac{135}{EI}\right)(6)(3) = \frac{1215\ kN\text{-}m^3}{EI}$$

$$\theta_A = \frac{\Delta_c + \Delta_{CA}}{6} = \frac{1}{6}\left(\frac{2666.25}{EI} + \frac{1215}{EI}\right)$$

$$\theta_A = \frac{646.875\ kN\text{-}m^2}{EI}$$

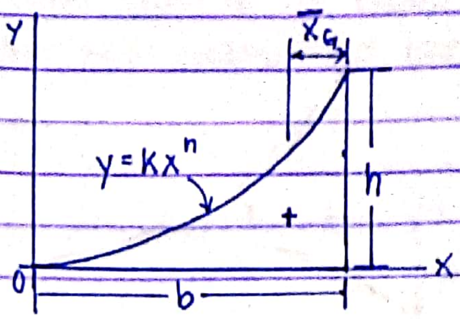
$$\theta_A = \frac{646.875\ kN\text{-}m^2}{(200\times10^6\ kN/m^2)(1040\times10^{-6}\ m^4)}$$

$$\theta_A = 0.003\ rad$$

Bending Moment Diagram by Parts

Basic Principles:

1. $M = (\Sigma M_L) = (\Sigma M_R)$

2. 

where: n = degree of curve

Area of Spandrel: $A_{spandrel} = \dfrac{bh}{n+1}$

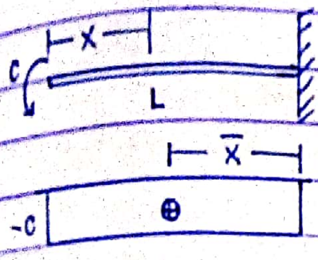
Centroid of Spandrel (location): $\bar{x}_G = \dfrac{b}{n+2}$

* Take each point load, uniform load, and moment load as cantilever loadings.

where:
- A = area of moment diagram
- M_x = moment about a section of distance x
- $\bar{x}$ = location of centroid
- degree = degree of power of moment diagram

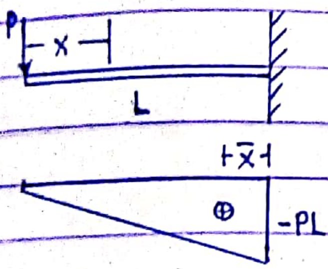
Common Loads :

1. couple or Moment Load



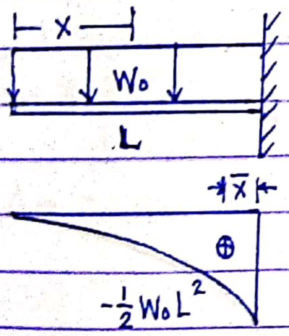
$$A = -CL$$
$$M_x = -C$$
$$\bar{x} = \frac{L}{2}$$
degree = 0

2. concentrated Load



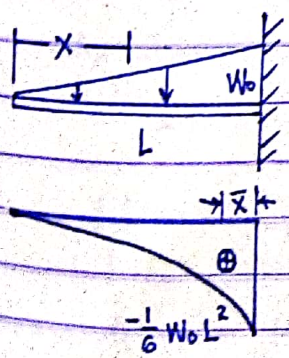
$$A = -\frac{PL^2}{2}$$
$$M_x = -Px$$
$$\bar{x} = \frac{L}{3}$$
degree = 1

3. Uniformly Distributed Load



$$A = -\frac{1}{6} W_0 L^3$$
$$M_x = -\frac{1}{2} W_0 x^2$$
$$\bar{x} = \frac{L}{4}$$
degree = 2

