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Hydraulics

Part: Open Channel Flow

Tutorial solutions

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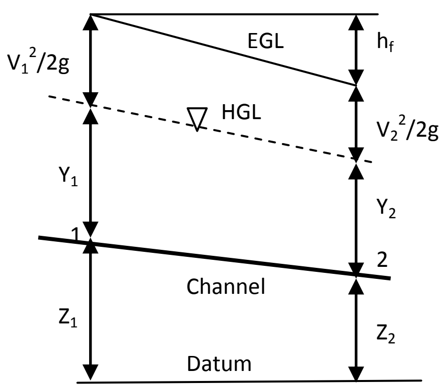
Uniform flow

1. Show that discharge through a channel with steady flow is given by

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(h - h_f\right)}$$

where A_1 and A_2 are the sectional areas of flow at the section 1 and 2 and h is the drop in water surface level between the sections and h_f is the loss of energy head between two sections.

Solution:



Applying Bernoulli's equation between points 1 and 2

$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + +h_f$

$[(Z_1 + y_1) - (Z_2 + y_2)] - h_f = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

$h - h_f = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

$V_2^2 - V_1^2 = 2g\left(h - h_f\right)$ (a)

From continuity equation

$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1}{A_2} V_1$

Substituting V_2 in a

$V_1^2 \left(\frac{A_1^2 - A_2^2}{A_2^2} \right) = 2g(h - h_f)$

$V_1 = \frac{A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g(h - h_f)}$

$Q = A_1 V_1$

$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g(h - h_f)}$

2. The velocity distribution in a wide river of 3m deep (y_0 =3m) is given by

$$v = 1 + 2\left(\frac{y}{y_0}\right)^{1/2}$$

Find α and β.

Solution:

$V = \frac{1}{A} \int_0^{y_0} v dA = \frac{1}{b y_0} \int_0^{y_0} v b dy$

$= \frac{1}{y_0} \int_0^{y_0} v dy = \frac{1}{3} \int_0^3 \left[1 + 2\left(\frac{y}{y_0}\right)^{1/2} \right] dy$

$= \frac{1}{3} \left[y + \frac{4}{3} \frac{y^{3/2}}{3^{1/2}} \right]_0^3 = \frac{7}{3}$

$\alpha = \int_0^{y_0} \frac{v^3 dA}{V^3 A} = \int_0^{y_0} \frac{v^3 b dy}{V^3 b y_0}$

$= \int_0^{y_0} \frac{v^3 dy}{V^3 y_0} = \int_0^3 \frac{\left[1 + 2\left(\frac{y}{y_0}\right)^{1/2} \right]^3 dy}{(7/3)^3 x 3}$

$= \frac{9}{343} \left[1 + 6\left(\frac{y}{y_0}\right)^{1/2} + 12 \frac{y}{y_0} + 8\left(\frac{y}{y_0}\right)^{3/2} \right] dy$

$= \frac{9}{343} \left[y + 4 \frac{y^{3/2}}{3^{1/2}} + 12 \frac{y^2}{6} + \frac{16}{5} \frac{y^{5/2}}{3^{3/2}} \right]_0^3$ = 1.118

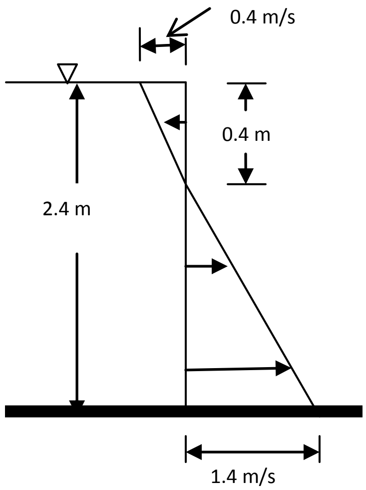
$\alpha = \int_0^{y_0} \frac{v^2 dA}{V^2 A} = \int_0^{y_0} \frac{v^2 b dy}{V^2 b y_0}$

$= \int_0^{y_0} \frac{v^2 dy}{V^2 y_0} = \int_0^3 \frac{\left[1 + 2\left(\frac{y}{y_0}\right)^{1/2} \right]^2 dy}{(7/3)^2 x 3}$

$= \frac{3}{49} \left[1 + 4\left(\frac{y}{y_0}\right)^{1/2} + 4 \frac{y}{y_0} \right] dy$

$= \frac{3}{49} \left[y + \frac{8}{3} \frac{y^{3/2}}{3^{1/2}} + 4 \frac{y^2}{6} \right]_0^3$ = 1.04

3. For the velocity distribution given in the figure below, find energy and momentum correction factors.



Solution:

y_0 = 2.4m

Mean velocity $(V) = \frac{1}{y_0}\int_0^{y_0} v dy = \frac{Area\ under\ v-y\ curve}{y_0}$

$V = \frac{\frac{1}{2}x1.4x2-\frac{1}{2}x0.4x0.4}{2.4}$ = 0.55 m/s

$\alpha = \frac{\int_0^{y_0} v^3 dy}{V^3 y_0} = \frac{Area\ under\ v^3-y\ curve}{V^3 y_0}$

$\alpha = \frac{\frac{1}{2}x(1.4^3)x2-\frac{1}{2}x(0.4^3)x0.4}{0.55^3 x2.4}$ = 6.84

$\beta = \frac{\int_0^{y_0} v^2 dy}{V^2 y_0} = \frac{Area\ under\ v^2-y\ curve}{V^2 y_0}$

$\beta = \frac{\frac{1}{2}x(1.4^2)x2-\frac{1}{2}x(0.4^2)x0.4}{0.55^2 x2.4}$ = 2.65

4. The velocity profile in a circular pipe is given by $\frac{v}{V_{max}} = \left(1 - \frac{r^2}{R^2}\right)$, where v = velocity at any radius r, v_{max} = maximum velocity and R = radius of pipe. Find average velocity, α and β.

Solution:

Mean velocity (V)=$\frac{\int vdA}{A} = \frac{\int_0^R v(2\pi r)dr}{\pi R^2}$

(Considering an elementary strip of thickness dr at a radial distance r from pipe center.)

$\bar{V} = \frac{\int_0^R V_{max}\left(1-\frac{r^2}{R^2}\right)(2\pi r)dr}{\pi R^2}$

$= \frac{2V_{max}}{R^2}\int_0^R \left(1 - \frac{r^2}{R^2}\right) rdr$

$= \frac{2V_{max}}{R^2}\int_0^R \left(r - \frac{r^3}{R^2}\right) dr$

$= \frac{2V_{max}}{R^2}\left(\frac{R^2}{2} - \frac{R^2}{4}\right)$

$= \frac{V_{max}}{2}$

$\alpha = \int_0^R \frac{v^3 dA}{V^3 A} = \int_0^R \frac{\left[V_{max}\left(1-\frac{r^2}{R^2}\right)\right]^3 (2\pi r) dr}{\left(\frac{V_{max}}{2}\right)^3 \pi R^2}$

$= \frac{16}{R^2}\int_0^R \left(1 - \frac{3r^2}{R^2} + \frac{3r^4}{R^4} - \frac{r^6}{R^6}\right) r dr$

$= \frac{16}{R^2}\left(\frac{R^2}{2} - \frac{3R^2}{4} + \frac{R^2}{2} - \frac{R^2}{8}\right)$

= 2

$\beta = \int_0^R \frac{v^2 dA}{V^2 A} = \int_0^R \frac{\left[V_{max}\left(1-\frac{r^2}{R^2}\right)\right]^2 (2\pi r) dr}{\left(\frac{V_{max}}{2}\right)^2 \pi R^2}$

$= \frac{8}{R^2}\int_0^R \left(1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right) r dr$

$= \frac{8}{R^2}\left(\frac{R^2}{2} - \frac{R^2}{2} + \frac{R^2}{6}\right)$

= 1.33

5. Water flows in a rectangular channel that is 10m wide at a depth of 1.5m. The channel slope is 0.0025. Taking Chezy's C = 120, compute water velocity, flow rate and bed shear stress.

Solution:
Width (b) =10m
Flow depth (y) =1.5m
Channel slope (S) = 0.0025
C = 120
Velocity (V) =?
Flow rate (Q) = ?
Bed shear stress (τ_0) = ?

A = by = 10x1.5 = 15 m^2
P = b + 2y = 10+2x1.5 = 13 m
R =A/P = 15/13 = 1.15m

$V = C\sqrt{RS} = 120\sqrt{1.15x0.0025}$ = 6.43 m/s
Q = A V = 15x6.43 = 96.45 m^3/s
$\tau_0 = \gamma RS$ = 9810x1.15x0.0025 = 28.2 N/m^2

6. A trapezoidal channel with bottom width 2m and side slope 2:1, flows with a depth of 0.7m in a slope of 0.0004. If n = 0.015, compute bed shear stress, shear velocity, Chezy's C and friction factor f.

Solution:
Bottom width (b) = 2m
Z:1 =2:1
Flow depth (y) = 0.7m
Slope (S) = 0.0004
Manning's n = 0.015
Bed shear stress (τ_0) = ?
Shear velocity (V_*) = ?
Chezy's C = ?
Friction factor (f) = ?

$A = (b + Zy)y = (2 + 2x0.7)0.7$ = 2.38m^2
$P = b + 2y\sqrt{1 + Z^2} = 2 + 2x0.7\sqrt{1 + 2^2}$ = 5.13m
R =A/P = 2.38/5.13 = 0.464m

$\tau_0 = \gamma RS$ = 9810x0.464x0.0004 = 1.82 N/m^2
$V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{1.82}{1000}}$ = 0.0426 m/s
$C = \frac{1}{n}R^{1/6} = \frac{1}{0.015}(0.464)^{1/6}$ = 59
$f = \frac{8gn^2}{R^{1/3}} = \frac{8x9.81x0.015^2}{(0.4640^{1/3}}$ = 0.023

7. A trapezoidal channel has a bottom width of 2.5m and depth of flow of 0.8m. The side slopes are 1.5:1. The channel is lined with bricks (K = 3mm). If the longitudinal slope of the channel is 0.0003, estimate (a) the average shear stress, (b) the hydrodynamic nature of the surface, (c) Chezy C by using f, (d) Manning's n, (e) the uniform flow discharge for cases c and d. Take υ = 10^{-6} m^2/s.

Solution:
Bottom width (b) = 2.5m
Z:1 =1.5:1
Flow depth (y) = 0.8m
Slope (S) = 0.0003
k = 3mm = 3x10^{-3} m
Bed shear stress (τ_0) = ?
Chezy's C = ?
Manning's n = ?
Discharge (Q) = ?

$A = (b + Zy)y = (2.5 + 1.5x0.8)0.8$ = 2.96m^2
$P = b + 2y\sqrt{1 + Z^2} = 2.5 + 2x0.8\sqrt{1 + 1.5^2}$ = 5.384m
R =A/P = 2.96/5.384 = 0.5947m

a) $\tau_0 = \gamma RS$ = 9810x0.5947x0.0003 = 1.75 N/m^2
b) $V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{1.75}{1000}}$ = 0.0418 m/s
$R_n = \frac{V_* k}{\upsilon} = \frac{0.0418x3x10^{-3}}{10^{-6}}$ = 125
As R_n>60, the boundary is hydrodynamically rough.
C) For rough boundary
$\frac{1}{\sqrt{f}} = 2log_{10}(k/4R) - 1.14$
$\frac{1}{\sqrt{f}} = 2log_{10}\left(\frac{3x10^{-3}}{4x0.5947}\right) - 1.14$
f = 0.02
$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8x9.81}{0.02}}$ = 63
d) $n = \frac{1}{C}R^{1/6} = \frac{1}{62.6}(0.5947)^{1/6}$ = 0.0146
(e) Using Chezy
$Q = AC\sqrt{RS} = 2.96x62.6\sqrt{0.5947x0.003}$ = 2.475 m^3/s
Using Manning,
$Q = \frac{1}{n}AR^{2/3}S^{1/2}$
$= \frac{1}{0.0146}2.96x(0.5947)^{2/3}x(0.0003)^{1/2}$ = 2.483 m^3/s

8. The cross-section of a stream could be approximated to a rectangular section of 6m bottom width. The stream is in a mountainous region and is formed by cobbles (d_{90} = 300mm). Estimate the discharge if the depth of flow is 1.5m and the bed slope is 0.001.

Solution:
Width (b) =6m
Flow depth (y) =1.5m
Channel slope (S) = 0.001
d_{90} = 300mm = 0.3m
Flow rate (Q) = ?
Manning's n = $\frac{d_{90}^{1/6}}{26} = \frac{0.3^{1/6}}{26}$ = 0.0315

A = by = 6x1.5 = 9 m^2
P = b + 2y = 6+2x1.5 = 9 m
R =A/P = 9/9 = 1m
Using Manning's equation

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$= \frac{1}{0.03146} 9x(1)^{2/3} x(0.001)^{1/2}$ = 9.03 m^3/s

9. A triangular channel of side slope 3:1 carries water at a flow rate of $10m^3/s$. If the depth of the flow is 1.32m and the channel is laid on slope of 0.0018, find Manning's n.

Solution:

Z:1 = 3:1

Water depth (y) =1.32m

Flow rate (Q) = $10m^3/s$

Channel slope (S) = 0.0018

Manning's n = ?

$A = Zy^2 = 3x1.32^2$ = 5.227m^2

$P = 2y\sqrt{1+Z^2} = 2x1.32\sqrt{1+3^2}$ = 8.348m

R = A/P = 5.227/8.348 = 0.626m

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$10 = \frac{1}{n} 5.227x(0.626)^{2/3} x(0.0018)^{1/2}$

n = 0.016

10. A circular pipe, for which n =0.014, is laid on a slope of 0.00023 and is to carry 2.56 m^3/s when the pipe flows at 80% of full depth. Determine the required diameter of pipe.

Solution:

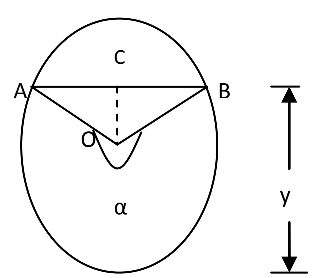
Discharge (Q) = 2.56 m^3/s

Manning's n = 0.014

Channel slope (S) = 0.00023

Flow depth (y) = 0.8D

Where D = Dia. of pipe



OC =0.8D-0.5D = 0.3D

<AOC = <BOC = arcCos(0.3D/0.5D) = 53.1^0

α = 360-2x53.1 = 253.8^0

$A = \frac{D^2}{8}(\alpha - Sin\alpha) = \frac{D^2}{8}(253.8x\pi/180 - Sin253.8)$ = 0.673D^2

$P = \frac{\alpha D}{2} = \frac{253.8x\frac{\pi}{180}D}{2}$ = 2.214D

R = A/P = $0.673D^2/2.214D$ = 0.304D

$Q = \frac{1}{n}AR^{2/3}S^{1/2}$

$2.56 = \frac{1}{0.014}0.673D^2x(0.304D)^{2/3}x(0.00023)^{1/2}$

D = 2.16m

11. A trapezoidal channel having bottom width = 6.5m and side slope 1.5:1, carries a discharge of 10.4 m^3/s. If the depth of flow is 1.15m and n = 0.013, compute the slope of the channel.

Solution:

Bottom width (b) = 6.5m

Z:1 =1.5:1

Discharge (Q) = 10.4 m^3/s

Flow depth (y) = 1.15m

Manning's n = 0.013

Slope of channel (S) = ?

$A = (b + Zy)y = (6.5 + 1.5x1.15)1.15$ = 9.46m^2

$P = b + 2y\sqrt{1 + Z^2} = 6.5 + 2x1.15\sqrt{1 + 1.5^2}$ = 10.65m

R =A/P = 9.46/10.65 = 0.888m

$Q = \frac{1}{n}AR^{2/3}S^{1/2}$

$10.4 = \frac{1}{0.013}9.46x(0.888)^{2/3}xS^{1/2}$

S = 0.00024

12. A rectangular channel carries water at a rate of 7.3 m^3/s, which is laid on a slope of 0.0004. If the depth of flow is 1.9m, compute the width of the channel. Take n = 0.011.

Solution:

Discharge (Q) = 7.3 m^3/s

Flow depth (y) =1.9m

Channel slope (S) = 0.0004

n = 0.011

Width (b) =?

A = by = 1.9b m^2

P = b + 2y = b+2x1.9 = b+3.8 m

R =A/P = 1.9b/b+3.8

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

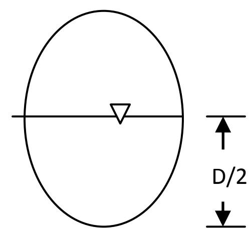
$$7.3 = \frac{1}{0.011}(1.9b)\left(\frac{1.9b}{b+3.8}\right)^{2/3}(0.0004)^{1/2}$$

$$\frac{b^{5/3}}{b+3.8} = 1.378$$

Solving for b

b = 4.23m

13. A sewer pipe is proposed to be laid on a slope of 1 in 2500 to carry a discharge of 1.5 m^3/s. What size of circular channel is used if the pipe runs half full. Take n = 0.015.



Solution:

Discharge (Q) =1.5 m^3/s

Slope (S) = 1/2500

n = 0.015

Diameter (D) = ?

When the pipe is half full

Flow depth (y) = D/2

C/S area (A) = $\frac{\pi D^2}{8}$

Wetted perimeter (P) = $\frac{\pi D}{2}$

Hydraulic radius (R) = $\frac{A}{P} = \frac{\frac{\pi D^2}{8}}{\frac{\pi D}{2}} = \frac{D}{4}$

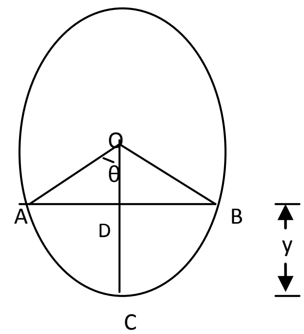
$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$1.5 = \frac{1}{0.015}\frac{\pi D^2}{8}(D/4)^{2/3}(1/2500)^{1/2}$$

D = 2.1m

14. A circular drainage pipe 0.8m in diameter conveys a discharge at a depth of 0.25m. If the pipe is laid on a slope of 1 in 900, calculate discharge. Take n = 0.015.

Solution:
Diameter (D) = 0.8m
Flow depth (y) = 0.25m
Slope (S) = 1/900
n = 0.015
Discharge (Q) =?



OA=OB=OC = 0.4m
OD = OC-CD = 0.4-0.25 = 0.15m
θ = arccos(0.15/0.4) = 1.186 rad = 68^0

Flow area (A) = Area of sector OACB- Area of triangle AOB

$$= \frac{2\theta}{2\pi}\frac{\pi D^2}{4} - 2x\frac{1}{2}x\frac{D}{2}Sin\theta x\frac{D}{2}Sin\theta$$

$$= \frac{D^2}{8}(2\theta - Sin2\theta) = \frac{0.8^2}{8}(2x1.186 - Sin2x1.186) = 0.134\text{m}^2$$

Wetted perimeter (P) = $\frac{2\theta}{2\pi}\pi D = \theta D$=1.186x0.8 = 0.949m
Hydraulic radius (R) = A/P = 0.134/0.949 = 0.141m

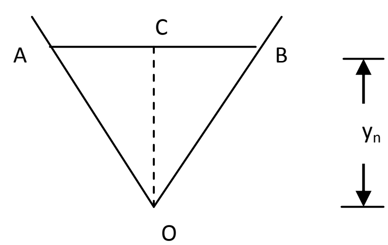
$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

$$= \frac{1}{0.015}0.134x(0.141)^{2/3}x(1/900)^{1/2}$$

= 0.08 m^3/s

15. Compute the normal depth in a triangular channel with bottom angle 60^0 when it carries a discharge of $3m^3/s$. Take n = 0.015 and S = 0.00022.

Solution:
Bottom angle (<AOB) = 60^0
Discharge (Q) = $3m^3/s$
n = 0.015
S = 0.00022
Normal depth (y_n) = ?