4. Uniformly Varying Load

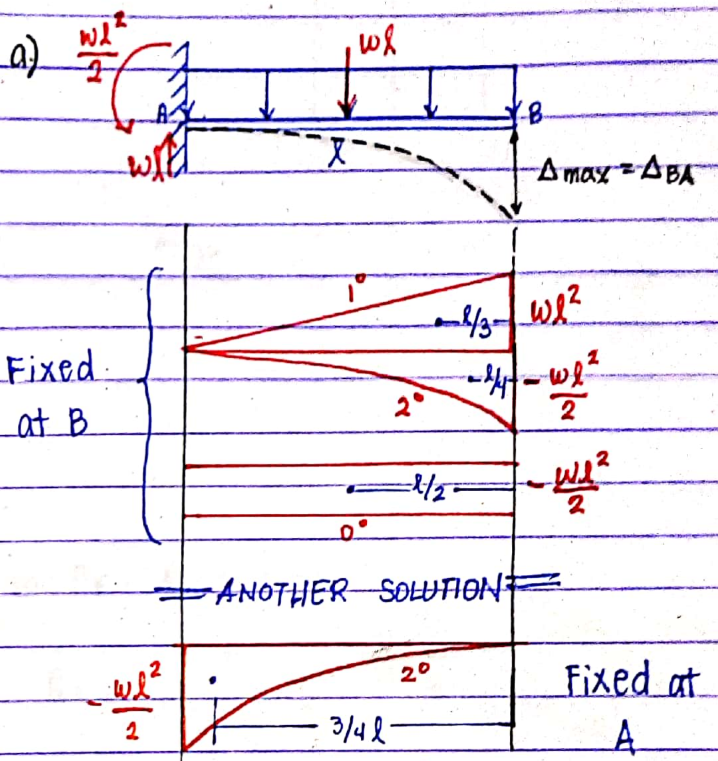


$$A = -\frac{1}{24} W_0 L^3$$
$$M_x = -\frac{W_0}{6L} x^2$$
$$\bar{x} = \frac{L}{5}$$
degree = 3

Sample Problem

22. Determine the free end deflection of a cantilever beam carrying a
 a] uniformly distributed load over the entire length of the beam
 b] free end concentrated load

a)



$$\Delta_{BA} = \Delta_{max} = A_{AB}\,\bar{x}_B / EI$$

$$\Delta_{BA} = \frac{1}{EI}\left[\frac{l}{2}(wl^2)\left(\frac{l}{3}\right) - \frac{l}{3}\left(\frac{wl^2}{2}\right)\left(\frac{l}{4}\right) - (l)\left(\frac{wl^2}{2}\right)\left(\frac{l}{2}\right)\right]$$

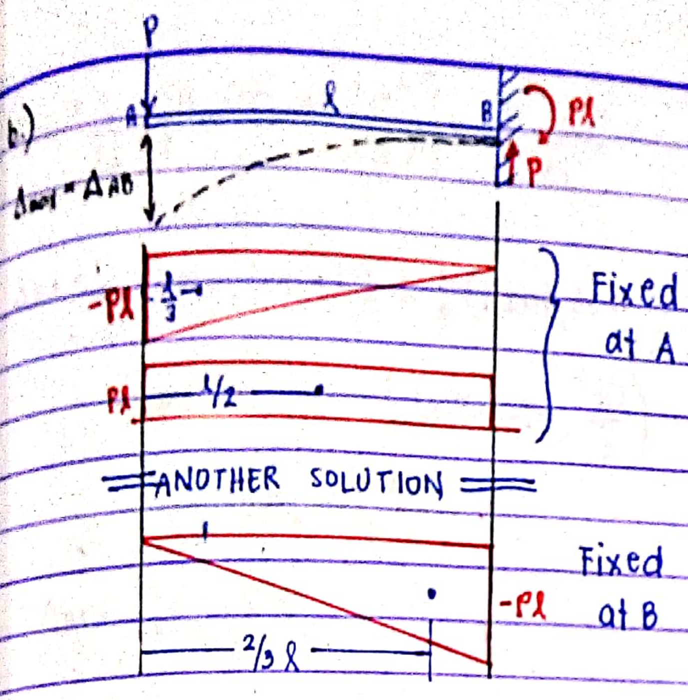
$$\Delta_{BA} = \left[\frac{1}{6} - \frac{1}{24} - \frac{1}{4}\right]\frac{wl^4}{EI} = \frac{-wl^4}{8EI}$$