AC =BC =y_n tan30 =0.577 y_n
Area (A) = $2x\frac{1}{2}0.577y_n x y_n = 0.577y_n^2$
P =$2y_n$/cos30 = 2.31y_n
R = A/P = 0.249 y_n

From Manning's equation

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$A R^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$0.577 y_n^2 (0.249 y_n)^{2/3} = \frac{0.015x3}{\sqrt{0.00022}}$$

y_n = 2.63m

16. A rectangular channel of 6.1m wide is laid on a slope of 0.00013. The channel carries discharge of 6.8m^3/s. Compute the normal depth of the channel and maximum shear stress on the bed taking n = 0.0149.

Solution:

Width (b) = 6.1m

Discharge (Q) = 6.8 m^3/s

Channel slope (S) = 0.00013

n = 0.0149

Normal depth (y_n) =?

Maximum bed shear stress (τ_0) = ?

A = b y_n = = 6.1 y_n m^2

P = b + 2 y_n = 6.1+2 y_n m

$$R = \frac{A}{P} = \frac{6.1 y_n}{6.1 + 2 y_n}$$

From Manning's equation

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$A R^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$6.1 y_n \left(\frac{6.1 y_n}{6.1 + 2 y_n}\right)^{2/3} = \frac{0.0149x6.8}{\sqrt{0.00013}}$$

$$\frac{y_n^{5/3}}{(6.1 + 2 y_n)^{2/3}} = 0.436$$

Solving for y_n

y_n = 1.465m

$$R = \frac{6.1 y_n}{6.1 + 2 y_n} = \frac{6.1x1.465}{6.1 + 2x1.465} = 0.989$$

$\tau_0 = \gamma R S$ = 9810x0.989x0.00013 = 1.26 N/m^2

17. A channel with a trapezoidal cross-section is to carry 25m^3/s. If S =0.00014, n = 0.015, b =6m and Z:1 =1.5:1, determine the normal depth of flow.

Solution:

Bottom width (b) = 6m

Z:1 =1.5:1

Discharge (Q) = 25 m^3/s

S = 0.00014

Manning's n = 0.015

Normal depth (y_n) = ?

$$A = (b + Zy_n)y_n = (6 + 1.5y_n)y_n = 6y_n + 1.5y_n^2$$
$$P = b + 2y_n\sqrt{1 + Z^2} = 6 + 2y_n\sqrt{1 + 1.5^2} = 6 + 3.6y_n$$

$$R = \frac{A}{P} = \frac{6y_n + 1.5y_n^2}{6 + 3.6y_n}$$

From Manning's equation

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$
$$AR^{2/3} = \frac{nQ}{\sqrt{S}}$$
$$6y_n + 1.5y_n^2\left(\frac{6y_n + 1.5y_n^2}{6 + 3.6y_n}\right)^{2/3} = \frac{0.015x25}{\sqrt{0.00014}}$$
$$\frac{(6y_n + 1.5y_n^2)^{5/3}}{(6 + 3.6y_n)^{2/3}} = 31.69$$

Solving for y_n

y_n =2.42m

18. A circular channel, 2.5m in diameter, is made of concrete (n = 0.014) and is laid on a slope of 1 in 200. Calculate the depth of uniform flow for a discharge of 15 m^3/s.

Solution:

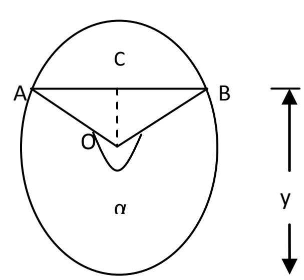
Discharge (Q) = 15 m^3/s

Manning's n = 0.014

Channel slope (S) = 1/200

Dia. of channel (D) = 2.5m

Depth of flow (y) = ?



$$A = \frac{D^2}{8}(\alpha - Sin\alpha) = \frac{2.5^2}{8}(\alpha - Sin\alpha) = 0.78(\alpha - Sin\alpha)$$
$$P = \frac{\alpha D}{2} = \frac{\alpha x2.5}{2} = 1.25\alpha$$
$$R = \frac{A}{P} = \frac{0.78(\alpha - Sin\alpha)}{1.25\alpha} = 0.624\frac{(\alpha - Sin\alpha)}{\alpha}$$

$Q = \frac{1}{n} AR^{2/3} S^{1/2}$

$15 = \frac{1}{0.014} x 0.78(\alpha - Sin\alpha) x (0.624 \frac{(\alpha - Sin\alpha)}{\alpha})^{2/3} x (1/200)^{1/2}$

$\frac{(\alpha - Sin\alpha)^{5/3}}{\alpha^{2/3}} = 5.214$

Solving

$\alpha = 3.945$ rad

$\alpha = 226^0$

<AOC = (360-226)/2 =67^0

OC=1.25cos67 = 1.22m

y = 1.25+1.22 = 2.47m

19. A rectangular channel (n = 0.02) carries flow of 25 m^3/s with the depth of flow equal to the width of channel. If bed slope = 0.0004, find the Froude number of the flow.

Solution:

Discharge (Q) = 25 m^3/s

n = 0.02

Depth of flow = y

Width = b

b = y

Bed slope (S) = 0.0004

Froude number (Fr) = ?

A = by =yxy = y^2

P = b+2y =y+2y = 3y

R= A/P= y/3

Using Manning's equation

$Q = \frac{1}{n} AR^{2/3} S^{1/2}$

$25 = \frac{1}{0.02} y^2 x (y/3)^{2/3} x (0.0004)^{1/2}$

y = 4.4m

b = 4.4m

A = 4.4x4.4 = 19.36 m^2

V = Q/A = 25/19.36 = 1.3m/s

$Fr = \frac{V}{\sqrt{gy}} = \frac{1.3}{\sqrt{9.81x4.4}}$ = 0.197

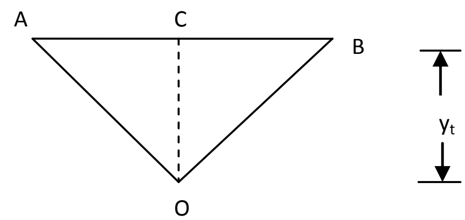
20. A triangular channel of apex angle 90^0 and a rectangular channel of the same material have the same bed slope. If the rectangular channel has the depth of flow equal to the width and the flow areas in both channels are the same, find the ratio of discharges in the rectangular and triangular channels respectively.

Solution:

Bed slope = S and Manning'n are same for both channel

Triangular channel

Bottom angle (<AOB) = 90^0



AC = BC = y_t tan45 = y_t

Area (A_t) = $2x\frac{1}{2}y_t x y_t = y_t^2$

$P_t = 2y_t/\cos 45 = 2.828y_t$

$R_t = A_t/P_t = 0.3536\ y_t$

Rectangular channel

Depth of flow = y

Width = b

b = y

A = by = yxy = y^2

P = b+2y = y+2y = 3y

R = A/P = y/3

$A_t = A$

$y_t^2 = y^2$

$y_t = y$

$Q_{triangular} = \frac{1}{n}A_t R_t^{2/3} S^{1/2}$

$= \frac{1}{n}y_t^2(0.3536y_t)^{2/3}S^{1/2}$

$= \frac{1}{n}y^2(0.3536y)^{2/3}S^{1/2}$

$= \frac{0.5}{n}y^{8/3}S^{1/2}$

$Q_{rectangular} = \frac{1}{n}AR^{2/3}S^{1/2}$

$= \frac{1}{n}y^2(y/3)^{2/3}S^{1/2}$

$= \frac{0.48}{n}y^{8/3}S^{1/2}$

$\frac{Q_{rectangular}}{Q_{triangular}}$ = 0.96

21. An open channel is to be designed to carry 1 m^3/s at a slope of 0.0065. Find the most economical cross-section for (a) a rectangular section, (b) a trapezoidal section, (c) a triangular section and (d) a semicircular section. Take n = 0.01.

Solution:

Discharge (Q) = 1 m^3/s

S =0.0065

n = 0.01

a. For most economical Rectangular section, b = 2y

A = by =$2y^2$

P = b+2y = 4y

R= A/P =0.5y

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$1 = \frac{1}{0.01} 2y^2 (0.5y)^{2/3} 0.0065^{1/2}$

y = 0.42m

b = 2x0.42 = 0.84m

b. For most economical Triangular section, Z:1 = 1:1

A= $Zy^2 = y^2$

$P = 2y\sqrt{z^2+1} = 2\sqrt{2}y$

R= A/P = $y/2\sqrt{2}$ = 0.353y

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$1 = \frac{1}{0.01} y^2 (0.353y)^{2/3} 0.0065^{1/2}$

y = 0.59m

c. For most economical Trapezoidal section

Z:1 = $\frac{1}{\sqrt{3}} : 1,\ b = 2y/\sqrt{3}$, P = 3b

A = (b+zy)y = $\left(\frac{2y}{\sqrt{3}} + \frac{y}{\sqrt{3}}\right) y = 1.732y^2$

R = A/P = 0.5y

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$1 = \frac{1}{0.01} 1.732y^2 (0.5y)^{2/3} 0.0065^{1/2}$

y = 0.44m

$b = 2x0.44/\sqrt{3}$ =0.51m

d. For most economical semi-circular section,

$A = \frac{\pi D^2}{8}, P = \frac{\pi D}{2}$

R = A/P = 0.25D

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

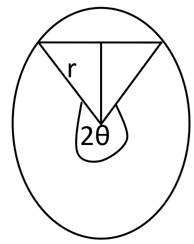
$1 = \frac{1}{0.01} \frac{\pi}{8} D^2 (0.25D)^{2/3} 0.0065^{1/2}$

D = 0.92m

y = D/2 = 0.46m

22. Find the slope at which a circular sewer of 1.5m diameter should be laid to provide the maximum velocity at a discharge of 0.75 m^3/s. Take n = 0.015.

Solution:
Radius (R) = 0.75m
Discharge (Q) = 0.75 m^3/s
n = 0.015
Slope (S) = ?
For maximum velocity
$\theta = 128.75^0$
$\theta = 128.75^0 = 128.75\frac{\pi}{180} = 2.247$ rad



$A = r^2\left(\theta - \frac{1}{2}sin2\theta\right) = 0.75^2\left(2.247 - \frac{1}{2}sin257.5^0\right)$ = 1.538 m^2
$P = 2r\theta = 2x0.75x2.247$ = 3.37m
Hydraulic radius (R) = A/P= 0.45m
$Q = \frac{1}{n}AR^{2/3}S^{1/2}$
$0.75 = \frac{1}{0.015}1.538x(0.45)^{2/3}S^{1/2}$
S = 0.000155

23. The wetted perimeter of a most efficient rectangular channel is 3.8m. If the channel has a value of n = 0.012, and slope of 1 in 2000, find the discharge and the state of flow (Subcritical/supercritical)

Solution:
Wetted perimeter of most efficient rectangular channel (P) = 3.8m
n = 0.012
Bed slope (S) = 1/2000
Discharge (Q) = ?
State of flow = ?
For most efficient rectangular channel, b = 2y where b = width, y = depth of flow
P = b+2y = 2y+2y = 4y
y = P/4 = 3.8/4 = 0.95m

A = by = $2y^2$ = $2x0.95^2$ = 1.805m^2
R = y/2 = 0.95/2 = 0.475m
Using Manning's equation
$Q = \frac{1}{n}AR^{2/3}S^{1/2}$
$= \frac{1}{0.012}1.805x(0.475)^{2/3}x(1/2000)^{1/2}$ = 2.05 m^3/s
V = Q/A = 2.05/1.805 = 1.135 m/s

Froude no. $(Fr) = \frac{V}{\sqrt{gy}} = \frac{1.135}{\sqrt{9.81x0.95}}$ = 0.37
As Fr<1, the flow is subcritical.

24. The area of cross-section of flow in a channel is $6m^2$. Calculate the dimensions of the most efficient section if the channel is (a) triangular, (b) rectangular and (c) trapezoidal (2:1). Which has the least perimeter?

Solution:
Flow area (A) = 6 m^2
a. For most efficient Triangular section, Z:1 = 1:1
A= $Zy^2 = y^2$
6 = y^2
y = 2.45m

b. For most economical Rectangular section, b = 2y
A = by =$2y^2$
6 =$2y^2$
y = 1.732m
b = 2x1.732 = 3.464m

c. For most economical Trapezoidal section
For most economical trapezoidal channel,
$b + 2zy = 2(\sqrt{1+z^2})y$
Given Z:1 = 2: 1
$b + 2x2y = 2(\sqrt{1+2^2})y$
b = 0.47y
A = (b+zy)y = (0.47y+2y)y = 2.47 y^2
6 = 2.47 y^2
y = 1.558m
b= 0.47x1.558=0.732m

Computing wetted perimeter
Triangular
$P = 2y\sqrt{1+Z^2} = 2x2.45\sqrt{1+2^2}$ = 6.929m
Rectangular
P = b+2y = 4y = 4x1.732 = 6.928m
Trapezoidal
$P = b + 2y\sqrt{1+Z^2} = 0.732 + 2x1.558\sqrt{1+2^2}$ = 7.699m
The rectangular channel has the least wetted perimeter.

25. Determine the most economical section of a trapezoidal channel with side slope of 2:1, carrying a discharge of 11.5m^3/s with a velocity of 0.75m/s. What should be the bed slope of the channel? Take n = 0.025.

Solution:
Z:1 = 2:1
n= 0.025
Discharge (Q) = 11.5m^3/s
Velocity (V) = 0.75m/s

For most economical trapezoidal channel,
$b + 2zy = 2\left(\sqrt{1 + z^2}\right)y$
$b + 2x2y = 2\left(\sqrt{1 + 2^2}\right)y$
b = 0.472y
A = (b+zy)y = (0.472y+2y)y = 2.472y^2
$P = b + 2y\sqrt{1 + Z^2} = 0.472y + 2y\sqrt{1 + 2^2}$ = 4.944y
R =A/P = 0.5y

Q = AV
11.5 = 2.472y^{2}x0.75
y = 2.49m
b = 0.472x2.49 = 1.17m

A = 2.472x 2.49^2 = 15.326 m^2
R = 0.5x2.49 = 1.245m

$Q = \frac{1}{n} AR^{2/3}S^{1/2}$
$11.5 = \frac{1}{0.025} 15.326x1.245^{2/3}S^{1/2}$
S = 0.00026

26. In a wide rectangular channel, if the normal depth is increased by 25%, find the increase in discharge.

Solution:
For wide rectangular channel, R = y_n
$Q_1 = \frac{1}{n} A_1 R_1^{2/3} S^{1/2} = \frac{1}{n} by_n (y_n)^{2/3} S^{1/2}$
For 25% increase in normal depth
$Q_2 = \frac{1}{n} A_2 R_2^{2/3} S^{1/2} = \frac{1}{n} bx1.25y_n (1.25y_n)^{2/3} S^{1/2}$
Taking the ratio

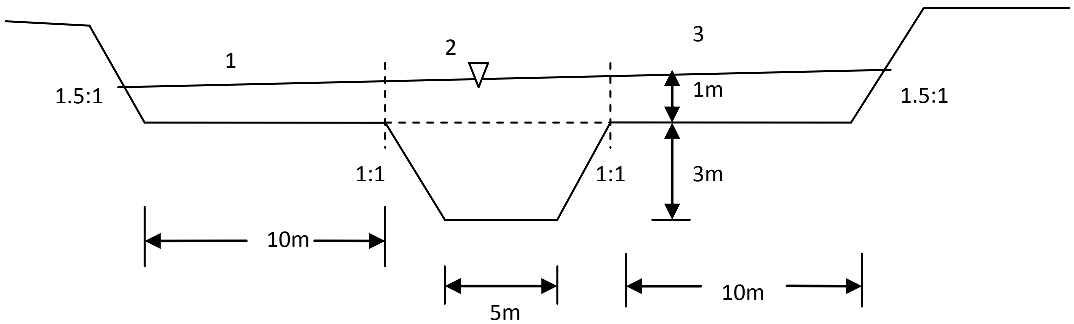
$\frac{Q_1}{Q_2} = 0.689$
$Q_2 = 1.45Q_1$
Hence discharge increases by 45%.

27. Two concrete pipes (Chezy's C = 100) carries flow from an open channel of 1.8m wide and 0.9m deep (Chezy's C = 120). The slope of both structures is 0.0009. (a) Determine the diameter of pipe. (b) Find the depth of water in the rectangular channel after it has become stabilized, if the slope is changed to 0.0016. (Use C = 120).

Solution:
C/s Area of channel (A_1) = 1.8x0.9 = 1.62m^2
Wetted perimeter of channel (P_1) = 1.8+2*0.9 = 3.6m
Hydraulic radius of channel (R_1) = 1.62/3.6 = 0.45m
Slope for both ($S_1 = S_2$) = 0.0009
C/S of pipe (A_2) = $\frac{\pi D^2}{4}$
Wetted perimeter of pipe (P_2) = πD
Hydraulic radius of pipe (R_2) = 0.25D
a)Chezy's C of pipe (C_2) = 100
Chezy's C of channel (C_1) = 120
Diameter of pipes (D) = ?
Qchannel = $A_1 C_1 \sqrt{R_1 S_1} = 1.62x120\sqrt{0.45x0.0009}$ = 3.91m^3/s
Qchannel = Q pipe1+Qpipe2
$A_1 C_1 \sqrt{R_1 S_1} = 2 A_2 C_2 \sqrt{R_2 S_2}$
$1.62x120\sqrt{0.45x0.0009} = 2x\frac{\pi D^2}{4}x100\sqrt{0.25Dx0.0009}$
D =1.22m

b) Q = 3.91m^3/s
Slope of channel (S) = 0.0016
C = 120
Depth (y) = ?
A = by = 1.8y
P = b+2y = 1.8+2y
$R = \frac{A}{P} = \frac{1.8y}{1.8+2y}$
$Q = AC\sqrt{RS}$
$3.91 = 1.8yx120\sqrt{\frac{1.8y}{1.8+2y}0.0016}$
$\frac{y^{3/2}}{(1.8+2y)^{1/2}} = 0.337$
Solving for y
y = 0.716m

28. For the compound section shown in the fig., find discharge when n = 0.015, S = 0.00016 for all parts of section.



Solution:

C/s area1 (A_1) $= \left[\frac{1}{2}x(1.5x1)x1 + 10x1\right]$ = 10.75 m^2

C/S area 2 (A_2) $= [(5 + 3x1 + 3x1)x1 + (5 + 1x3)x3]$ = 35 m^2

C/S area 3 (A_3) $= \left[\frac{1}{2}x(1.5x1)x1 + 10x1\right]$ = 10.75 m^2

Wetted perimeter for sub-area1 (P_1) = $\left(1.0\sqrt{1 + 1.5^2}\right) + 10$ = 11.8m

Wetted perimeter for sub-area2 (P_2) = $\left(5 + 2x3x\sqrt{1 + 1^2}\right)$ = 13.48m

Wetted perimeter for sub-area1 (P_3) = $\left(1.0\sqrt{1 + 1.5^2}\right) + 10$ = 11.8m

Hydraulic radius for sub-area1 (R_1) =10.75/11.8 = 0.911m

Hydraulic radius for sub-area1 (R_1) =35/13.48 = 2.6m

Hydraulic radius for sub-area1 (R_3) =10.75/11.8 = 0.911m

n = n_1= n_2= n_3 = 0.015

S = S_1 = S_2 = S_3 = 0.00016

Discharge (Q) = $Q_1 + Q_2 + Q_3$

$= \frac{1}{n}A_1R_1^{2/3}S^{1/2} + \frac{1}{n}A_2R_2^{2/3}S^{1/2} + \frac{1}{n}A_3R_3^{2/3}S^{1/2}$

= $\frac{1}{0.015}10.75x0.911^{2/3}x0.00016^{1/2} + \frac{1}{0.015}x35x2.6^{2/3}x0.00016^{1/2} +$
$\frac{1}{0.015}10.75x0.911^{2/3}x0.00016^{1/2}$

= 72.84 m^3/s

29. Show that the discharge formula for a trapezoidal channel having n = 0.012 carrying maximum discharge is

$$Q = 105y^{8/3}\left(\sqrt{1+Z^2} - Z/2\right)S^{1/2}$$

Solution:

For most economical trapezoidal channel,

$b + 2zy = 2\left(\sqrt{z^2+1}\right)y$ (a)

A = (b+zy)y (b)

R = 0.5y

From a,

$b + zy = 2y\left(\sqrt{1+z^2} - z/2\right)$ (c)

From a and c

$A = 2y^2\left(\sqrt{1+z^2} - z/2\right)$

$Q = \frac{1}{n}AR^{2/3}S^{1/2}$

$Q = \frac{1}{0.012}2y^2\left(\sqrt{1+z^2} - z/2\right)(0.5y)^{2/3}S^{1/2}$

$Q = 105y^{8/3}\left(\sqrt{1+z^2} - z/2\right)S^{1/2}$

30. A lined rectangular channel with n = 0.015 is 4.5m wide and has a flow depth of 2m with bed slope of 1 in 1600. Retaining the rectangular shape of the channel section and the same total area of lining, to what maximum extent can discharge be increased without changing the slope.

Solution:

Width (b) = 4.5m

Flow depth (y) = 2m

n = 0.015

Bed slope (S) = 1/1600

A_1 = by = 4.5x2 = 9 m^2

P_1 = b + 2y = 4.5+2x2 = 8.5 m

R_1 =A/P = 9/8.5 = 1.058m

Discharge through the channel

$Q_1 = \frac{1}{n}A_1R_1^{2/3}S^{1/2} = \frac{1}{0.015}9x(1.058)^{2/3}(1/1600)^{1/2}$ = 15.57 m^3/s

Second case is the case of the economical channel. In this case b = 2y

Area of lining per m length is same in both cases. i.e. wetted perimeter = constant

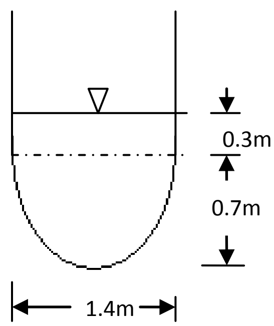
So, P_2 = 8.5m

P_2= b_2+2y_2

$8.5 = 2y_2 + 2y_2$
$y_2 = 2.125$m
$b_2 = 2y_2 = 4.25$

$A_2 = b_2y_2 = 4.25x2.125 = 9.03$ m^2
$P_2 = b_2 + 2y_2 = 4.25+2x2.125 = 8.5$ m
$R_2 = A_2/P_2 = 9.03/8.5 = 1.062$m
Discharge through the channel
$Q_2 = \frac{1}{n} A_2 R_2^{2/3} S^{1/2} = \frac{1}{0.015} 9.03x(1.062)^{2/3}(1/1600)^{1/2}$ = 15.66 m^3/s
Increase in discharge = 15.66-15.57 = 0.09 m^3/s

31. A channel has a vertical wall 1.4m apart and semi-circular of radius 0.7m invert. If the depth of flow is 1m, bed slope is 1/2500, find the discharge using Chezy's formula. Take C = 50.



Solution:
Radius (r) = 0.7m
Bed slope (S) = 1/2500
C = 50
Discharge (Q) = ?

C/S area (A) = $\frac{\pi x 0.7^2}{2} + 1.4x0.3$ = 1.1897 m^2
Wetted perimeter (P) = $\pi x 0.7 + 0.3x2$ = 2.799m
Hydraulic radius (R) = A/P = 1.1897/2.799 = 0.425m

$Q = CA\sqrt{RS}$
$= 50x1.1897\sqrt{0.425x1/2500}$ = 0.7756 m^3/s

32. Find the hydraulic exponent for trapezoidal channel.

Solution:

$$N = \frac{2y}{3A}\left[5T - 2R\frac{dP}{dy}\right]$$

For trapezoidal channel,

$$A = (b + Zy)y$$
$$P = b + 2y\sqrt{1 + Z^2}$$
$$\frac{dP}{dy} = 2\sqrt{1 + Z^2}$$
$$R = \frac{A}{P} = \frac{(b + Zy)y}{b + 2y\sqrt{1 + Z^2}}$$
$$T = (b + 2Zy)$$

Substituting the values in the equation of N

$$N = \frac{2y}{3(b + Zy)y}\left[5(b + 2Zy) - 2\frac{(b + Zy)y}{b + 2y\sqrt{1 + Z^2}}2\sqrt{1 + Z^2}\right]$$

$$N = \frac{10}{3}\frac{(b + 2Zy)}{(b + Zy)} - \frac{8}{3}\frac{y\sqrt{1 + Z^2}}{b + 2y\sqrt{1 + Z^2}}$$

$$N = \frac{10(1 + 2Zy/b)}{3(1 + Zy/b)} - \frac{8}{3}\frac{(y/b)\sqrt{1 + Z^2}}{\left(1 + (2y/b)\sqrt{1 + Z^2}\right)}$$

33. Obtain an expression for the normal depth of the most efficient (a) rectangular channel and (b) trapezoidal channel.

Solution:

a) For the most efficient rectangular channel

$b = 2y_n$

$A = by = 2\ y_n^2$

$P = b+2y = 4\ y_n$

$R = A/P = y_n\ /2$

Using Manning's equation

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$
$$AR^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$2{y_n}^2 x \left(\frac{y_n}{2}\right)^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$y_n = 1.09\left(\frac{nQ}{\sqrt{S}}\right)^{3/8}$$

b) For the most efficient trapezoidal channel

$b + 2zy = 2\left(\sqrt{z^2+1}\right)y$ (a)

$Z:1 = \frac{1}{\sqrt{3}}:1$ (b)

From a and b,

$$b + \frac{2}{\sqrt{3}}y = 2\left(\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right)y, \qquad b = \frac{2y}{\sqrt{3}}$$

Wetted perimeter (P) = $b + 2y\sqrt{z^2+1} = \frac{2y}{\sqrt{3}} + 2y\left(\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}\right) = \frac{6y}{\sqrt{3}}$

$A = (b + Zy_n)y_n = \left(\frac{2y_n}{\sqrt{3}} + \frac{y_n}{\sqrt{3}}\right)y_n = \sqrt{3}{y_n}^2 = 1.732{y_n}^2$

R = A/P = $y_n/2$

Using Manning's equation

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

$$AR^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$1.732{y_n}^2 x \left(\frac{y_n}{2}\right)^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$y_n = 0.9678\left(\frac{nQ}{\sqrt{S}}\right)^{3/8}$$

34. A trapezoidal channel having side slope of 1:1 has to carry a flow of $15m^3/s$. The bed slope is 1 in 1000. Chezy's C is 45 if the channel is unlined and 70 if the channel is lined with concrete. The cost per m^3 of excavation is 3 times cost per m^2 of lining. Find which arrangement is economical.

Solution:

Discharge (Q) = $15m^3/s$

Z:1 = 1:1

Bed slope (S) = 1/1000

For channel of best section,

$b + 2zy = 2y\left(\sqrt{1+z^2}\right)$

$b + 2y = 2y\left(\sqrt{1+1^2}\right)$

b = 0.8284y

A = (b+zy)y = (0.8284y+y)y = 1.8284 y^2

cost per m^2 of lining = p, cost per m^3 of excavation = 3p

case a: Unlined channel

C = 70

$Q = AC\sqrt{RS}$

$15 = 1.8284y^2x45\sqrt{\frac{y}{2}x\frac{1}{1000}}$

y = 2.31m

A = $1.8284 x 2.31^2 = 9.756 m^2$

Cost of excavation for 1 m length = 9.756x1x3p = 29.268p

case b: Lined canal

C = 70

$Q = AC\sqrt{RS}$

$15 = 1.8284y^2x70\sqrt{\frac{y}{2}x\frac{1}{1000}}$

y = 1.94m

A = $1.8284 x 1.94^2 = 6.881 m^2$

$P = b + 2y\left(\sqrt{1+z^2}\right) = 0.8284x1.94 + 2x1.94\left(\sqrt{1+1^2}\right)$ = 7.094m

For 1m length

cost of excavation = 6.881x1x3p = 20.643p

cost of lining = 7.094x1xp = 7.094p

Total cost = 27.737p (<29.268p)

The lined channel is more economical.

Specific energy

1. A rectangular channel 3.5m wide carries a discharge of 11.4 m^3/s and has its specific energy of 2m of water. Calculate alternate depths and corresponding Froude numbers.

Solution:
Width (b) = 3.5m
Discharge (Q) = 11.4 m^3/s
Specific energy (E) = 2m
Alternate depths (y_1, y_2) =?
Froude no. (Fr_1, Fr_2) = ?

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$
$$2 = y + \frac{11.4^2}{2x9.81(3.5y)^2}$$
$$2 = y + \frac{0.54}{y^2}$$

Solving for y
y_1 = 0.62m, y_2 = 1.84m
$$V_1 = \frac{Q}{by_1} = \frac{11.4}{3.5x0.62} = 5.25\text{m/s}$$
$$V_2 = \frac{Q}{by_2} = \frac{11.4}{3.5x1.84} = 1.77\text{m/s}$$
$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{5.25}{\sqrt{9.81x0.62}} = 2$$
$$Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.77}{\sqrt{9.81x1.84}} = 0.4$$

2. A rectangular channel 9m wide carries 5.73 m^3/s when flowing 1.41m deep. Compute specific energy. Is the flow subcritical or supercritical?

Solution:
Width (b) = 9m
Discharge (Q) = 5.73 m^3/s
Flow depth (y) = 1.41m
Specific energy (E) = ?

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$
$$E = 1.41 + \frac{5.73^2}{2x9.81x(9x1.41)^2} = 1.42\text{m}$$

q = Q/b = 5.73/9 = 0.636 $m^3/s/m$

Critical depth for rectangular channel is

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.636^2}{9.81}\right)^{1/3} = 0.345\text{m}$$

As $y>y_c$, the flow is subcritical.

3. Water flows at 3.5 m/s in a rectangular channel at a depth of 0.6m. Find the critical depth for (a) this specific energy, (b) this rate of discharge, (c) type of flow and alternate depth for this discharge.

Solution:

Velocity (V) = 3.5m/s

Flow depth (y) = 0.6m

Critical depth (y_c) = ?

Alternate depth (y_1) =?

a)Specific energy is

$$E = y + \frac{V^2}{2g} = 0.6 + \frac{3.5^2}{2x9.81} = 1.224\text{m}$$

$$y_c = \frac{2}{3}E = \frac{2}{3}1.224 = 0.82\text{m}$$

b) Discharge per unit width (q) = Vy = 3.5x0.6 = 2.1 $m^3/s/m$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.1^2}{9.81}\right)^{1/3} = 0.766\text{m}$$

c) As $y<y_c$, the flow is supercritical.

For q = 2.1 $m^3/s/m$

$$E = y + \frac{q^2}{2gy^2} = 0.6 + \frac{2.1^2}{2x9.81x0.6^2} = 1.224\text{m}$$

$$E = y_1 + \frac{q^2}{2gy_1^2}$$

$$1.224 = y_1 + \frac{2.1^2}{2gy_1^2}$$

$$1.224 = y_1 + \frac{0.2247}{y_1^2}$$

Solving for by y_1

Alternate depth (y_1) = 0.999m

4. For a constant specific energy of 1.98m, what maximum flow can occur in a rectangular channel 3m wide?

Solution:

Specific energy (E) = 1.98m

Width (b) = 3m

Maximum flow (Q_{max}) = ?
Critical depth

$y_c = \frac{2}{3}E = \frac{2}{3}x1.98$ = 1.32m

Critical velocity (V_c) = $\sqrt{gy_c} = \sqrt{9.81x1.32}$ = 3.6m/s

$Q_{max} = A_c V_C$ = (3x1.32)x3.6 = 14.25 m^3/s

5. Calculate the critical depth (y_c) and corresponding specific energy (E_C) for the following different shapes of channel when Q = 7.5 m^3/s.

(a) Rectangular channel with b = 4m
(b) Triangular channel with Z:1 = 0.5:1
(c) Trapezoidal channel with b = 3.6m, Z:1 =2:1

Solution:
Discharge (Q) = 7.5 m^3/s

a) Rectangular channel
b= 4m
q = Q/b =7.5/4 = 1.875 $m_3/s/m$

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.875^2}{9.81}\right)^{1/3}$ = 0.71m

$E_c = 1.5y_c$ = 1.5x0.71 = 1.065m

b) Triangular channel
Z:1 = 0.5:1
At critical depth,

$\frac{A_c^3}{T} = \frac{Q^2}{g}$

$\frac{(Zy_c^2)^3}{2Zy_c} = \frac{Q^2}{g}$

$y_c = \left(\frac{2Q^2}{gZ^2}\right)^{1/5} = \left(\frac{2x7.5^2}{9.81x0.5^2}\right)^{1/5}$ = 2.15m

$E_c = 1.25y_c$ = 1.25x2.15 = 2.68m

c) Trapezoidal channel
b = 3.6m, Z:1 =2:1
At critical depth,

$\frac{A_c^3}{T} = \frac{Q^2}{g}$

$\frac{[(b+Zy_c)y_c]^3}{b+2Zy_c} = \frac{Q^2}{g}$

$\frac{[(3.6+2y_c)y_c]^3}{3.6+4y_c} = \frac{7.5^2}{9.81}$

$\frac{[(3.6+2y_c)y_c]^3}{3.6+4y_c} = 5.733$

Solving for y_c by trial
y_c = 0.669m

$A_c = (b + Zy_c)y_c$ = (3.6+2x0.669)x0.669 = 3.3 m^2
$V_c = Q/A_c$ = 7.5/3.3 = 2.27m/s

$E_c = y_c + \frac{V_c^2}{2g} = 0.669 + \frac{2.27^2}{2x9.81}$ = 0.931m

6. A rectangular channel 7.5m wide carries 12m^3/s with a velocity 1.5 m/s. Compute specific energy. Also find depth of flow when specific energy is minimum. What will be the value of critical velocity as well as minimum specific energy?

Solution:
Channel width (b) = 7.5m
Discharge (Q) = 12m^3/s
Velocity (V) = 1.5m/s
Specific energy (E) = ?
Critical depth (y_c) = ?
Critical velocity (V_c) = ?
Minimum specific energy (E_C) = ?

A = Q/V = 12/1.5 = 8 m^2
Depth of flow (y) = A/b = 8/7.5 = 1.067m
$E = y + \frac{V^2}{2g} = 1.067 + \frac{7.5^2}{2x9.81}$ = 1.18m

q= Q/b = 12/7.5 = 1.6 m^3/s/m
$y_c = \left(\frac{q^2}{g}\right)^{1/3}$ =$\left(\frac{1.6^2}{9.81}\right)^{1/3}$ = 0. 639m

$V_c = \sqrt{gy_c} = \sqrt{9.81x0.639}$ = 2.503m/s
$E_c = 1.5\ y_c$ = 1.5x0.639 = 0.9585m

7. Water flows down a wide rectangular channel of concrete (n = 0.014) laid on a slope of 1 in 500. Find the depth and flow rate for critical condition.

Solution:
Slope (S) = 1/500
n = 0.014

Critical depth (y_c) = ?
Discharge (Q) = ?

For wide rectangular channel, R = y

$V_c = \frac{1}{n} R^{2/3} S^{1/2} = \frac{1}{n} {y_c}^{2/3} S^{1/2}$

Also, $V_c = \sqrt{g y_c}$

Equating

$\frac{1}{n} {y_c}^{2/3} S^{1/2} = \sqrt{g y_c}$

$\frac{1}{0.014} {y_c}^{2/3} (1/500)^{1/2} = {y_c}^{1/2} \sqrt{9.81}$

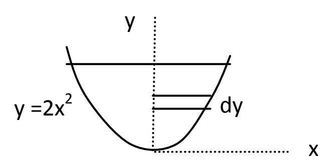
y_c = 0.886m

$y_c^3 = \frac{q^2}{g}$

$(0.886)^3 = \frac{q^2}{g}$

q = 2.6 m^3/s/m

8. Calculate the critical depth of the parabolic channel as shown in the fig., if Q = 4 m^3/s.



dA = xdy

Solution:

y = 2x²

$x = \sqrt{y/2}$

$A = 2 \int_0^{y_c} x dy = 2 \int_0^{y_c} (\sqrt{y/2}) dy = 0.94\, y_c^{3/2}$

T = 2x = $2\sqrt{y_c/2} = 1.414 y_c$

At critical depth,

$\frac{A_c^3}{T} = \frac{Q^2}{g}$

$\frac{\left(0.94 y_c^{3/2}\right)^3}{1.414 y_c} = \frac{4^2}{9.81}$

y_c = 1.33m

9. A trapezoidal channel has a bottom width of 6m, side slopes 1:1, and flows at a depth of 1.8m. For n = 0.015 and a discharge of 10.1 m^3/s, calculate (a) the normal slope, (b) the critical depth and critical slope for 10.1 m^3/s, and (c) the critical slope at normal depth of 1.8m.

Solution:
Bottom width (b) = 6m
Z:1 = 1:1
Flow depth (y) = 1.8m
n= 0.015
Discharge (Q) = 10.1 m^3/s

a) Normal slope (S) = ?
$A = (b + Zy)y = (6 + 1x1.8)1.8$ = 14.04m^2
$P = b + 2y\sqrt{1 + Z^2} = 6 + 2x1.8\sqrt{1 + 1^2}$ = 11.09m
R =A/P = 14.04/11.09 = 1.266m

$Q = \frac{1}{n} AR^{2/3}S^{1/2}$
$10.1 = \frac{1}{0.015} 14.04x(1.266)^{2/3}xS^{1/2}$
S = 0.0001

b)Critical slope (S_c) = ?
Critical depth (y_c) = ?

At critical depth,
$\frac{A_c^3}{T} = \frac{Q^2}{g}$
$\frac{[(b+Zy_c)y_c]^3}{b+2Zy_c} = \frac{Q^2}{g}$
$\frac{[(6+y_c)y_c]^3}{6+2y_c} = \frac{10.1^2}{9.81}$
$\frac{[(6+y_c)y_c]^3}{6+2y_c} = 10.39$

Solving for y_c
y_c = 0.637

$A_c = (b + Zy_c)y_c$ = (6+1x0.637)x0.637 = 4.227 m^2
$P_c = b + 2y_c\sqrt{1 + Z^2} = 6 + 2x0.637\sqrt{1 + 1^2}$ = 7.8m
R_C= 4.227/7.8 = 0.542m

$Q = \frac{1}{n} A_c R_c^{2/3} S_c^{1/2}$
$10.1 = \frac{1}{0.015} 4.227x(0.542)^{2/3} S_c^{1/2}$

S_c = 0.003

c) Normal depth (y_n) = 1.8m
From a
A =14.04 m^2, R =1.266m
T = b+2zy_c = 6+2x1x1.8 =9.6m

$$V_c = \sqrt{\frac{gA}{T}} = \sqrt{\frac{9.81x14.04}{9.6}} = 3.78 \text{ m/s}$$
$$V_c = \frac{1}{n} R_c^{2/3} S_c^{1/2}$$
$$3.79 = \frac{1}{0.015} x1.226^{2/3} \, S_c^{1/2}$$
S_c =0.0024

10. A rectangular channel (n = 0.012) is laid on a slope of 0.036 and carries 16.24 m^3/s. For critical flow conditions, what width is required?

Solution:
Slope (S) = 0.036
Discharge (Q) = 16.24 m^3/s
n = 0.012
Width (b) = ?