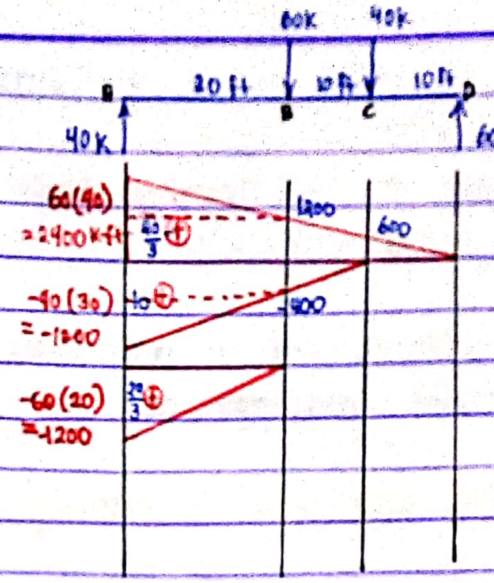
Another Solution:

$$\Delta_{max} = \frac{A_{AB}\,\bar{x}_B}{EI} = \frac{1}{EI}\left[\frac{l}{3}\left(\frac{wl^2}{2}\right)\left(\frac{3l}{4}\right)\right]$$

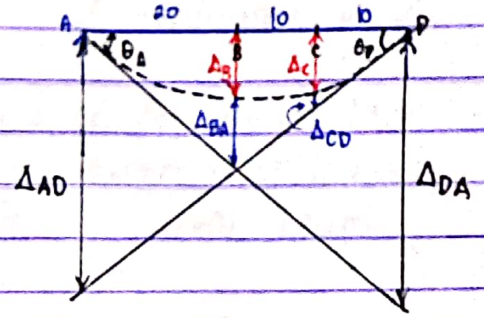
$$\Delta_{max} = \frac{wl^4}{8EI}$$

b)





Find :
Δ_B
Δ_C
θ_A
θ_D



$$\Delta_{max} = \Delta_{AB} = (Area)_{AB}\ \bar{X}_A / EI$$

$$\Delta_{AB} = \frac{1}{EI}\left[\frac{\ell}{2}(P\ell)\left(-\frac{\ell}{3}\right) + \ell(P\ell)\left(\frac{\ell}{2}\right)\right]$$

$$\Delta_{AB} = \left[-\frac{1}{6} + \frac{1}{2}\right]\frac{P\ell^3}{EI} = \frac{P\ell^3}{3EI}$$

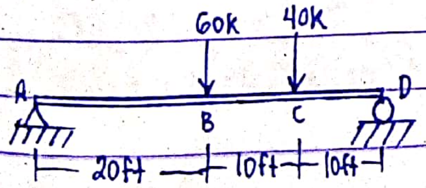
Another Solution :

$$\Delta_{AB} = \Delta_{max} = (Area)_{AB}\ \bar{X}_A / EI$$

$$\Delta_{AB} = \frac{1}{EI}\left[\frac{\ell}{2}(P\ell)\left(\frac{2}{3}\ell\right)\right]$$

$$\Delta_{AB} = \Delta_{max} = \frac{P\ell^3}{3EI}$$

23. Use moment diagram by parts to find Δ_B, Δ_C, θ_A, and θ_D.



EI = constant
E = 1,800 ksi
I = 46,000 in^4

$$\tan\theta_A = \theta_A = \frac{\Delta_{DA}}{40} \ ; \quad \theta_D = \frac{\Delta_{AD}}{40}$$

$$\theta_A = \frac{\Delta_{BA} + \Delta_B}{20} \qquad \theta_D = \frac{\Delta_{CD} + \Delta_C}{10}$$

$$\Delta_{AD} = \frac{1}{EI}\left[\frac{1}{2}(2400)(40)\left(\frac{40}{3}\right) - \frac{1}{2}(1200)(30)(10) - \frac{1}{2}(1200)(20)\left(\frac{20}{3}\right)\right]$$

$$\Delta_{AD} = \frac{380,000}{EI}$$

$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(2400)(40)\left(\frac{80}{3}\right) - \frac{1}{2}(1200)(30)(30) - \frac{1}{2}(1200)(20)\left(20 + \frac{40}{3}\right)\right]$$

$$\Delta_{DA} = 340,000 / EI$$

$$\Delta_{CD} = \frac{1}{EI}\left[\frac{1}{2}(10)(600)\left(\frac{10}{3}\right)\right] = \frac{10,000}{EI}$$

$$\Delta_{BA} = \frac{1}{EI}\left[(1200)(20)(10) + \frac{1}{2}(1200)(20)\left(\frac{40}{3}\right) - (400)(20)(10) - \frac{1}{2}(800)(20)\left(\frac{40}{3}\right) - \frac{1}{2}(1200)(20)\left(\frac{40}{3}\right)\right]$$

$$\Delta_{BA} = \frac{160,000}{3EI}$$

$$\theta_A = \frac{340,000/EI}{40} = \frac{8500}{EI} \; (k\text{-}ft^2)$$

$$\theta_A = \frac{8500 \; k\text{-}ft^2 \, (12)^2 (in/ft)^2}{(1800 \; ksi)(46,000 \; in^4)}$$

$$\theta_A = 0.015 \; rad$$

$$\theta_D = \frac{380,000/EI}{40} = \frac{9500}{EI}$$

$$\theta_D = \frac{9500 \; k\text{-}ft^2 \, (12)^2 (in/ft)^2}{(1800 \; ksi)(46,000 \; in^4)}$$

$$\theta_D = 0.017 \; rad$$

$$\Delta_B = 20\,\theta_A - \Delta_{BA}$$

$$\Delta_B = 20\left[\frac{8500}{EI}\right] - \left[\frac{160,000}{3EI}\right] = \frac{350,000}{3EI}$$

$$\Delta_B = \frac{350,000 \; k\text{-}ft^3 \, (12)^3}{3(1800 \; ksi)(46,000 \; in^4)} = 2.43 \; in \downarrow$$

$$\Delta_C = 10\,\theta_D - \Delta_{CD}$$

$$\Delta_C = 10\left[\frac{9500}{EI}\right] - \frac{10,000}{EI} = \frac{85,000}{EI}$$

$$\Delta_C = \frac{85000 \; k\text{-}ft^3 \, (12)^3}{(1800 \; ksi)(46000 \; in^4)} = 1.77 \; in \downarrow$$

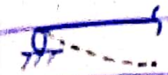
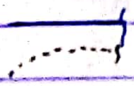
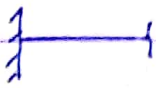
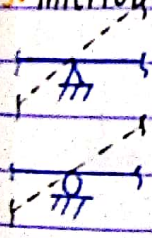
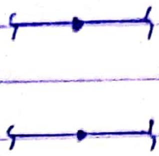
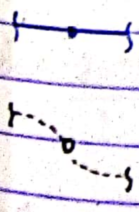
Conjugate Beam Method

- slope on realbeam = shear on conjugate beam
- deflection on real beam = moment on conjugate beam

Properties of Conjugate Beam

1. length of actual beam = length of conjugate beam
2. load on conjugate beam = $\frac{M}{EI}$ diagram of loads in real beam
3. simple support remains same in actual beam & conjugate beam
4. fixed end in real beam = free end in conjugate beam
5. point of zero shear in conjugate beam = point of zero slope in real beam
6. point of max moment in conjugate beam = point of max deflection in real beam

Table of Supports:

Real Beam		Conjugate Beam	
1. Hinge Support 	$\theta \neq 0$, $\Delta = 0$	Hinge Support 	$S \neq 0$, $M = 0$
2. Roller Support 	$\theta \neq 0$, $\Delta = 0$	Roller Support 	$S \neq 0$, $M = 0$
3. Fixed Support 	$\theta = 0$, $\Delta = 0$	Free End 	$S = 0$, $M = 0$
4. Free End 	$\theta \neq 0$, $\Delta \neq 0$	Fixed Support 	$S \neq 0$, $M \neq 0$
5. Interior Support 	$\theta \neq 0$ and continuous, $\Delta = 0$	Internal Hinge 	$S \neq 0$ and continuous, $M = 0$
6. Internal Hinge 	$\theta \neq$ and discontinuous, $\Delta \neq 0$	Interior Support 	$S \neq 0$ and discontinuous, $M \neq 0$