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{16.24^2}{b^2 g}\right)^{1/3} = \frac{2.99}{b^{2/3}}$$
$$A_c = b y_c = b\frac{2.99}{b^{2/3}} = 2.99 b^{1/3}$$
$$P_c = b + 2y_c = b + 2\frac{2.99}{b^{2/3}} = \frac{b^{5/3}+5.98}{b^{2/3}}$$
$$R_c = \frac{A_c}{P_c} = \frac{2.99b^{1/3}}{\frac{b^{5/3}+5.98}{b^{2/3}}} = \frac{2.99b}{b^{5/3}+5.98}$$

$$Q = \frac{1}{n} A_c R_c^{2/3} S^{1/2}$$
$$16.24 = \frac{1}{0.012} 2.99 b^{1/3} \left(\frac{2.99b}{b^{5/3}+5.98}\right)^{2/3} 0.036^{1/2}$$
$$\frac{b}{(b^{5/3}+5.98)^{2/3}} = 0.1655$$
Solving for b
b = 0.568m

11. Water flows through a rectangular channel, 3m wide, with a velocity of 1.25m/s. The depth of flow is 0.5m. Compute (a) specific energy, (b) specific force, (c) alternate depth, (d) conjugate depth, (e) critical depth for the discharge and (f) type of flow.

Solution:
Width of channel (b) = 3m
Velocity (V) = 1.25m/s
Depth (y) = 0.5m
(a) Specific energy $(E) = y + \frac{V^2}{2g} = 0.5 + \frac{1.25^2}{2x9.81}$ = 0.579m
(b) Q = AV = (3x0.5)X1.25 = 1.875m^3/s
Specific force $(F) = \frac{Q^2}{gA} + A\bar{Z} = \frac{1.875^2}{9.81x(3x0.5)} + (3x0.5)x0.25$ = 0.614m
(c) Alternate depth (y_1) = ?
For y = 0.5m, Froude number $(Fr) = \frac{V}{\sqrt{gy}} = \frac{1.25}{\sqrt{9.81x0.5}}$ = 0.56. The flow is subcritical.
The alternate depth should be in supercritical flow, i.e. it is less than 0.5m.

$$E = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2gA_1^2}$$

$$0.579 = y_1 + \frac{1.875^2}{2x9.81x(3y_1)^2}$$

$$0.579 = y_1 + \frac{0.0199}{y_1^2}$$

Solving for y_1
y_1 = 0.2435m
(d) Conjugate depth (y_2) = ?

$$F = \frac{Q^2}{gA_2} + A_2\overline{Z_2}$$

$$0.614 = \frac{1.875^2}{9.81x3y_2} + 3y_2xy_2/2$$

$$0.614 = \frac{0.119457}{y_2} + 1.5y_2^2$$

Solving for y_2
y_2 = 0.2208m
(e) Critical depth $(y_c) = \left(\frac{Q^2}{b^2g}\right)^{1/3} = \left(\frac{1.875^2}{3^2x9.81}\right)^{1/3}$ = 0.3414m
(f) As $y>y_c$, the flow is subcritical.

12. At what depths may 1m^3/s flow in a trapezoidal channel 2m wide and having side slopes 1:1 if the specific energy is 0.5m? What would be the corresponding slopes required to sustain uniform flow if n = 0.015? Also find the minimum specific energy required to carry this discharge.

Solution:
Discharge (Q) = 1m^3/s
Width (b) = 2m
Z:1 = 1:1, n = 0.015
Specific energy (E) = 0.5m
Depths = ?
Slope (S) = ?

Minimum specific energy (E_c) = ?

A = (b+zy)y = (2+y)y

$P = b + 2y\sqrt{1+Z^2} = 2 + 2y\sqrt{1+1^2}$ = 2+2.828y

$E = y + \frac{Q^2}{2gA^2}$

$0.5 = y + \frac{1^2}{2g[(2+y)y]^2}$

$0.5 = y + \frac{0.0509}{y^4+4y^3+4y^2}$

Solving for y for alternate depths

y = 0.1837m, 0.46m

For y = 0.1837m

A = (2+y)y = (2+0.1837)0.1837 = 0.4011 m^2

P = 2+2.828y = 2+2.828x0.1837 = 2.3922

R = A/P = 0.4011/2.3922 = 0.1676

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$1 = \frac{1}{0.015} 0.4011x(0.1676)^{2/3} S^{1/2}$

S = 0.0151

For y = 0.46m

A = (2+y)y = (2+0.46)0.46 = 1.1316 m^2

P = 2+2.828y = 2+2.828x0.46 = 3.3

R = A/P = 1.1316/3.3 = 0.343

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$1 = \frac{1}{0.015} 1.1316(0.343)^{2/3} S^{1/2}$

S = 0.000732

Finding critical depth

At critical depth,

$\frac{A_c^3}{T} = \frac{Q^2}{g}$

$\frac{[(b+Zy_c)y_c]^3}{b+2Zy_c} = \frac{Q^2}{g}$

$\frac{[(2+y_c)y_c]^3}{2+2y_c} = \frac{1^2}{9.81}$

$\frac{[(2+y_c)y_c]^3}{2+2y_c} = 0.102$

Solving

y_c = 0.2803m

$A_c = (b + Zy_c)y_c$ =(2+1x0.2803)0.2803 = 0.639 m^2

$E_c = y_c + \frac{Q^2}{2gA_c^2} = 0.2803 + \frac{1^2}{2gx0.639^2}$ = 0.405m

13. Find at what bed slope a 4m wide rectangular channel be led so that the flow is critical at a normal depth of 1.25m, with Manning's coefficient (n) = 0.015.

Solution:
Channel width (b) = 4m
Normal depth (y) =1.25m
n = 0.015
Channel slope (S) = ?

A = by = 4x1.25=5 m^2
P = b + 2y = 4+2x1.25 = 6.5 m
R =A/P = 0.769
The flow is critical at depth y = 1.25m.
For rectangular channel, the critical depth is given by

$y = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$

$y^3 = \frac{Q^2}{b^2 g}$

$1.25^3 = \frac{Q^2}{4^2 x 9.81}$

Q =17.5 m^3/s

From Manning's equation

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$17.5 = \frac{1}{0.015} 5x(0.769)^{2/3} S^{1/2}$

S = 0.0039

14. Show that alternate depths and critical depth in a rectangular channel is related by the following equation.

$y_c = \left(\frac{2y_1^2 y_2^2}{y_1+y_2}\right)^{1/3}$ and $E = \frac{y_1^2+y_1 y_2+y_2^2}{y_1+y_2}$

Solution:

$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$ (a)

$y_c^3 = \frac{q^2}{g} = \frac{V_1^2 y_1^2}{g}$

$\frac{V_1^2}{g} = \frac{y_c^3}{y_1^2}$ (b)

Similarly,

$\frac{V_2^2}{g} = \frac{y_c^3}{y_2^2}$ (c)

From a, b and c

$y_1 + \frac{y_c^3}{2y_1^2} = y_2 + \frac{y_c^3}{2y_2^2}$

$y_1 - y_2 = y_c^3 \left[\frac{1}{2y_2^2} - \frac{1}{2y_1^2}\right]$

$y_1 - y_2 = y_c^3 \left[\frac{y_1^2 - y_2^2}{2y_1^2 2y_2^2}\right]$

$y_c = \left(\frac{2y_1^2 y_2^2}{y_1 + y_2}\right)^{1/3}$

Also, $\frac{q^2}{g} = \frac{2y_1^2 y_2^2}{y_1 + y_2}$ (d)

$E = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2}$ (e)

From d and e

$E = y_1 + \frac{1}{2y_1^2}\frac{2y_1^2 y_2^2}{y_1 + y_2}$

$E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}$

15. If F_n is the Froude no. for normal depth y_n of a rectangular channel, show that

$$\frac{y_c}{y_n} = (F_n)^{2/3}$$

Solution:

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{V^2 y_n^2}{g}\right)^{1/3}$

$y_c = \frac{V^{2/3} y_n^{2/3}}{g^{1/3}}$

Dividing by y_n

$\frac{y_c}{y_n} = \frac{V^{2/3}}{g^{1/3} y_n^{1/3}}$

$\frac{y_c}{y_n} = \left(\frac{V}{\sqrt{gy_n}}\right)^{2/3}$

$\frac{y_c}{y_n} = (F_n)^{2/3}$

16. In a flow through rectangular channel, show that the Froude numbers at alternate depths is related by

$$\left(\frac{F_{r2}}{F_{r1}}\right)^{2/3} = \frac{2 + F_{r2}^2}{2 + F_{r1}^2}$$

Solution:

$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$

$y_1 \left(1 + \frac{V_1^2}{2gy_1}\right) = y_2 \left(1 + \frac{V_2^2}{2gy_2}\right)$

As $Fr = V/\sqrt{gy}$, above expression reduces to

$y_1\left(1+\frac{F_{r1}^2}{2}\right)=y_2\left(1+\frac{F_{r2}^2}{2}\right)$

$\frac{y_1}{y_2}=\frac{2+F_{r2}^2}{2+F_{r1}^2}$ (a)

Also, $F_r^2=\frac{V^2}{gy}=\frac{q^2}{gy^3}$

$F_{r1}^2=\frac{q^2}{gy_1^3}$ and $F_{r2}^2=\frac{q^2}{gy_2^3}$

Taking ratio

$\left(\frac{y_1}{y_2}\right)^3=\left(\frac{F_{r2}}{F_{r1}}\right)^2$ (b)

From a and b

$\left(\frac{F_{r2}}{F_{r1}}\right)^{2/3}=\frac{2+F_{r2}^2}{2+F_{r1}^2}$

17. Determine the d/s depth in a horizontal rectangular channel in which the bottom rises 0.15m, if discharge is 8.4 m^3/s, the width is 3.6m and the u/s depth is 1.2m. What would be the d/s depth if the rise is 0.3m.

Solution:

Discharge (Q) = 8.4m^3/s

Width (b) = 3.6m

Rise of bottom (ΔZ) = 0.15m

u/s depth (y_1) = 1.2m

d/s depth (y_2) = ?

u/s velocity (V_1) = Q/A = 8.4/(3.6x1.2) = 1.94m/s

Froude no. at 1 $(Fr_1)=\frac{V_1}{\sqrt{gy_1}}=\frac{1.94}{\sqrt{9.81x1.2}}$ = 0.56

As $Fr_1<1$, the u/s flow is subcritical and the transition will cause a drop in water level.

q=Q/b = 8.4/3.6 = 2.33 $m^3/s/m$

$y_c=\left(\frac{q^2}{g}\right)^{1/3}=\left(\frac{2.33^2}{9.81}\right)^{1/3}$ = 0.82m

a) E_{min} = 1.5y_c =1.5x0.82 = 1.23m

Specific energy at u/s (E_1) = $y_1+\frac{V_1^2}{2g}=y_1+\frac{q^2}{2gy_1^2}=1.2+\frac{2.33^2}{2x9.81x1.2^2}$ = 1.39m

D/s specific energy = E_2

$E_2=E_1-\Delta Z$

= 1.39-0.15

E_2 =1.24>E_{min} . The u/s depth will remain unchanged.

$E_2 = y_2 + \frac{q^2}{2gy_2^2}$

$1.24 = y_2 + \frac{q^2}{2gy_2^2}$

$1.24 = y_2 + \frac{2.33^2}{2x9.81y_2^2}$

$1.24 = y_2 + \frac{0.277}{y_2^2}$

Solving for y_2

(y_2>y_c and y_2<y_1-ΔZ)

y_2 = 0.89m

D/s depth = 0.89m

b) Rise of bottom (ΔZ) = 0.3m

As computed in a, E_1 = 1.39m, E_{min} = 1.23m

$E_2 = E_1 - \Delta Z$

= 1.39-0.3

E_2 =1.09<E_{min}, which is not possible. So E_2 = E_{min} and y_2 = y_c = 0.82m

18. A rectangular channel contracts from a width of 3m to 2.5m in a short transition section. If the discharge is 6.5 m^3/s, and the u/s depth is 0.4m, determine the d/s depth.

Solution:

Width at 1 (b_1) = 3m

Width at 2 (b_2) =2.5m

Discharge (Q) = 6.5 m^3/s

u/s depth (y_1) = 0.4m

u/s velocity (V_1) = Q/A_1 = 6.5/(3X0.4) = 1.587m/s

Froude no. at 1 $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{1.587}{\sqrt{9.81x0.9}}$ = 5.41

As Fr_1>1, the u/s flow is supercritical and the transition will cause a rise in water level.

a) d/s depth (y_2) = ?

q_1 = Q/b_1 = 6.5/3 = 2.167 m^3/s/m

q_2 = Q/b_2 = 6.5/2.5 = 2.6 m^3/s/m

$y_{c2} = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{2.6^2}{9.81}\right)^{1/3}$ = 0.883m

E_{c2} = 1.5 y_{c2} = 1.5x0.883 = 1.324m.

Specific energy at u/s (E_1) = $y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q_1^2}{2gy_1^2} = 0.4 + \frac{2.167^2}{2x9.81x0.4^2}$ = 1.896m

D/s specific energy = E_2 = E_1 = 1.896m

As E_2>E_{c2}, the u/s depth remains unchanged.

$E_2 = 1.896$

$y_2 + \frac{q_2^2}{2gy_2^2} = 1.896$

$y_2 + \frac{2.6^2}{2x9.81xy_2^2} = 1.896$

$y_2 + \frac{0.344}{y_2^2} = 1.896$

Solving for y_2

$y_2 = 0.496m$

D/s depth = 0.496m

19. A wide rectangular channel carries a flow of $2.7m^3/s/m$, the depth of flow being 1.5m. (a) Calculate the minimum rise in the floor at a section required to produce critical flow condition. (b) What is the corresponding fall in water level?

If the loss of energy over the rise is 10% of upstream velocity head, what will be the minimum rise in the floor to produce critical flow?

Solution:

Flow (q) = $2.7m^3/s/m$

Depth of flow (y_1) = 1.5m

Velocity (V_1) =q/y_1 = 2.7/1.5 = 1.8m/s

a) Rise in floor level (ΔZm) =?

Critical depth (y_c) = $\left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.7^2}{9.81}\right)^{1/3}$ = 0.906m

Specific energy at u/s (E_1) =$y_1 + \frac{V_1^2}{2g} = 1.5 + \frac{1.8^2}{2x9.81}$ = 1.665m

Critical depth occurs at section 2.

At section 2, specific energy (E_2) = E_{min} = $1.5y_c$ = 1.5x0.906 = 1.359m

$E_1 - \Delta Zm = E_2$

$\Delta Zm = E_1 - E_{min}$ = 1.665-1.359 = 0.306m

b) Fall in water level (Δh) = ?

$\Delta h = y_1 - (y_c + \Delta Zm) = 1.5 - (0.906 + 0.306)$ = 0.288m

Head loss over hump (h_L) = $0.1\frac{V_1^2}{2g} = 0.1x\frac{1.8^2}{2x9.81}$ = 0.0165m

Minimum height of hump $(\Delta Zm) = E_1 - E_{min} - h_L$ = 1.665-1.359-0.0165 = 0.2895m

20. In a 3.4m wide rectangular channel laid at a slope of 0.0003, uniform flow occurs at a depth of 2m. Find how high a hump can be raised on the channel bed without causing a change in the u/s depth. If the u/s depth is to be raised to 2.35m, what should be the height of hump? Take n = 0.015

Solution:
u/s depth (y_1) = 2m
Channel width (b) = 3.4m
Bed slope (S) = 0.0003
n = 0.015

C/s area (A) = by_1 = 3.4x2 = 6.8 m^2
Wetted perimeter (P) = b+2y_1 = 3.4+2x2 = 7.4m
R = A/P = 6.8/7.4 = 0.92m

Discharge (Q) = $\frac{1}{n}AR^{2/3}S^{1/2} = \frac{1}{0.015}6.8x0.92^{2/3}0.0003^{1/2}$ = 7.43 m^3/s
q = Q/b = 7.43/3.4 = 2.185 m3/s/m

a) Height of hump (Δz) =?
Critical depth (y_c) = $\left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.185^2}{9.81}\right)^{1/3}$ = 0.786m
Specific energy at u/s (E_1) =$y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} = 2 + \frac{2.185^2}{2x9.81x2^2}$ = 2.06m
Critical depth occurs at section 2.
At section 2, specific energy (E_2) = E_{min} = 1.5y_c = 1.5x0.786 = 1.18m

$E_1 - \Delta z = E_2$
$\Delta z = E_1 - E_2$ = 2.06-1.18 = 0.88m

b)u/s depth (y_{1a}) is increased to 2.35m keeping Q same.
Height of hump ($\Delta z1$) =?
Specific energy at u/s (E_1) =$y_{1a} + \frac{V_{1a}^2}{2g} = y_{1a} + \frac{q^2}{2gy_{1a}^2} = 2.35 + \frac{2.185^2}{2x9.81x2.35^2}$ = 2.394m
From above, y_c = 0.786m
Critical depth occurs at section 2.
At section 2, specific energy (E_2) = E_{min} = 1.5y_c = 1.5x0.786 = 1.18m

$E_1 - \Delta z1 = E_2$
$\Delta z1 = E_1 - E_2$ = 2.394-1.18 = 1.214m

21. A rectangular channel is 2.1m wide and carries a flow of $3m^3/s$ at a depth of 0.9m. A contraction of the channel is proposed at a certain section. Compute the depth at contraction and the smallest allowable contracted width that will not affect the u/s flow condition.

Solution:
Channel width (b) = 2.1m
Flow rate (Q) = $3m^3/s$
u/s flow depth (y_1) = 0.9m
Depth at contraction (y_c) = ?
Minimum width at contraction (B_m) = ?

C/s area (A) = by_1 = 2.1x0.9 = $1.89m^2$
u/s velocity (V_1) = Q/A = 3/1.89 = 1.587m/s
Froude no. at 1 $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{1.587}{\sqrt{9.81x0.9}}$ = 0.53
As $Fr_1<1$, the u/s flow is subcritical and the transition will cause a drop in water level.

Specific energy at u/s (E_1) $= y_1 + \frac{V_1^2}{2g} = 0.9 + \frac{1.587^2}{2x9.81}$ = 1.028m
At the contacted section, specific energy (E_2) = E_1 = 1.028m
$E_c = E_2$
Critical depth at contracted section (y_c)= $\frac{2}{3}E_c = \frac{2}{3}1.028$ = 0.685m

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{gB_m^2}\right)^{1/3}$$

$$y_c^3 = \frac{Q^2}{gB_m^2}$$

$$0.685^3 = \frac{3^2}{9.81B_m^2}$$

B_m = 1.69m

22. Water flows in a 4m wide rectangular channel at a depth of 2.1m and velocity of 2 m/s. The channel is contracted to 1.9m and bed raised by 0.55m in a given reach. What will be the depth in the u/s reach?

Solution:
Width of channel (b) = 4m
Depth of flow (y_1) = 2.1m
Velocity (V_1) = 2m/s
Width of contraction (B_c) = 1.9m
Rise (Δz) =0.55m
Upstream depth (y_{1a}) = ?

Discharge (Q) = 4x2.1x2 = 16.8 m^3/s
q_1 = 16.8/4 = 4.2 $m^3/s/m$

Froude number $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{2}{\sqrt{9.81x2.1}}$ = 0.44

As the flow is subcritical, the depth in transition will decrease.

Specific energy (E_1) =$y_1 + \frac{V_1^2}{2g} = 2.1 + \frac{2^2}{2x9.81}$ = 2.304m

Discharge/width at contraction (qa) = 16.8/1.9 = 8.842 $m^3/s/m$

If the flow in contraction is critical, then critical depth at contraction (y_c) is

$$y_c = \left(\frac{qa^2}{g}\right)^{1/3} = \left(\frac{8.842^2}{9.81}\right)^{1/3} = 1.997\text{m}$$

E_{min} = 1.5 y_c = 1.5x1.997 = 2.995m

$E_2 = E_1 - \Delta Z = 2.304 - 0.55$ = 1.754m

Since $E_2 < E_{min}$, the flow is not possible. So, the flow at 2 is critical and the u/s depth will rise.

$$E_{1a} = E_{min} + \Delta Z$$

$$y_{1a} + \frac{q^2}{2gy_{1a}^2} = 3.545$$

$$y_{1a} + \frac{4.2^2}{2gy_{1a}^2} = 3.545$$

$$y_{1a} + \frac{0.899}{y_{1a}^2} = 3.545$$

Solving for y_{1a}

y_{1a} = 3.47m

23. A 6m wide rectangular channel carries a discharge of 30m^3/s at a depth of 2.7m.

(a) Calculate the width at which the channel should be contracted so that the depth in the contracted section is critical. (b) What will be the depth at the contracted section if width there is 5m? (c) What will be the depth of flow in the u/s and in the contracted section if the width of channel is reduced to 3.1m?