Sign Convention:

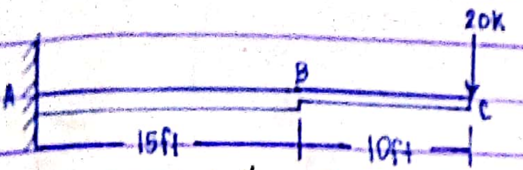
- (+) M/EI diagram of real beam = (+) loads on conjugate beam
- (+) shear in conjugate beam = (+) (↺) slope of real beam
- (+) bending moment in conjugate beam = (+) deflection of real beam.

Procedure for Analysis:

1. Construct the M/EI diagram for the real beam subjected to real loading.
2. Determine the conjugate beam corresponding to the given real beam.
3. Apply the M/EI diagram as load on conjugate beam.
4. Calculate the reactions at the supports of the conjugate beam by applying the equations of equilibrium and condition (if any).
5. Determine the shears and bending moments at those points on the conjugate beam where slopes and deflections are desired on the real beam.

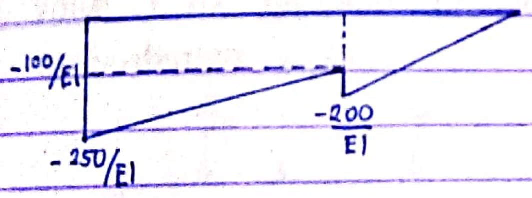
Sample Problem:

24. Determine the slopes and deflections at points B and C of the cantilever beam shown by the conjugate beam method

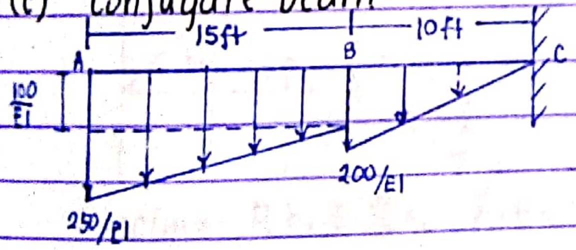


(a) Real Beam

(b) M/EI Diagram ($k\text{-}ft/EI$ with $I = 3000\ in^4$)



(c) Conjugate Beam



$$+\uparrow S_B = \frac{1}{EI}\left[-100(15) - \frac{1}{2}(150)(15)\right] = \frac{-2625\ k\text{-}ft^2}{EI}$$

$$\theta_B = \frac{-2625}{EI} = \frac{2625\ (12)^2}{(29{,}000)(3{,}000)} = -0.0043\ rad$$

$$+\circlearrowright M_B = \frac{1}{EI}\left[-100(15)(7.5) - \frac{1}{2}(150)(15)(10)\right] = \frac{-22500\ k\text{-}ft^3}{EI}$$

$$\Delta_B = \frac{-22500}{EI} = \frac{-22500\ (12)^3}{(29{,}000)(3{,}000)} = 0.45\ in \downarrow$$

$$+\uparrow S_C = \frac{1}{EI}\left[-100(15) - \frac{1}{2}(150)(15) - \frac{1}{2}(200)(10)\right] = \frac{3625\ k\text{-}ft^2}{EI}$$

$$\theta_C = \frac{-3625\ k\text{-}ft^2}{EI} = \frac{-3625\ (12)^2}{(29{,}000)(3{,}000)} = 0.006\ rad$$

$$+\circlearrowright M_C = \frac{1}{EI}\left[-100(15)(17.5) - \frac{1}{2}(150)(15)(20) - \frac{1}{2}(200)(10)(6.67)\right] = \frac{-55420\ k\text{-}ft^3}{EI}$$

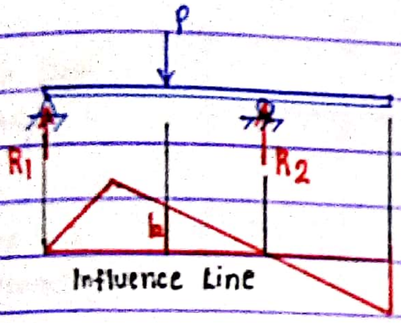
$$\Delta_C = \frac{-55420}{EI} = \frac{55420\ (12)^3}{(29000)(3000)} = 1.1\ in \downarrow$$

Course Outcome 4

Influence Lines

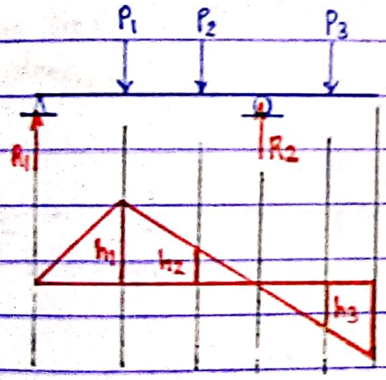
Properties of Influence Lines:

1. Value of the function due to single concentrated load



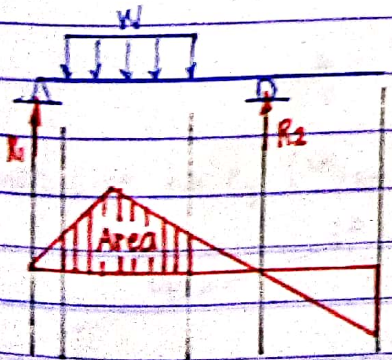
Function = Ph

2. Value of the function due to multiple concentrated loads



Function = $P_1 h_1 + P_2 h_2 - P_3 h_3$

3. Value of function due to uniformly distributed load



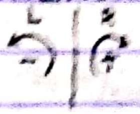
Function = w (Area)

Conversions:

1. Reaction Influence Line
   R.I.L. = push ↑

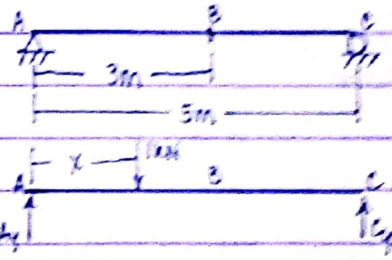
2. Shear Influence Line
   S.I.L. =

3. Moment Influence Line
   M.I.L. =

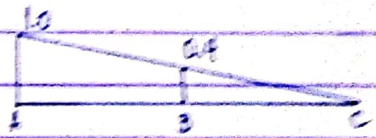


Examples:

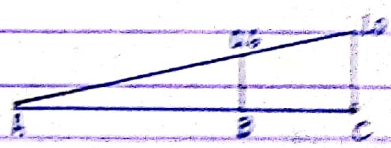
1.



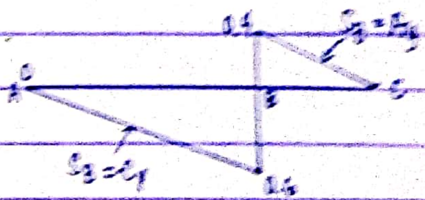
Influence Line for Ay (kN/kN)



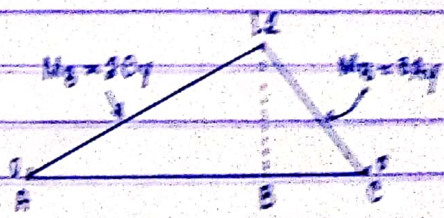
Influence Line for Cy (kN/kN)



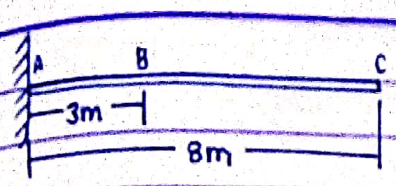
Influence Line for SB (kN/kN)

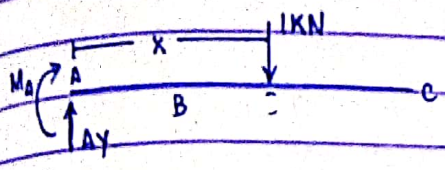


Influence Line for MB (kN-m/kN)

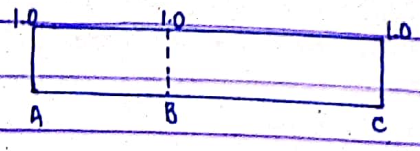


2.