Solution:

Channel width (b) = 6m

Flow rate (Q) = 30m^3/s

Flow depth (y_1) = 2.7m

Velocity at 1 $(V_1) = \frac{Q}{A_1} = \frac{30}{6x2.7}$ = 1.85m/s

Froude no. at 1 $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{1.85}{\sqrt{9.81x2.7}}$ = 0.36

As $Fr_1<1$, the u/s flow is subcritical and the transition will cause a drop in water level.

a) Minimum width at contraction (b_m) = ?

Discharge per unit width at 1 (q_1) = 30/6 = 5$m^3/s/m$

Specific energy at u/s (E_1) =$y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q_1^2}{2gy_1^2} = 2.7 + \frac{5^2}{2x9.81x2.7^2}$ = 2.875m

At the contacted section, specific energy (E_2) = E_1 = 2.875m

E_c = E_2

Critical depth at contracted section (y_c)= $\frac{2}{3}E_c = \frac{2}{3}x2.875$ = 1.916m

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{gB_m^2}\right)^{1/3}$$

$y_c^3 = \frac{Q^2}{gB_m^2}$

$1.916^3 = \frac{30^2}{9.81B_m^2}$

B_m = 3.61m

b) Width (b_2) = 5m

Depth at contracted section (y_2) = ?

As $b_2 > b_m$(critical width), the flow in the contracted section will not be critical.

At u/s, $y_{c1} = \left(\frac{q_1^2}{g}\right)^{1/3} = \left(\frac{5^2}{9.81}\right)^{1/3}$ = 1.365m

Discharge per unit width at 2 (q_2) = Q/b_2 = 30/5 = 6m^3/s/m

$E_1 = E_2$

$2.875 = y_2 + \frac{q_2^2}{2gy_2^2}$

$2.875 = y_2 + \frac{6^2}{2x9.81xy_2^2}$

$2.875 = y_2 + \frac{1.835}{y_2^2}$

Solving for y_2 ($y_2 > y_c$, $y_2 < y_1$)

y_2 = 2.605m (>y_c)

c) Width (b_2) = 3.1m

Depth at contracted section (y_2) = ?

Depth at u/s section (y_{1a}) = ?

Discharge per unit width (q_2) = Q/b_2 = 30/3.1 = 9.677m^3/s/m

As $b_2 < b_m$, the flow cannot take place with the given specific energy ($E_2 < E_{min}$). So the flow in the contracted section should be critical.

$y_c = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{9.677^2}{9.81}\right)^{1/3}$ = 2.121m

E_{min} = E_2 = 1.5y_c = 1.5x2.121 = 3.181m

However, since E_1 = 2.875m, the flow cannot occur unless E_{1a} =E_{min} = 3.181m

$y_{1a} + \frac{q_1^2}{2gy_{1a}^2} = 3.181$

$y_{1a} + \frac{5^2}{2gy_{1a}^2} = 3.181$

$y_{1a} + \frac{1.274}{y_{1a}^2} = 3.181$

Solving for y_{1a} ($y_{1a} > y_1$)

U/s depth = 3.043m

Depth at contracted section = critical depth = 2.121m

24. A 4.9m wide rectangular channel carries a discharge of $20m^3/s$ at a depth of 2m.
(a) Calculate the depth of flow over a hump of 0.3m on the bed. (b) What will be the minimum rise in the bed level required to obtain critical depth over the rise? (c) What will be the water depths u/s and over the hump if the bed level is raised by 0.7m?

Solution:
Channel width (b) = 4.9m
Discharge (Q) = $20m^3/s$
U/s depth (y_1) = 2m
Discharge per unit width (q) = 20/4.9 = $4.08m^3/s/m$
Velocity at 1 $(V_1) = \frac{Q}{A_1} = \frac{20}{4.9x2}$ = 2.04 m/s
Froude no. at 1 $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{2.04}{\sqrt{9.81x2}}$ = 0.46
As $Fr_1<1$, the u/s flow is subcritical and the transition will cause a drop in water level.
Specific energy at u/s (E_1) $= y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} = 2 + \frac{4.08^2}{2x9.81x2^2}$ = 2.212m
$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{4.08^2}{9.81}\right)^{1/3}$ = 1.193m
Critical depth occurs at section 2.
At section 2, specific energy (E_2) = E_{min} = $1.5y_c$ = 1.5x1.193 = 1.7895m

a) Rise (Δz) =0.3m
$E_2 = E_1 - \Delta Z$ = 2.212-0.3 = 1.912m
$E_2>E_{min}$, the flow is sub-critical.
Depth of flow over hump (y_2) = ?
$E_1 - \Delta z = E_2$
$2.212 - 0.3 = y_2 + \frac{q^2}{2gy_2^2}$
$1.912 = y_2 + \frac{4.08^2}{2gy_2^2}$
$1.912 = y_2 + \frac{0.848}{y_2^2}$
Solving for y_2 ($y_2>y_c$, $y_2<y_1$)
y_2 =1.566m

b) Minimum rise in bed level (ΔZm) =?
Critical depth occurs at section 2.
As calculated above, Specific energy at u/s (E_1) = 2.212m and minimum specific energy at section 2 (E_2) = E_{min} = 1.7895m

$\Delta Zm = E_1 - E_{min}$ = 2.212-1.7895 = 0.4225m

c) Rise (Δz) =0.7m
Water depth over hump (y_2) = ?

U/s water depth (y_{1a}) = ?

$E_2 = E_1 - \Delta Z$ = 2.212-0.7 = 1.512m

As $E_2<E_{min}$, specific energy at 1 is inadequate for water to flow at critical depth at section 2. Hence the depth at section 1 increases to y_{1a} and depth at section 2 is critical.

$y_2 = y_c$ = 1.193m and $E_2 = E_{min}$

$$E_{1a} = E_{min} + \Delta Z$$

$$y_{1a} + \frac{q^2}{2gy_{1a}^2} = 1.7895 + 0.7$$

$$y_{1a} + \frac{4.08^2}{2gy_{1a}^2} = 2.4895$$

$$y_{1a} + \frac{0.848}{y_{1a}^2} = 2.4895$$

Solving y_{1a} by trial and error ($y_{1a}>y_1$)

Solving for y_{1a}

y_{1a} = 2.333m

U/s depth = 2.333m

25. The width of a rectangular channel is reduced gradually from 3m to 2m and the floor is raised by 0.3m at a given section. When the approaching depth of flow is 2.05m, what rate of flow will be indicated by a drop of 0.2m in the water surface elevation at the contracted section?

Solution:

U/s depth (y_1) = 2.05m

ΔZ = 0.3m

Depth at contracted section (y_2) = 2.05-0.3-0.2 = 1.55m

Width at u/s (b_1) = 3m

Width at contracted section (b_2) = 2m

Discharge (Q) = ?

$$E_1 = E_2 + \Delta Z$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + 0.3$$

$$y_1 + \frac{Q^2}{2gA_1^2} = y_2 + \frac{Q^2}{2gyA_2^2} + 0.3$$

$$2.05 + \frac{Q^2}{2x9.81x(3x2.05)^2} = 1.55 + \frac{Q^2}{2x9.81x(2x1.55)^2} + 0.3$$

Q = 7.11 m^3/s

26. A flow of $30m^3/s$ is carried in a 5m wide rectangular channel at a depth of 1.0m. Find the slope necessary to sustain uniform flow at this depth if n = 0.012. What change in roughness would produce uniform critical flow at this discharge on this given slope?

Solution:
Discharge (Q) = 30m^3/s
Width (b) =5m
Flow depth (y) =1.0m
n= 0.012
Channel slope (S) = ?
Change in roughness for uniform critical condition = ?

A = by = 5x1.0 = 5 m^2
P = b + 2y = 5+2x1.0 = 7 m
R =A/P = 5/7 = 0.714m

$Q = \frac{1}{n} AR^{2/3} S^{1/2}$

$30 = \frac{1}{0.012} 5x(0.714)^{2/3} S^{1/2}$
S = 0.008

For critical flow
Critical depth (y_c) = $\left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{B^2 g}\right)^{1/3} = \left(\frac{30^2}{5^2 x 9.81}\right)^{1/3}$ = 1.54m
A_c = by_c = 5x1.54 = 7.7 m^2
P_c = b + 2yc = 5+2x1.54 = 8.08 m
R_c =A/P = 7.7/8.08 = 0.953m
Finding Manning's n with S = 0.008

$$Q = \frac{1}{n} AR^{2/3} S^{1/2}$$

$$30 = \frac{1}{n} 7.7x(0.953)^{2/3} 0.008^{1/2}$$

n = 0.022

Change in roughness = 83% increase

27. A 3m wide rectangular channel carries 3 m^3/s of water at a depth of 1m. If the width is to be reduced to 2m and bed raised by 10cm, what would be the depth of flow in the contracted section? What maximum rise in the bed level of the contracted section is possible without affecting the depth of flow upstream of transition? Neglect loss of energy in transition.

What would be the change in water surface elevations if the rise in bed is 30cm?

Solution:
Width of channel at 1 (b_1) = 3m
Width of channel at 1 (b_2) = 2m

Discharge (Q) = 3 m^3/s
Depth of channel at 1 (y_1) = 1m
Rise in bed level at 2 (Δz) =0.1m
Velocity at 1 $(V_1) = \frac{Q}{A_1} = \frac{3}{3x1}$ = 1m/s
Froude no. at 1 $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{1}{\sqrt{9.81x1}}$ = 0.32
As Fr_1<1, the u/s flow is subcritical and the transition will cause a drop in water level.

Critical depth at 2 (y_{c2}) = $\left(\frac{Q^2}{b_2^2 g}\right)^{1/3} = \left(\frac{3^2}{2^2x9.81}\right)^{1/3}$ = 0.6121m
Minimum specific energy (E_{c2}) = 1.5 y_{c2} = 1.5x0.6121= 0.9181m

(a) Depth of flow in the contracted section (y_2) = ?
Specific energy at u/s (E_1) =$y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2g(b_1y_1)^2} = 1 + \frac{3^2}{2x9.81x(3x1)^2}$ = 1.051m
$E_2 = E_1 - \Delta z$ = 1.051-0.1 = 0.951
As E_2>E_{c2}, the u/s depth will remain unchanged

$E_2 = y_2 + \frac{Q^2}{2g(b_2y_2)^2}$
$0.951 = y_2 + \frac{3^2}{2x9.81(2xy_2)^2}$
$0.951 = y_2 + \frac{0.11467}{y_2^2}$
Solving for y_2 (y_2<y_1, y_2>y_c)
y_2 = 0.7436m

b) Maximum rise in bed level at 2 (Δzm) = ?
At critical state, the rise in bed level will be maximum, without affecting the u/s depth.
E_2 = E_{C2}
$E_1 = E_c + \Delta zm$
$\Delta zm = E_1 - E_c$ = 1.051-0.9181 = 0.133m

c) Rise in bed level at 2 (Δz) =0.3m
$E_2 = E_1 - \Delta z$ = 1.051-0.3 = 0.751
As E_2<E_{c2}, the flow is not possible. So, the d/s depth should be critical and the u/s depth will increase to y_{1a}.
y_2 = y_{c2} =0.6121m , E_2= E_{c2} = 0.9181m

$E_{1a} = E_{c2} + \Delta Z$
$y_{1a} + \frac{Q^2}{2gA_1^2} = 0.9181 + 0.3$
$y_{1a} + \frac{3^2}{2g(y_{1a}x3)^2} = 1.2181$

$y_{1a} + \frac{0.0509}{y_{1a}^2} = 1.2181$

Solving for y_{1a} (y_{1a}>y_1)

y_{1a} = 1.174m

28. A rectangular channel 2m wide has a flow of 2.4 m^3/s at a depth of 1.0m. Determine if critical depth occurs (a) a section where a hump of ΔZ = 20cm high is installed across the bed, (b) a side wall constriction (no hump) reducing the channel width to 1.7m, and (c) both the hump and side wall constriction combined. Will the upstream depth be affected for case (c)? If so, to what extent? Neglect head losses of the hump and constriction cause by friction, expansion and contraction.

Solution:

Width of channel (b) = 2m

Discharge (Q) = 2.4 m^3/s

Flow depth (y_1) = 1.0m

Specific energy at section 1 (E_1) = $y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2g(by_1)^2} = 1.0 + \frac{2.4^2}{2x9.81(2x1)^2}$ = 1.0734m

(a) ΔZ = 20cm = 0.2m

Critical depth is given by

$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{2.4^2}{2^2 x9.81}\right)^{1/3}$ = 0.527m

E_c =1.5 y_c =1.5x0.527 = 0.7905m

At critical condition, $E_2 = E_C$

The height of hump for critical condition $(\Delta Zc) = E_1 - E_2 = E_1 - E_c$

= 1.0734-0.7905 = 0.2829m

As $\Delta Z < \Delta Zc$, the critical depth does not occur.

(b) b_2 = 1.7m

Critical depth is given by

$y_{c2} = \left(\frac{Q^2}{b_2^2 g}\right)^{1/3} = \left(\frac{2.4^2}{1.7^2 x9.81}\right)^{1/3}$ = 0.588m

E_{c2} =1.5 y_c =1.5x0.588 = 0.882m

For constricted flow

$E_1 = E_2$

E_2 = 1.0734m

As E_2>E_{c2}, the critical depth does not occur.

(c) ΔZ = 20cm = 0.2m and b_2 = 1.7m

As computed in (b), E_{c2} = 0.882m

At critical condition, $E_2 = E_{C2}$

The height of hump for critical condition $(\Delta Zc) = E_1 - E_2 = E_1 - E_{c2}$

= 1.0734-0.882 = 0.1914m

As $\Delta Z > \Delta Zc$, the flow is not possible and the depth at 2 should be critical.

In case (c), $E_2 = E_1 - \Delta Z$ = 1.0734-0.2 = 0.8734m

As $E_2 < E_{c2}$, specific energy at 1 is inadequate for water to flow. Hence the u/s will be affected. The u/s depth should rise to y_{1a} and depth at section 2 should be critical.

$y_2 = y_{c2}$ = 0.588m and $E_2 = E_{c2}$ = 0.882m. E1a = u/s specific energy

$$E_{1a} = E_2 + \Delta Z$$
$$E_{1a} = Ec_2 + \Delta Z$$
$$y_{1a} + \frac{Q^2}{2g(by_{1a})^2} = 0.882 + 0.2$$
$$y_{1a} + \frac{2.4^2}{2g(2xy_{1a})^2} = 1.082$$
$$y_{1a} + \frac{0.0734}{y_{1a}^2} = 1.082$$

Solving for y_{1a}

y_{1a} = 1.01m

29. A discharge of 16 m^3/s flows with a depth of 2m in a 4m wide rectangular channel. At a downstream section, the width is reduced to 3.5m and the channel bed is raised by 0.35m. To what extent will the surface elevation be affected by these changes?

Solution:

Width of channel at 1 (b_1) = 4m

Width of channel at 1 (b_2) = 3.5m

Discharge (Q) = 16 m^3/s

Depth of channel at 1 (y_1) = 2m

Rise in bed level at 2 (Δz) =0.35m

Velocity at 1 $(V_1) = \frac{Q}{A_1} = \frac{16}{4x2}$ = 2m/s

Froude no. at 1 $(Fr_1) = \frac{V_1}{\sqrt{gy_1}} = \frac{2}{\sqrt{9.81x2}}$ = 0.45

As $Fr_1 < 1$, the u/s flow is subcritical and the transition will cause a drop in water level.

Critical depth at 2 (y_{c2}) = $\left(\frac{Q^2}{b_2^2 g}\right)^{1/3} = \left(\frac{16^2}{3.5^2 x 9.81}\right)^{1/3}$ = 1.287m

Minimum specific energy (E_{c2}) = 1.5 y_{c2} = 1.5x1.287= 1.9305m

Specific energy at u/s (E_1) =$y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2g(b_1 y_1)^2} = 2 + \frac{16^2}{2x9.81x(4x2)^2}$ = 2.2038m

$E_2 = E_1 - \Delta z$ = 2.2038-0.35 = 1.8538m

As $E_2 < E_{c2}$, the flow is not possible. So, the d/s depth should be critical and the u/s depth will increase to y_{1a}.

$y_2 = y_{c2}$ =1.287m , $E_2 = E_{c2}$ = 1.9305m

$$E_{1a} = E_{c2} + \Delta Z$$

$y_{1a} + \frac{Q^2}{2gA_1^2} = 1.9305 + 0.35$

$y_{1a} + \frac{16^2}{2g(y_{1a}x4)^2} = 2.2805$

$y_{1a} + \frac{0.8155}{y_{1a}^2} = 2.2805$

Solving for y_{1a} ($y_{1a}>y_1$)

y_{1a} = 2.0946m

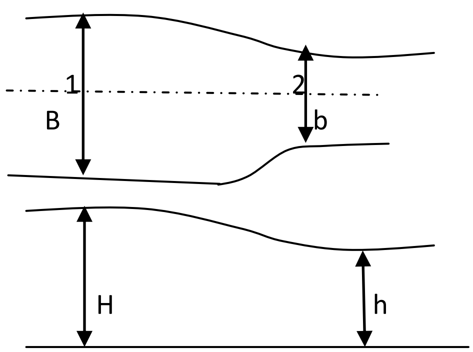
Upstream elevation will rise by 0.0946m.

30. A venturiflume in a rectangular channel of width B has the throat width of b. The depth of liquid at the entry is H and at the throat is h. Prove that the following relation exists for the discharge and width ratio:

$Q = 3.13bH^{3/2}\left(\frac{h}{H}\right)^{3/2}$

$\frac{b}{B} = \sqrt{3}\frac{h}{H} - \sqrt{2}\left(\frac{h}{H}\right)^{3/2}$

Solution:



At the contraction, the flow is critical.

So, $V_2 = \sqrt{gh}$

Discharge (Q) = A_2V_2

$= bh\sqrt{gh}$

$= 3.13bh^{3/2}$

$= 3.13bh^{3/2}\frac{H^{3/2}}{H^{3/2}}$

Rearranging

$Q = 3.13bH^{3/2}\left(\frac{h}{H}\right)^{3/2}$ (a)

Expression for b/B

$E_1 = E_2$

$E_2 = E_C = 1.5h$

So, $E_1 = 1.5h$

$H + \frac{V_1^2}{2g} = 1.5\text{h}$ (I)

From continuity equation

$A_1V_1 = A_2V_2$

$BH\ V_1 = bh\ V_2$

$BHV_1 = bh\sqrt{gh}$

$V_1 = \frac{bh}{BH}\sqrt{gh}$

Substituting the value of V_1 in I

$H + \frac{b^2h^2}{B^2H^2}\frac{h}{2} = 1.5h$

$\frac{b^2h^3}{2B^2H^2} = 1.5h - H$

$\frac{b^2}{B^2} = 3\frac{H^2}{h^2} - 2\frac{H^3}{h^3}$

$\frac{b}{B} = \left[3\frac{H^2}{h^2} - 2\frac{H^3}{h^3}\right]^{1/2}$

(Expanding by neglecting higher order terms)

$\frac{b}{B} = \sqrt{3}\frac{h}{H} - \sqrt{2}\left(\frac{h}{H}\right)^{3/2}$ (b)

Gradually varied flow

1. A rectangular channel 7.5m wide has a uniform depth of flow of 2m and has a bed slope of 1 in 3000. If due to weir constructed at the downstream end of the channel, water surface at a section is raised by 0.75m, determine the slope of water surface (a) with respect to channel bottom and (b) with respect to horizontal at this section. Take n = 0.02 and use dynamic equation of GVF.

Solution:
Width (b) = 7.5m
Uniform flow depth (y) = 2m
Bed slope (S_0) = 1/3000
n = 0.02

A = by = 7.5x2= 15 m^2
P = b + 2y = 7.5+2x2 = 11.5 m
R =A/P = 15/11.5 = 1.304m
T = b = 7.5m
Discharge through the channel $(Q) = \frac{1}{n} A R^{2/3} S_0^{1/2}$
$= \frac{1}{0.02} 15(1.304)^{2/3}(1/3000)^{1/2}$ = 16.34 m^3/s

Computing energy slope (S_f) at the weir
y1 = 2+0.75 =2.75m
A1 = by1 = 7.5x2.75= 20.625 m^2
V= Q/A1 = 16.34/20.625 = 0.8 m/s

P1 = b + 2y 1= 7.5+2x2.75 = 13 m
R1 =A1/P1 = 20.625/13 = 1.586
$S_f = \frac{n^2 V^2}{R1^{4/3}} = \frac{0.02^2 0.8^2}{1.586^{4/3}}$ = 0.000138

Slope of water surface with respect to channel bottom $\left(\frac{dy}{dx}\right) = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A1^3}} = \frac{(1/3000) - 0.000138}{1 - \frac{16.34^2 x 7.5}{9.81 x 20.625^3}}$ = 0.0002

For rising water surface profile,
Slope of water surface with respect to horizontal $(S_w) = S_0 - \frac{dy}{dx}$ = (1/3000)-0.0002 = 0.00013

2. In a 1.8m wide rectangular channel (bottom slope = 0.002, n = 0.014), water flows at 6.72 m^3/s. A temporary dam increases the depth to 2.73m. What type of flow profile is created behind the dam?

Solution:

Width of channel (b) = 1.8m

Bottom slope (S) = 0.002

n = 0.014

Discharge (Q) = 6.72 m^3/s

Depth at dam (y) = 2.73m

q= Q/b = 6.72/1.8 = 3.73 $m^3/s/m$

Finding critical depth (y_c) and normal depth (y_n)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.73^2}{9.81}\right)^{1/3} = 1.123\text{m}$$

A = by_n = 1.8y_n

P = b+2y_n = 1.8+2y_n

$$R = \frac{A}{P} = \frac{1.8y_n}{1.8+2y_n}$$

For normal depth

$$AR^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$1.8y_n\left(\frac{1.8y_n}{1.8+2y_n}\right)^{2/3} = \frac{0.014x6.72}{\sqrt{0.002}}$$

$$\frac{y_n^{5/3}}{(1.8+2y_n)^{2/3}} = 0.789$$

Solving for y_n

y_n = 1.67m

y = 2.73m, y_c = 1.123m, y_n = 1.67m

As $y_n > y_c$, the slope is mild.

For $y > y_n > y_c$, the profile is M_1.

3. Water flows at a rate of 7.5m^3/s in a 2m wide rectangular channel having bottom slope = 0.009. If a dam at d/s increases the depth to 4m, examine the water surface profile u/s from the dam. Take n = 0.015.

Solution:

Width of channel (b) = 2m

Bottom slope (S) = 0.009

n = 0.015

Discharge (Q) = 7.5 m^3/s

Depth at dam (y) = 4m

q= Q/b = 7.5/2 = 3.5 $m^3/s/m$

Finding critical depth (y_c) and normal depth (y_n)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.5^2}{9.81}\right)^{1/3} = 1.076\text{m}$$

A = by_n = $2y_n$

P = $b+2y_n$ = $2+2y_n$

$$R = \frac{A}{P} = \frac{2y_n}{2+2y_n}$$

For normal depth

$$AR^{2/3} = \frac{nQ}{\sqrt{S}}$$

$$2y_n\left(\frac{2y_n}{2+2y_n}\right)^{2/3} = \frac{0.015x7.5}{\sqrt{0.009}}$$

$$\frac{y_n^{5/3}}{(1+y_n)^{2/3}} = 0.593$$

Solving for y_n

y_n = 0.955m

y = 4m, y_c = 1.076m, y_n = 0.955m

As $y_n < y_c$, the slope is steep.

For $y > y_c > y_n$, the profile is S_1.

4. A 3m wide rectangular channel (n = 0.0139) carries water at a rate of 9.53 m^3/s. There is an abrupt change of slope from 0.0016 (mild) to 0.0150 (steep). Flow depth is 1.3m at u/s and 0.7m at d/s. Identify the type of water surface profile u/s and d/s. Also sketch the profile.

Solution:

Width of channel (b) = 3m

Bottom slope u/s (S1) = 0.0016

Bottom slope d/s (S2) = 0.015

n = 0.0139

Discharge (Q) = 9.53 m^3/s

q= Q/b = 9.53/3 = 3.176 $m^3/s/m$

Finding critical depth (y_c) and normal depth (y_n)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.176^2}{9.81}\right)^{1/3} = 1.009\text{m}$$

A = by_n = $3y_n$

P = $b+2y_n$ = $3+2y_n$

$R = \frac{A}{P} = \frac{3y_n}{3+2y_n}$

Normal depth at u/s

$AR^{2/3} = \frac{nQ}{\sqrt{S1}}$

$3y_{n1}\left(\frac{3y_{n1}}{3+2y_{n1}}\right)^{2/3} = \frac{0.0139x9.53}{\sqrt{0.0016}}$

$\frac{y_{n1}^{5/3}}{(3+2y_{n1})^{2/3}} = 0.531$

Solving for y_{n1}

y_{n1} = 1.38m

Normal depth at d/s

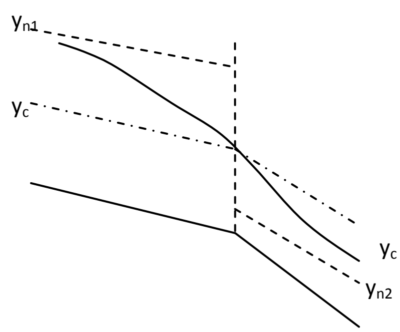
$AR^{2/3} = \frac{nQ}{\sqrt{S1}}$

$3y_{n2}\left(\frac{3y_{n2}}{3+2y_{n2}}\right)^{2/3} = \frac{0.0139x9.53}{\sqrt{0.015}}$

$\frac{y_{n2}^{5/3}}{(3+2y_{n2})^{2/3}} = 0.173$

Solving for y_{n2}

y_{n2} = 0.62m



At u/s, y = 1.3m, y_{n1} =1.38m y_c = 1.009m

At d/s, y = 0.7m, y_{n2} =0.62m y_c = 1.009m

At u/s y_{n1}>y>y_c, the profile is M_2.

At d/s y_c>y>y_{n2}, the profile is S_2.

The flow is subcritical u/s and supercritical d/s. At the break, critical flow occurs.

5. A 1.8m rectangular channel carries a discharge of 1.8m^3/s at a bed slope of 0.004. At a certain section, the depth of flow is 1m. Predict the type of profile. Take n = 0.013.

Solution:

Channel width (b) = 1.8m

Discharge (Q) = 1.8m^3/s

Bed slope (S_0) = 0.004

n= 0.013

Flow depth (y) = 1m

Type of profile = ?

Length of the profile = ?

q= Q/b = 1.8/1.8 = 1 $m^3/s/m$

Finding critical depth (y_c) and normal depth (y_n)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1^2}{9.81}\right)^{1/3} = 0.467\text{m}$$

A = by_n = $1.8y_n$

P = $b+2y_n$ = $1.8+2y_n$

$$R = \frac{A}{P} = \frac{1.8y_n}{1.8+2y_n}$$

For normal depth

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}}$$

$$1.8y_n\left(\frac{1.8y_n}{1.8+2y_n}\right)^{2/3} = \frac{0.013x1.8}{\sqrt{0.004}}$$

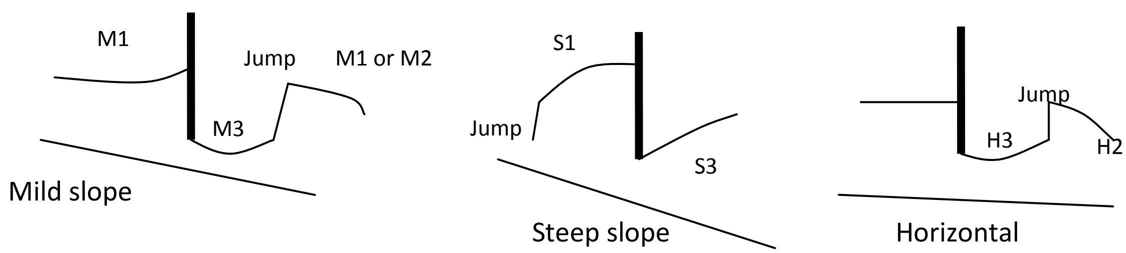
$$\frac{y_n^{5/3}}{(1.8+2y_n)^{2/3}} = 0.1389$$

Solving for y_n

y_n = 0.255m

Here $y>y_c>y_n$, the profile is S_1.

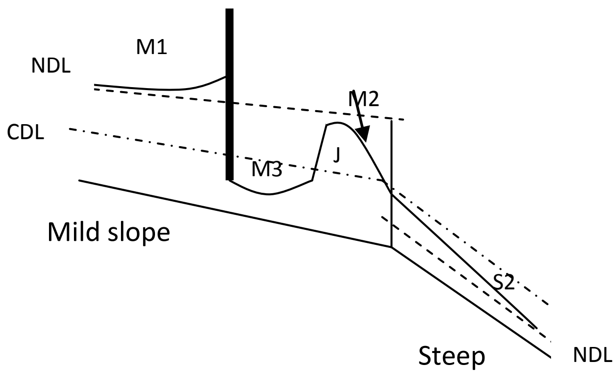
6. Sketch the flow profile having sluice gate as shown in the figures.



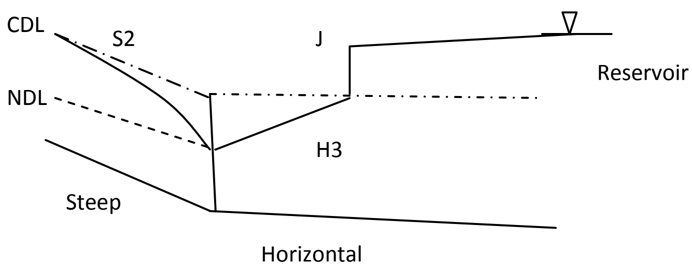
(I) (II) (III)

7. Sketch the flow profile having series of channels as shown in the figures.

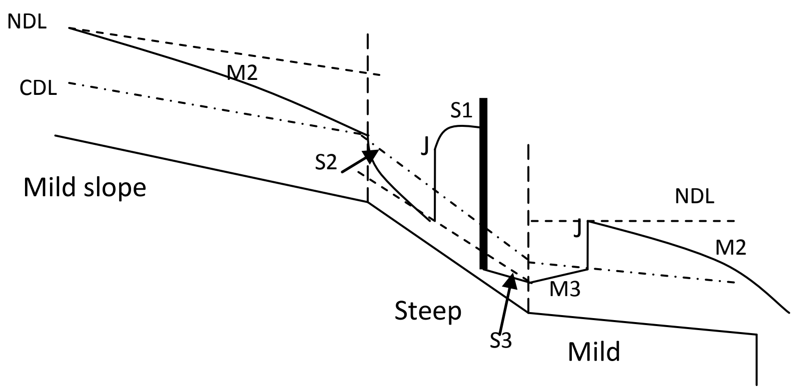
I.



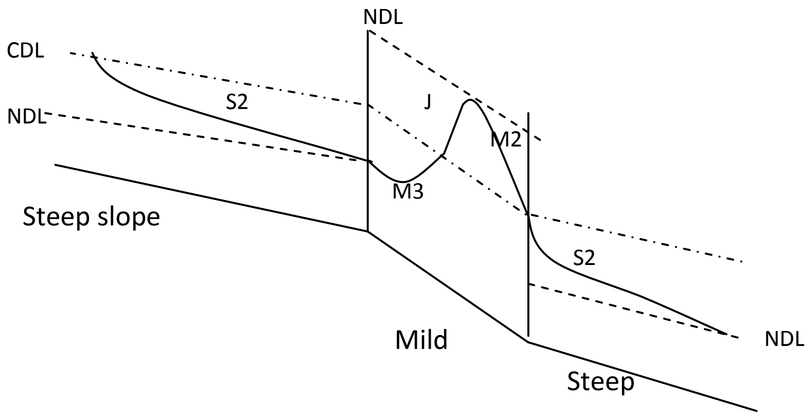
II.



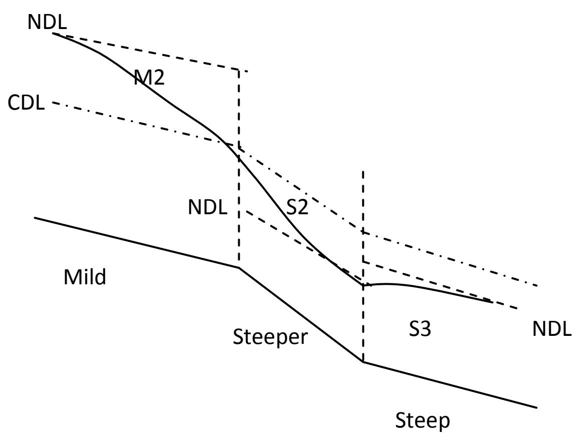
III.



IV.



V.



8. A rectangular channel is 4.5m wide and is to carry $14m^3/s$ of water. The bottom slope of the channel is 0.0015. It discharges into a stream at which the depth of flow is 3m. Calculate the distance from the channel outlet to the point where normal depth would occur under this condition. Take n = 0.013. Use direct step method and solve in one step.

Solution:
Channel width (b) = 4.5m
Discharge (Q) = $14m^3/s$
Bed slope (S_0) = 0.0015
n= 0.013
Flow depth at outlet (y) = 3m
Distance from outlet to point at which depth = normal depth (y_n) = ?

Finding normal depth
$A = by_n = 4.5y_n$

P = b+2y_n = 4.5+2y_n

$R = \frac{A}{P} = \frac{4.5y_n}{4.5+2y_n}$

For normal depth

$AR^{2/3} = \frac{nQ}{\sqrt{S_0}}$

$4.5y_n\left(\frac{4.5y_n}{4.5+2y_n}\right)^{2/3} = \frac{0.013x14}{\sqrt{0.0015}}$

$\frac{y_n^{5/3}}{(4.5+2y_n)^{2/3}} = 0.383$

Solving for y_n

y_n = 1.22m

Here y_n = y_1 =1.22m and y_2 = y = 3m

Direct step method in tabular form

y	A	P	R	V	E	Sf	$\bar{S_f}$	Δx	x
1.22	5.49	6.94	0.791	2.55	1.551	0.0015			0
3	13.5	10.5	1.286	1.04	3.055	0.00013	0.000816	2198	2198

Formulae used in the table

A= by, P = b+2y, R=A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\bar{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \bar{S_f}}$, x = cumulative sum of Δx

The distance from outlet to point at which depth is equal to normal depth = 2198m

9. A rectangular channel of 2m wide carries a discharge of 2.35 m^3/s. The depth of flow in the channel varies from 1.2m at one section to 0.9m at a section 200m d/s. Find the bed slope of the channel. n = 0.012. Use direct step method.

Solution:

Channel width (b) = 2m

Discharge (Q) =2.35 m^3/s

y_1 =1.2m

y_2 = 0.9m

Δx = 200m

n = 0.012

Bed slope (S_0) = ?

y	A	P	R	V	E	S_f	$\bar{S_f}$
1.2	2.4	4.4	0.545	0.98	1.249	0.00031	
0.9	1.8	3.8	0.474	1.31	0.987	0.00066	0.000487

Formulae used in the table

A= by, P = b+2y, R=A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\bar{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

The variables A to $\bar{S_f}$ are computed in the table above

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \bar{S_f}}$

$200 = \frac{1.249 - 0.987}{S_0 - 0.000487}$

S_0 = 0.0018

10. A rectangular concrete channel 4m wide has a slope of 9x10^{-4}. It carries a flow of 18 m^3/s and has a depth of 2.3m at one section. By using direct step method and taking one step, compute the depth 300m downstream. Take n = 0.012. What is the type of surface curve obtained?

Solution:

Channel width (b) = 4m

Discharge (Q) =18m^3/s

y_1 =2.3m

Δx = 300m

n = 0.012

Bed slope (S_0) = 9x10^{-4}

y_2 = ?

Finding critical depth (y_c)

$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{18^2}{4^2 x 9.81}\right)^{1/3}$ = 1.27m

Finding normal depth

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$AR^{2/3} = \frac{nQ}{S_0^{1/2}}$

$4y_n \left(\frac{4y_n}{4+2y_n}\right)^{2/3} = \frac{0.012x18}{(9x10^{-4})^{1/2}}$

$\frac{(4y_n)^{5/3}}{(4+2y_n)^{2/3}} = 7.2$

Solving for y_n

y_n = 1.85m

y_n>y_c: Mild slope

Section 1

A_1= by_1 = 4x2.3 = 9.2 m^2

P_1 = b+2y_1 = 4+2x2.3 = 8.6m

R_1=A_1/P_1 = 9.2/8.6 = 1.0697

V_1= Q/A_1 = 18/9.2 = 1.956 m/s

$E_1 = y_1 + \frac{V_1^2}{2g} = 2.3 + \frac{1.956^2}{2g}$ = 2.495

$S_{f1} = \frac{n^2 V_1^2}{R_1^{4/3}} = \frac{0.012^2 1.956^2}{1.0697^{4/3}} = 0.000504$

Section2

$A_2 = by_2 = 4y_2$

$P_2 = b+2y_2 = 4+2y_2$

$R_2 = A_2/P_2 = 4y_2/4+2y_2$

$V_2 = Q/A_2 = 18/4y_2$

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{(18/4y_2)^2}{2g} = y_2 + 1.032/y_2^2$$

$$S_{f1} = \frac{n^2 V_1^2}{R_1^{4/3}} = \frac{0.012^2 (18/4y_2)^2}{\left(\frac{4y_2}{4+2y_2}\right)^{4/3}} = \frac{0.0466}{(4y_2)^{10/3}}(4+2y_2)^{4/3}$$

$$\overline{S_f} = \frac{1}{2}\left[S_{f1} + S_{f2}\right]$$

$$\overline{S_f} = \frac{1}{2}\left[0.000504 + \frac{0.0466}{(4y_2)^{10/3}}(4+2y_2)^{4/3}\right] = \left[0.000252 + \frac{0.0233}{(4y_2)^{10/3}}(4+2y_2)^{4/3}\right]$$

$$\Delta x = \frac{E_2 - E_1}{S_0 - \overline{S_f}}$$

$$300 = \frac{y_2 + 1.032/y_2^2 - 2.495}{9x10^{-4} - \left[0.000252 + \frac{0.0233}{(4y_2)^{10/3}}(4+2y_2)^{4/3}\right]}$$

$$y_2 + \frac{1.032}{y_2^2} - 2.495 = 0.1944 - \frac{6.99}{(4y_2)^{10/3}}(4+2y_2)^{4/3}$$

$$y_2 + \frac{1.032}{y_2^2} + \frac{6.99}{(4y_2)^{10/3}}(4+2y_2)^{4/3} = 2.6894$$

Solving for y_2 ($y_2 > y_1$ for backwater)

$y_2 = 2.46$m

As $y > y_n > y_c$: the profile is M_1.

11. A trapezoidal channel has a constant bed slope of 0.0015, b=3m and Z:1 = 1:1. A control gate increases the depth immediately u/s to 4m when discharge is 19m^3/s. Compute GVF profile from the gate to the point at which the depth = 3m. Use direct step method and perform computation for water depth at 0.1m interval. Take n = 0.017.