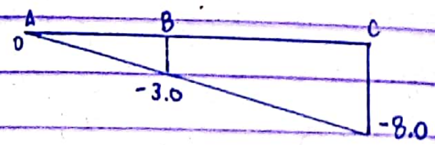




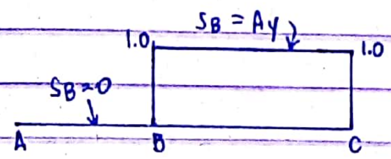
Influence Line for Ay (KN/KN)



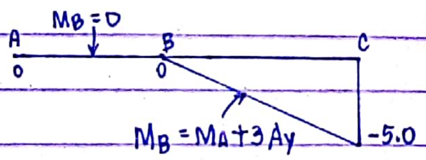
Influence Line for MA (KN·m/KN)



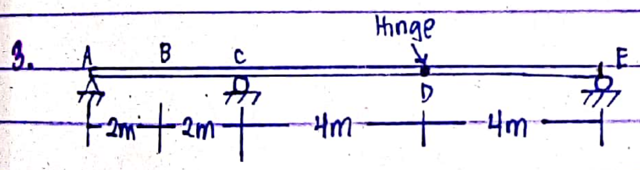
Influence Line for SB (KN/KN)

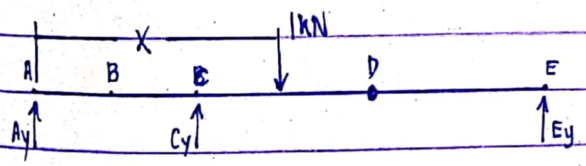


Influence Line for MB (KN-m/KN)

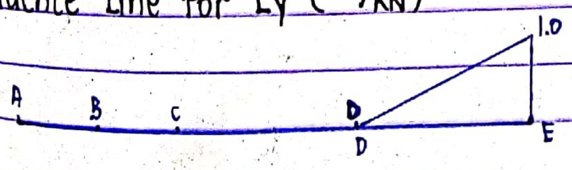


3.

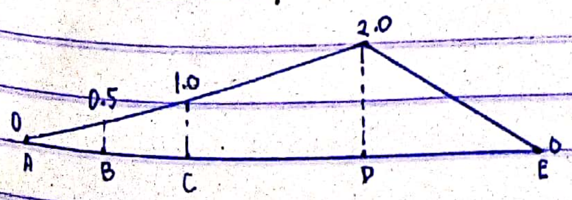




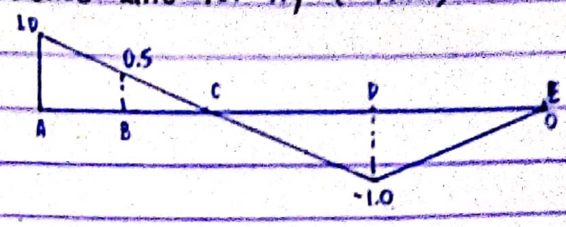
Influence Line for Ey (KN/KN)



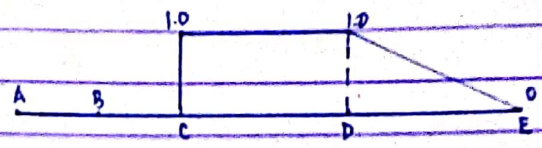
Influence Line for Cy (KN/KN)



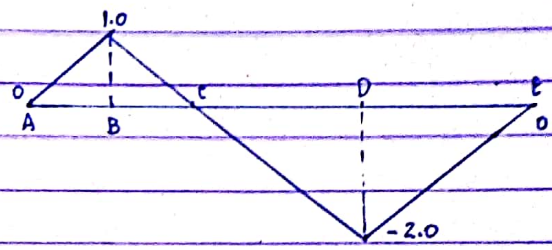
Influence Line for Ay (KN/KN)



Influence Line for SCR (KN/KN)



Influence Line for MB (KN-m/KN)



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