Solution:

Bottom width (b) = 3m

Z:1 = 1:1

Bed slope (S_0) = 0.0015

Discharge (Q) = 19m^3/s

n =0.017

GVF Computation using direct step method

y	A	P	R	V	E	Sf	Sfb	dx	x
4	28	14.314	1.956	0.679	4.0235	0.000054			0
3.9	26.91	14.031	1.918	0.706	3.9254	0.000060	0.0000570	-68.0	68.0
3.8	25.84	13.748	1.880	0.735	3.8276	0.000067	0.0000635	-68.1	136.1
3.7	24.79	13.465	1.841	0.766	3.7299	0.000075	0.0000710	-68.3	204.4
3.6	23.76	13.182	1.802	0.800	3.6326	0.000084	0.0000795	-68.5	272.9
3.5	22.75	12.899	1.764	0.835	3.5356	0.000095	0.0000895	-68.8	341.7
3.4	21.76	12.617	1.725	0.873	3.4389	0.000107	0.0001010	-69.1	410.8
3.3	20.79	12.334	1.686	0.914	3.3426	0.000120	0.0001135	-69.4	480.3
3.2	19.84	12.051	1.646	0.958	3.2467	0.000136	0.0001280	-69.8	550.1
3.1	18.91	11.768	1.607	1.005	3.1515	0.000155	0.0001455	-70.4	620.5
3	18	11.485	1.567	1.056	3.0568	0.000177	0.0001660	-71.0	691.4

Formulae used in the table

$A = (b + Zy)y,\ P = b + 2y\sqrt{1 + Z^2}$, R =A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[(S_f)_{i-1} + (S_f)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$, x = cumulative sum of Δx

12. A long rectangular channel 8m wide, bed slope 1:5000 and n = 0.015 conveys discharge of 40m^3/s. A sluice gate raises depth immediately u/s to 5.0m. Compute GVF profile from the gate to a distance of 1000m at 500m interval using standard step method. Take bed elevation at sluice gate = 100m, next station = 100.5m and last station = 100.7m

Solution:

Channel width (b) = 8m

Bed slope (S_0) = 1/5000 = 0.0002

n = 0.015

Discharge (Q) = 40m^3/s

Formulae to be used

A= by, P = b+2y, R=A/P

H= Z+ V^2/2g (where Z = water surface elevation)

$S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[(S_f)_{i-1} + (S_f)_i\right]$

$h_f = \Delta x \overline{S_f}$

H(2) = $(H(2))_{i-1} + h_f + h_e$

Computation of GVF by standard step method

st	Trial	Z	y	A	P	R	V	H(1)	S_f	$\bar{S_f}$	Δx	h_f	h_e	H(2)
A		105	5	40	18	2.2222	1	105.051	0.000078					105.051
B	1	105.400	4.9	39.2	17.8	2.2022	1.02	105.453	0.000082	0.000080	500	0.0398		105.091
	2	105.350	4.85	38.8	17.7	2.1921	1.031	105.404	0.000084	0.000083	500	0.0414		105.132
	3	105.300	4.8	38.4	17.6	2.1818	1.042	105.355	0.000086	0.000085	500	0.0426		105.175
	4	105.200	4.7	37.6	17.4	2.1609	1.064	105.258	0.000091	0.000089	500	0.0444		105.219
	5	105.210	4.71	37.68	17.42	2.163	1.062	105.267	0.000091	0.000091	500	0.0454		105.265
C	1	105.370	4.67	37.36	17.34	2.1546	1.071	105.428	0.000093	0.000092	500	0.0458		105.310
	2	105.350	4.65	37.2	17.3	2.1503	1.075	105.409	0.000094	0.000093	500	0.0466		105.357
	3	105.345	4.65	37.16	17.29	2.1492	1.076	105.404	0.000094	0.000094	500	0.0469		105.404

Final values

Station	Water surface elev.
A	105
B	105.21
C	105.345

13. A rectangular channel (n = 0.011) is 1.5m wide and carries 1.7 m^3/s of water. The bed slope is 0.0005., and at section 1 the depth is 0.9m. Find the distance to section 2, where the depth is 0.75m. Use direct integration method (Bresse's method).

Solution:
Channel width (b) = 1.5m
Discharge (Q) = 1.7 m^3/s
Bed slope (S_0) = 0.0005
n = 0.011
Flow depth at section 1 (y1) = 0.9m
Flow depth at section 2 (y2) = 0.75m
Distance to section 2 (L) = ?

q = Q/b = 1.7/1.5 = 1.133 $m^3/s/m$
Finding critical depth (y_c) and normal depth (y_n)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.133^2}{9.81}\right)^{1/3} = 0.508\text{m}$$

A = by_n = $1.5y_n$
P = $b+2y_n$ = $1.5+2y_n$

$$R = \frac{A}{P} = \frac{1.5y_n}{1.5+2y_n}$$

For normal depth

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}}$$

$$1.5y_n\left(\frac{1.5y_n}{1.5+2y_n}\right)^{2/3} = \frac{0.011x1.7}{\sqrt{0.0005}}$$

$$\frac{y_n^{5/3}}{(1.5+2y_n)^{2/3}} = 0.425$$

Solving for y_n
y_n = 0.985m
$u_1 = y_1/y_n$ = 0.9/0.985 = 0.913
$u_2 = y_2/y_n$ = 0.75/0.985 = 0.761

$$\varphi(u_1) = \frac{1}{6}ln\frac{u_1^2+u_1+1}{(u_1-1)^2} - \frac{1}{\sqrt{3}}tan^{-1}\frac{\sqrt{3}}{2u_1+1}$$

$$\varphi(u_1) = \frac{1}{6}ln\frac{0.913^2+0.913+1}{(0.913-1)^2} - \frac{1}{\sqrt{3}}tan^{-1}\frac{\sqrt{3}}{2x0.913+1} = 0.109$$

$$\varphi(u_2) = \frac{1}{6}ln\frac{u_2^2+u_2+1}{(u_2-1)^2} - \frac{1}{\sqrt{3}}tan^{-1}\frac{\sqrt{3}}{2u_2+1}$$

$$\varphi(u_2) = \frac{1}{6}ln\frac{0.761^2+0.761+1}{(0.761-1)^2} - \frac{1}{\sqrt{3}}tan^{-1}\frac{\sqrt{3}}{2x0.761+1} = -0.078$$

$L = \frac{y_n}{S_0}\left[(u_2 - u_1) - \left(1 - \left(\frac{y_c}{y_n}\right)^3\right)(\varphi(u_2) - \varphi(u_1))\right]$

$L = \frac{0.985}{0.005}\left[(0.761 - 0.913) - \left(1 - \left(\frac{0.508}{0.985}\right)^3\right)(-0.768 - 0.109)\right]$

= 16.4m

14. A very wide rectangular channel with horizontal bed terminates in a vertical drop. If it discharges 3.5 $m^3/s/m$ of water, estimate depth at the drop. How long on upstream depth will be 2.5m? Take n = 0.017. Use direct step method and solve in 3 steps.

Solution:

Discharge per unit width (q) = 3.5 $m^3/s/m$

Depth at drop = ?

Finding critical depth (y_c)

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.5^2}{9.81}\right)^{1/3}$ = 1.0768m

Depth at drop is assumed to be equal to critical depth, although it is a brink depth slightly less than critical depth. Hence, depth at drop = 1.0768m

Computing length by direct step method (S_0 = 0)

y	R	V	E	S_f	$\bar{S_f}$	Δx	x
1.07685	1.07685	3.25	1.6152	0.002766			0
1.5	1.5	2.33	1.7767	0.000914	0.00184	-87.7866	87.7866
2	2	1.75	2.1561	0.000351	0.000632	-599.836	687.622
2.5	2.5	1.4	2.5999	0.000167	0.000259	-1712.95	2400.57

Distance at which water depth becomes 2.5 m is 2400.57m

Formulae used in the table

R=y (For wide rectangular channel)

V= q/y, E = y +$V^2/2g$, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\bar{S_f} = \frac{1}{2}\left[(S_f)_{i-1} + (S_f)_i\right]$, $\Delta x = \frac{E_i - E_{i-1}}{S_0 - \bar{S_f}}$

15. A wide rectangular channel having a bottom slope of 5×10^{-3} has a constant value of Chezy's coefficient equal to 76. If discharge per unit width is 4.65 m^3/s and if the channel joins a reservoir in which water depth is 3m above channel bed, find the length of surface curve. Use direct step method taking maximum of 5 steps.

Solution:

Discharge per unit width (q) = 4.65 $m^3/s/m$
Bottom slope (S_0) = $5x10^{-3}$
C = 76
R=y (For wide rectangular channel)
Finding critical depth (y_c)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{4.65^2}{9.81}\right)^{1/3} = 1.3\text{m}$$

Finding normal depth (y_n)

$$Q = AC\sqrt{RS}$$

$$Q = by_nC\sqrt{y_nS}$$

$$y_n = \left(\frac{q}{C\sqrt{S}}\right)^{2/3} = \left(\frac{4.65}{76\sqrt{5x10^{-3}}}\right)^{2/3} = 0.9\text{m}$$

$y_c > y_n$: Steep slope
As the channel joins a reservoir, the profile is backwater curve (S_1). So the depth decreases from d/s to u/s. Computation is done from d/s to the point at which critical depth occurs.

Computation by Direct step method taking 5 steps

y	R	V	E	S_f	$\bar{S_f}$	Δx	x
3	3	1.55	3.1225	0.000139			0.0
2.8	2.8	1.66	2.9406	0.000171	0.000155	-37.54	37.54
2.5	2.5	1.86	2.6763	0.00024	0.000206	-55.11	92.65
2	2	2.325	2.2755	0.000468	0.000354	-86.27	178.92
1.5	1.5	3.1	1.9898	0.001109	0.000789	-67.84	246.76
1.3	1.3	3.576923	1.9521	0.001704	0.001407	-10.49	257.25

Length of surface curve = 257.25m

Formulae used in the table

V= q/y, E = y +V^2/2g, $S_f = \frac{V^2}{C^2R}$, $\bar{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$, $\Delta x = \frac{E_i - E_{i-1}}{S_0 - \bar{S_f}}$

16. A rectangular channel conveying a discharge of 30 m^3/s is 12m wide with a bed slope 1 in 6000 and having Manning's n = 0.025. The depth of flow at a section is 1.5m. Find how far upstream or downstream of this section the depth of flow will be 2m. Find also the type of profile. Use direct step method for calculation and take only two steps for calculation.

Solution:
Discharge (Q) = 30 m^3/s
Channel width (b) = 12m
Bed slope (S_0) = 1/6000
n = 0.025

Computation by direct step

y	A	p	R	V	E	S_f	$\bar{S_f}$	Δx	x
1.5	18	15	1.2	1.67	1.642	0.001367			0
1.8	21.6	15.6	1.385	1.39	1.898	0.000782	0.001075	-281.9	281.9
2	24	16	1.5	1.25	2.08	0.000569	0.000676	-357.6	639.5

Formulae used in the table

$A = by,\ P = b + 2y$, R =A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\bar{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \bar{S_f}}$, x = cumulative sum of Δx

At a distance of 639.5m, the depth is 2m.

Critical depth is given by

$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{30^2}{12^2 x 9.81}\right)^{1/3}$ = 0.86m

Computing normal depth

$Q = \frac{1}{n} AR^{2/3} S^{1/2}$

$AR^{2/3} = \frac{nQ}{\sqrt{S}}$

$by_n \left(\frac{by_n}{b+2y_n}\right)^{2/3} = \frac{nQ}{\sqrt{S}}$

$12y_n \left(\frac{12y_n}{12+2y_n}\right)^{2/3} = \frac{0.025x30}{\sqrt{1/6000}}$

$\frac{y_n^{5/3}}{(12+2y_n)^{2/3}} = 0.9236$

Solving for y_n

y_n = 3.03m

As $y_n > y_c$, the channel slope is Mild.

As $y_n > y > y_c$, the profile is M2. Therefore, 2m depth occurs at a distance of 639.5m u/s of section 1.

17. A wide rectangular channel conveys a discharge of 5 m^3/s with a bed slope of 1 in 3600 with Manning's coefficient (n) = 0.02. If the depth at a section is 3.5m, determine how far upstream or downstream of the section, the depth would vary within 5% of the normal depth. Find the nature of the profile and make calculation with direct step method and take only 2 steps for calculation.

Solution:

Discharge per unit width (q) = 5 m^3/s (Considering unit width)

Bed slope (S_0) = 1/3600

n = 0.02

For wide rectangular channel, hydraulic radius (R) = y

Finding critical depth (y_c)

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{5^2}{9.81}\right)^{1/3}$ = 1.366m

Computing normal depth (y_n)

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$Q = \frac{1}{n} (b y_n) y_n^{2/3} S_0^{1/2}$

$y_n = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} = \left(\frac{0.02x5}{\sqrt{1/3600}}\right)^{3/5}$ = 2.93m

As $y_n > y_c$, the channel slope is Mild.

Depth at section 1 (y_1) = 3.5m

As $y > y_n > y_c$, the flow profile is M_1 (backwater). So the depth reduces u/s of section 1.

At last section, the depth is 5% more than y_n.

1.05 y_n = 3.07m

Formulae used in the table

R=y, V= q/y, E = y +V^2/2g, $S_f = \frac{n^2 V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$, $\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$

Computation by Direct step method taking 2 steps

y	R	V	E	S_f	$\overline{S_f}$	Δx	x
3.5	3.5	1.43	3.604	0.000154			0
3.3	3.3	1.515	3.417	0.000187	0.000171	-1745	1745
3.07	3.07	1.63	3.205	0.000238	0.000213	-3241	4986

At a distance of 4986m upstream, the depth varies within 5% of normal depth.

18. The normal depth in a 10m wide rectangular channel having a bottom slope 0.001 is 2m. The Manning's n is 0.02. The construction of bridge raises the upstream water level by 1m. Determine the distance from the bridge where the flow depth is 2.5m.

Solution:

Width (b) = 10m

Channel bed slope (S_0) = 0.001

n = 0.02

Normal depth (y_n) =2m

Discharge

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$= \frac{1}{0.02} (10x2) x \left(\frac{10x2}{10+2x2}\right)^{2/3} x (0.001)^{1/2}$

= 40.1 m^3/s

Computation by direct step taking single step

y	A	p	R	V	E	S_f	$\overline{S_f}$	Δx	x
3	30	16	1.875	1.33	3.09	0.000306			0
2.5	25	15	1.66	1.6	2.63	0.000521	0.000414	-785	-785

Distance from the bridge where flow depth is 2.5m = 785m

Formulae used in the table

$A = by,\ P = b + 2y$, R =A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[(S_f)_{i-1} + (S_f)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$, x = cumulative sum of Δx

Hydraulic jump

1. Water flows through a 6m wide rectangular channel. The upstream depth is 0.6m. Will hydraulic jump occur for (a) Q = 5.6 m^3/s and (b) 17.92 m^3/s? Compute sequent depth if jump occurs.

Solution:
Channel width (b) = 6m
u/s depth (y_1) = 0.6m

a) Q = 5.6 m^3/s
u/s Velocity (V_1) = $\frac{Q}{by_1} = \frac{5.6}{6x0.6}$ = 1.55 m/s

u/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{1.55}{\sqrt{9.81x0.6}}$ = 0.64
For hydraulic jump to occur, the u/s flow should be supercritical. As F_{r1}<1, the flow is sub-critical and the jump cannot occur.

a) Q = 17.92 m^3/s
u/s Velocity (V_1) = $\frac{Q}{by_1} = \frac{17.92}{6x0.6}$ = 4.97 m/s

u/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{4.97}{\sqrt{9.81x0.6}}$ = 2.04
For hydraulic jump to occur, the u/s flow should be supercritical. As F_{r1}>1, the flow is supercritical and the jump occurs.

Sequent depth (y_2) = ?
$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$
$= \frac{0.6}{2}\left(-1 + \sqrt{1 + 8x2.04^2}\right)$ = 1.46m

2. Water flows through a 3.6m wide rectangular channel at a rate of 9.15 m^3/s at a velocity of 6m/s. Will hydraulic jump occur? If so, compute the energy lost in the jump.

Solution:
Channel width (b) = 3.6m
u/s velocity (V_1) = 6m/s
Discharge (Q) = 9.15 m^3/s

A = Q/V_1 = 9.15/6 = 1.525 m^2
u/s depth (y_1) = A/b = 1.525/3.6 = 0.42 m

u/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{6}{\sqrt{9.81x0.42}}$ = 2.95>1.

As Fr_1>1, the flow is supercritical at u/s and the hydraulic jump will occur.

Sequent depth (y_2)

$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$= \frac{0.42}{2}\left(-1 + \sqrt{1 + 8x2.95^2}\right)$ = 1.55m

Energy lost (ΔE) is

$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$

$= \frac{(1.55-0.42)^3}{4x0.42x1.55}$ = 0.55m

3. Water flows at a rate of 20 m^3/s through a rectangular section of 4m wide creating a hydraulic jump. The upstream depth of flow is 1.2m. Compute (a) upstream Froude number, (b) downstream depth, (c) downstream Froude number, (d) energy lost, (e) power in the jump (f) height of jump, (g) length of jump, and (h) efficiency of jump.

Solution:

Channel width (b) = 4m

u/s depth (y_1) = 1.2m

Discharge (Q) = 20 m^3/s

a) u/s Froude number (F_{r1}) = ?

u/s Velocity (V_1) = $\frac{Q}{by_1} = \frac{20}{4x1.2}$ = 4.16 m/s

u/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{4.16}{\sqrt{9.81x1.2}}$ = 1.2

b) d/s depth (y_2)= ?

$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$= \frac{1.2}{2}\left(-1 + \sqrt{1 + 8x1.2^2}\right)$ = 1.52m

c) d/s Froude number (F_{r2}) = ?

d/s Velocity (V_2) = $\frac{Q}{by_2} = \frac{20}{4x1.52}$ = 3.29 m/s

d/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{3.29}{\sqrt{9.81x1.52}}$ = 0.85

d) Energy lost (ΔE) =?

$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$

$= \frac{(1.52-1.2)^3}{4x1.2x1.52}$ = 0.0045m

e) Power in the jump (P) = ?

$P = \gamma Q(\Delta E)$ = 9810x20x0.0045 = 883W

f) Height of jump (Hj) = ?

$H_j = y_2 - y_1$ = 1.52-1.2 = 0.32m

g) Length of jump (L_j) =?

$L_j = 6\ H_j$ = 6x0.32 = 1.92m

h) Efficiency of jump (η) =?

$E_1 = y_1 + \frac{V_1^2}{2g} = 1.2 + \frac{4.16^2}{2x9.81}$ = 2.082m

$E_2 = y_2 + \frac{V_2^2}{2g} = 1.52 + \frac{3.29^2}{2x9.81}$ = 2.07m

$\eta = \frac{E_2}{E_1} = \frac{2.07}{2.082}$ = 0.99

4. Water flows over a concrete spillway into a rectangular channel 9m wide through a hydraulic jump. The depths before and after jump are 1.55m and 3.08m respectively. Find the critical depth, rate of flow in the channel and energy lost.

Solution:

Channel width (b) = 9m

Depth before jump (y_1) = 1.55m

Depth after jump (y_2) =3.08m

Critical depth (y_c) =?

Rate of flow (Q) = ?

Energy lost (ΔE) =?

Finding u/s Froude number

$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$3.08 = \frac{1.55}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

F_{r1} = 1.72

$F_{r1} = \frac{V_1}{\sqrt{gy_1}}$

$1.72 = \frac{V_1}{\sqrt{9.81x1.55}}$
V_1 = 6.7m/s

Q = A_1 V_1 = (9x1.55)x6.7 = 93.46 m^3/s
q = Q/b = 93.46/9 = 10.38 $m^3/s/m$

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{10.38^2}{9.81}\right)^{1/3}$ = 2.22m

$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$

$= \frac{(3.08-1.55)^3}{4x1.55x3.08}$ = 0.187m

5. If, in a hydraulic jump occurring on a horizontal rectangular channel, the Froude number before the jump is 10 and the energy loss is 5m, compute (a) sequent depth, (b) discharge per unit width, and (c) Froude number after jump.

Solution:
Froude no. before jump (F_{r1}) = 10
Energy loss (ΔE) = 5m
Sequent depths (y_1, y_2) =?
Discharge per unit width (q) =?
Froude no. after jump (F_{r2}) = ?

Finding u/s Froude number
$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$
$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8x10^2}\right)$
y_2 = 13.65 y_1

$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$
$5 = \frac{(13.65y_1-y_1)^3}{4y_1x13.65y_1}$
y_1 = 0.13m
y_2 = 13.65 x0.13 = 1.77m

$F_{r1} = \frac{V_1}{\sqrt{gy_1}}$
$10 = \frac{V_1}{\sqrt{9.81x0.13}}$
V_1 = 11.3m/s

$q = V_1 y_1 = 11.3x0.13 = 1.47\ m^3/s/m$

$V_2 = q/y_2 = 1.47/1.77 = 0.83 m/s$

$F_{r2} = \frac{V_2}{\sqrt{g y_2}} = \frac{0.83}{\sqrt{9.81 X 1.77}} = 0.2$

6. In a hydraulic jump occurring in a rectangular channel, the discharge per unit width is 2 $m^3/s/m$. If the energy loss through the jump is 1.75m, compute the sequent depths.

Solution:
Discharge per unit width (q) = 2 $m^3/s/m$
Energy loss (ΔE) = 1.75m
Sequent depths (y_1, y_2) =?
Solving by trial and error: Assume different value of F_{r1} and check whether the computed value is almost equal to given value of ΔE.

Fr_1	y_2/y_1	y_1	y_2	ΔE
3	3.772	0.356	1.345	0.503
4	5.179	0.294	1.524	1.037
5	6.589	0.254	1.671	1.680
5.1	6.730	0.250	1.684	1.749

Formulae
$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$
$F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{q}{y_1 \sqrt{g y_1}}$. Get y_1. ($y_1 = \left(\frac{q}{F_{r1}\sqrt{g}}\right)^{2/3}$
y_2 = column2xcolumn3
$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$

For F_{r1}= 5.1, computed ΔE is equal to given ΔE. With this Froude no., the sequent depths are 0.25m and 1.684m.

7. At the toe of the hydraulic jump, the Froude number and the depth of flow are 9 and 0.4m respectively. Compute the specific energy at the heel of the jump.

Solution:
Froude no. before jump (F_{r1}) = 10
Depth of flow before jump (y_1) = 0.4
Specific energy at heel of jump (E_2) = ?

$F_{r1} = \frac{V_1}{\sqrt{gy_1}}$

$9 = \frac{V_1}{\sqrt{9.81x0.4}}$

V_1 = 17.82m/s

u/s specific energy $(E_1) = y_1 + \frac{V_1^2}{2g} = 0.4 + \frac{17.82^2}{2x9.81}$ = 17.58m

$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

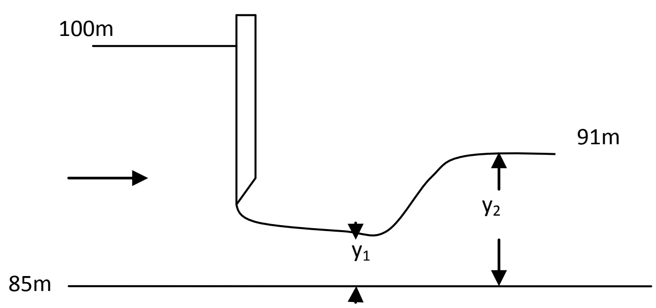
$= \frac{0.4}{2}\left(-1 + \sqrt{1 + 8x9^2}\right)$ = 4.9m

$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$

$= \frac{(4.9-0.4)^3}{4x0.4x4.9}$ = 11.62m

$E_2 = E_1 - \Delta E$ = 17.58-11.62 = 5.96m

8. An opening of a reservoir is controlled by a sluice gate. The opening is 1.3m. Water surface elevation u/s of gate is 100m. If the bed elevation is 85m and the tailwater elevation is 91m, what kind of jump will form? What are the sequent depths? Assume coefficient of contraction = 0.75 and coefficient of velocity = 0.95 for the sluice gate.



Solution: Head u/s (h) =100-85 = 15m

Tail water depth = 91-85 = 5m

Coefficient of contraction (C_c) = 0.75

Coefficient of velocity (C_v) = 0.95

Coefficient of discharge (C_d) = C_c C_v = 0.75x0.95 = 0.7125

Type of jump = ?

Sequent depths (y_1, y_2) = ?

Velocity u/s (V_1) = $C_v\sqrt{2gh} = 0.95\sqrt{2x9.81x15}$ = 16.29m/s

Discharge per unit width (q) = $a\ C_d\sqrt{2gh} = 1.3x1x0.7125\sqrt{2x9.81x15}$ = 15.88 m^3/s/m

Depth before jump (y_1) = q/V_1 = 16.29/15.88 = 0.975m

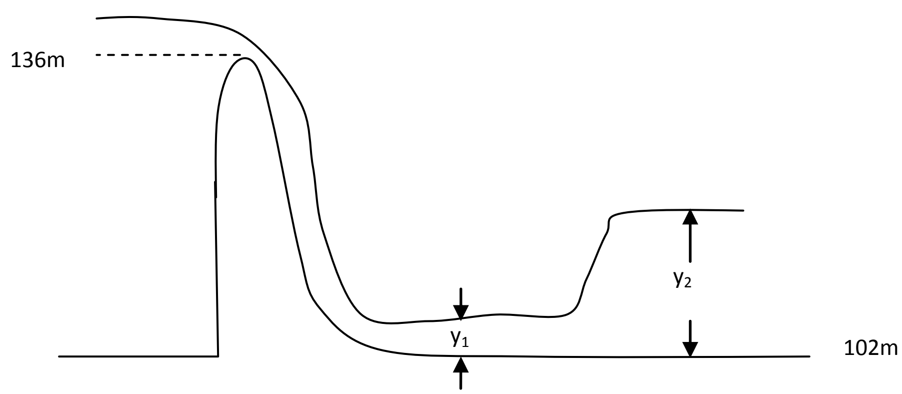
$F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{16.29}{\sqrt{9.81x0.975}}$ = 5.27

As F_{r1} is in between 4.5 and 9, steady jump occurs.

$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8F_{r1}^2}\right)$

$= \frac{0.975}{2}\left(-1+\sqrt{1+8x5.27^2}\right)$ = 6.8m

9. An overflow spillway has its crest at elevation 136m and a horizontal apron at an elevation of 102m on the d/s side. Estimate the tailwater elevation required to form a hydraulic jump when the elevation of the energy line just u/s of the spillway crest is 138m. Assume C_d = 0.735m for the spillway. Neglect energy loss due to flow over the spillway.



Solution:

Head (H) = 138-136 = 2m

Coefficient of discharge (C_d) = 0.735

Specific energy at u/s of spillway (E_u) = 138-102 = 36m (neglecting velocity head)

Tailwater elevation = ?

Discharge per unit width (q) = $\frac{2}{3}C_d\sqrt{2g}H^{3/2} = \frac{2}{3}x0.735\sqrt{2x9.81}x2^{3/2}$ = 6.139 m^3/s/m

$E_u = E_1$

$36 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2}$

$36 = y_1 + \frac{6.139^2}{2x9.81y_1^2}$

$36 = y_1 + \frac{1.92}{y_1^2}$

Solving

$y_1 = 0.232$m

$V_1 = q/ y_1 = 6.139/0.232 = 26.46$m/s

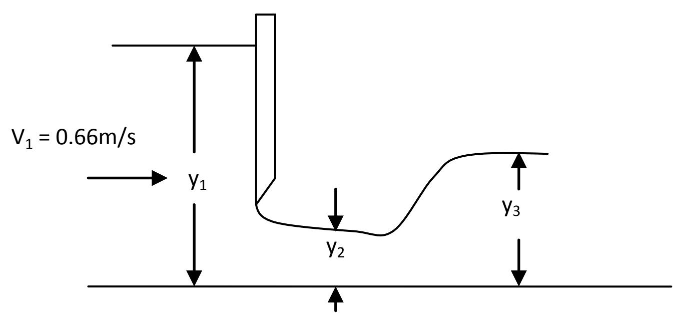
$F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{26.46}{\sqrt{9.81x0.232}} = 17.5$

$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$= \frac{0.232}{2}\left(-1 + \sqrt{1 + 8x17.5^2}\right) = 5.627$m

Tailwater elevation = 102+5.627 = 107.627m

10. Consider the flow under the sluice gate of the fig. If y_1 = 2.7m and all losses are neglected except the dissipation in the jump, calculate y_2 and y_3 and the percent dissipation ($100x\Delta E/E_2$). The channel is horizontal and wide.



Solution:

y_1 = 2.7m, V_1 =0.66m/s

$q = V_1 y_1 = 0.66x2.7 = 1.782$ m^3/s/m

$E_1 = E_2$

$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$

$2.7 + \frac{1.782^2}{2x9.81x2.7^2} = y_2 + \frac{1.782^2}{2x9.81y_2^2}$

$2.722 = y_2 + \frac{0.16}{y_2^2}$

Solving

$y_2 = 0.255$m

$V_2 = q/y_2 = 1.782/0.255 = 6.98$m/s

$F_{r2} = \frac{V_2}{\sqrt{gy_2}} = \frac{6.98}{\sqrt{9.81x0.255}}$ = 4.4

$y_3 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

$= \frac{0.255}{2}\left(-1 + \sqrt{1 + 8x4.4^2}\right)$ = 1.464m

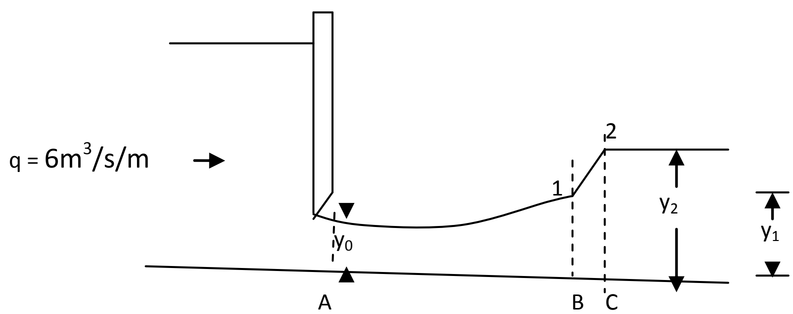
$\Delta E = \frac{(y_3 - y_2)^3}{4y_2y_3}$

$= \frac{(1.464-0.255)^3}{4x0.255x1.464}$ = 1.18m

$E_2 = y_2 + \frac{V_2^2}{2g} = 1.464 + \frac{6.98^2}{2x9.81}$ = 2.7m

Percent dissipation = $\frac{1.18}{2.7} x100$ = 43.7%

11. Water flows under a sluice gate in a very wide rectangular at the rate of 6m^3/s/m. Given Bed slope = 1 in 900, n = 0.015, depth at vena-contracta (y_1) = 1m, verify whether hydraulic jump occurs and also find the location of location jump downstream of the gate. Use direct step method and solve in 1 step.



Solution:

Discharge (q) = 6m^3/s/m

n = 0.015

Bed slope (S_0) = 1/900

Depth at vena-contracta (y_0) = 1m

Location of jump (AB) = ?

For wide rectangular channel R = y

Velocity at vena-contracta (V_0) = q/y_0 =6/1 = 6 m/s

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{6^2}{9.81}\right)^{1/3}$ = 1.542m

Finding normal depth

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$Q = \frac{1}{n} b y_n {y_n}^{2/3} S_0^{1/2}$

$q = \frac{1}{n} {y_n}^{5/3} S_0^{1/2}$

$6 = \frac{1}{0.015} {y_n}^{5/3} (1/900)^{1/2}$

y_n = 1.814m

As $y_n > y_c$, M3 profile will be formed d/s of gate.

At vena-contracta

$F_r = \frac{V_0}{\sqrt{g y_0}} = \frac{6}{\sqrt{9.81x1}}$ = 1.915>1. The flow is supercritical. ($y_0 < y_c$)

At 2, the depth becomes equal to normal depth.

$y_2 = y_n$ = 1.814m

$V_2 = q/y_2$ = 6/1.814 = 3.307m/s

$F_{r2} = \frac{V_2}{\sqrt{g y_2}} = \frac{3.307}{\sqrt{9.81x1.814}}$ = 0.78 <1. The flow is subcritical.

As the flow changes from supercritical to subcritical, hydraulic jump will occur.

Finding depth before jump (y_1)

$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

$= \frac{1.814}{2}\left(-1 + \sqrt{1 + 8x0.78^2}\right)$ = 1.289m

Length of jump (BC) = 6(y_2-y_1) = 6(1.814-1.289) = 3.15m

Finding length AB by direct step method with y_0 = 1m and y_1 = 1.289, S_0 = 1/900 (single step)

y	R	V	E	S_f	$\overline{S_f}$	Δx
1	1	6.000	2.835	0.0081		
1.289	1.289	4.654771	2.393	0.003475	0.005788	94.42

AB = 94.42m

The jump starts at a distance of 94.42m d/s of vena-contracta.

Formulae used in the table

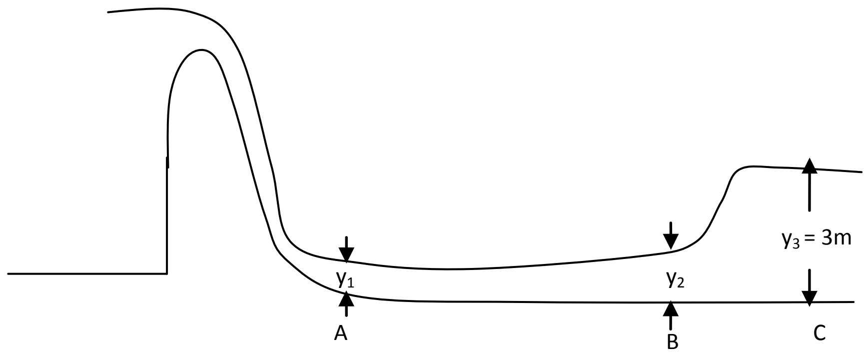
R=y (For wide rectangular channel)

V= q/y, E = y +V^2/2g, $S_f = \frac{n^2 V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$

12. After flowing over the concrete spillway of a dam, 252 m^3/s then passes over a level concrete apron (n= 0.013). The velocity of the water at the bottom of the spillway is 12.6m/s and the width of apron is 54m. Condition will produce a hydraulic jump, the depth in the channel below the apron being 3m. In order that the jump be contained on the apron, (a) how long should the apron be built (length ABC) ?

(Use direct step method to compute AB and hydraulic jump equation to compute BC) (b) How much energy is lost from the foot of the spillway (A) to the d/s of the jump (C).



Solution:
Discharge (Q) = 252 m^3/s
Velocity at 1 (V_1) = 12.6 m/s
Width of apron (b) = 54m
Depth at 3 (y_3) = 3m
n = 0.013
Length of apron = ?
Loss of energy between A and C = ?

Discharge per unit width (q) = Q/b = 252/54 = 4.67 $m^3/s/m$
Depth at 1 (y_1) = q/V_1 = 4.67/12.6 = 0.37m

$F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{12.6}{\sqrt{9.81x0.37}}$ = 6.61
As Fr_1>1, the flow at 1 is supercritical.

Velocity at 3 (V_3) = q/y_3 = 4.67/3 = 1.56m/s

$F_{r3} = \frac{V_3}{\sqrt{gy_3}} = \frac{1.56}{\sqrt{9.81x3}}$ = 0.287
As Fr_1<1, the flow at 3 is subcritical.
As the flow changes from supercritical to subcritical, hydraulic jump will occur. As the floor is horizontal. H3 profile will occur before the jump.

Finding y_2
$y_2 = \frac{y_3}{2}\left(-1 + \sqrt{1 + 8F_{r3}^2}\right)$
$= \frac{3}{2}\left(-1 + \sqrt{1 + 8x0.287^2}\right)$ = 0.43m

Finding length AB by direct step method with y_1 = 0.37m and y_2 = 0.43m, S_0 = 0 (horizontal)

y	A	P	R	V	E	S_f	$\overline{S_f}$	Δx	AB
0.37	19.98	93.96	0.213	12.6	8.462	0.211			
0.43	23.22	100.44	0.231	10.85	6.433	0.140	0.176	11.54	11.54

The jump starts at a distance of 11.54m from the foot of spillway.

Formulae used in the table

A= by, P = b+2y, R=A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[(S_f)_{i-1} + (S_f)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$

Length of jump (BC) = 6(y_3-y_2) = 6(3-0.43) = 15.42m

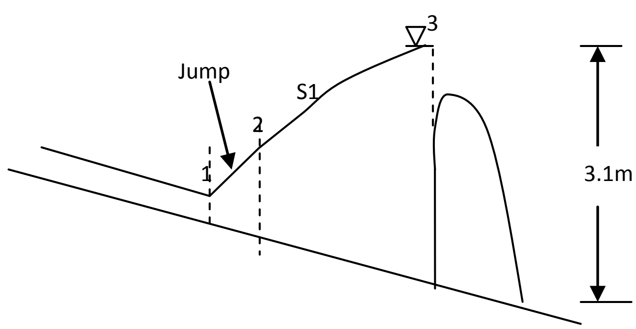
Length of apron = AB+BC = 11.54+15.42 = 26.96m

$E_A = y_1 + \frac{V_1^2}{2g} = 0.37 + \frac{12.6^2}{2x9.81}$ = 8.462m

$E_C = y_3 + \frac{V_3^2}{2g} = 3 + \frac{1.56^2}{2x9.81}$ = 3.124m

Energy lost between A and C = 8.462-3.124 = 5.338m

13. The depth of uniform flow in a rectangular channel 5m wide (n = 0.014, bed slope = 1 in 30) is 0.54m. A low dam raises the water to a depth of 3.1m. Find whether a hydraulic jump takes place and if so at what distance upstream of the dam.



Solution:

Width of channel (b) = 5m

Depth at 1 (y_1) = 0.54m
n = 0.014, bed slope (S_0) = 1/30
Depth at dam (y_3) = 2.1m

A_1 = b y_1 = 5x0.54 = 2.7 m^2
P_1 = b+2y_1 = 5+2x0.54 = 6.08m
R_1 = A_1/P_1 = 2.7/6.08 = 0.44m
Velocity at 1 (V_1) = $\frac{1}{n}R_1^{2/3}S_0^{1/2} = \frac{1}{0.014}x0.44^{2/3}x(1/30)^{2/3}$ = 7.54m/s
Discharge (Q) = A_1 V_1 = 2.7x7.54 = 20.358 m^3/s
q = Q/b = 20.358/5 = 4.0716 $m^3/s/m$
$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{4.0716^2}{9.81}\right)^{1/3}$ = 1.191m

$F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{7.54}{\sqrt{9.81x0.54}}$ = 3.27
As Fr_1>1, the flow at 1 is supercritical.
As $y_3 > y_c$, the flow at dam is subcritical.
As the flow changes from supercritical to subcritical, the hydraulic jump occurs.

Finding y_2
$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8F_{r1}^2}\right)$
$= \frac{0.54}{2}\left(-1+\sqrt{1+8x3.27^2}\right)$ = 2.24m

Finding distance between 2 and 3 by direct step method with y_1 = 0.54m and y_2 = 2.24m

y	A	P	R	V	E	S_f	$\overline{S_f}$	Δx
0.54	2.7	6.08	0.444	7.54	3.4376	0.032889		
2.24	11.2	9.48	1.181	1.818	2.4084	0.000518	0.016704	61.9

Distance between 2 and 3 = 61.9m.
Length of jump = 6(y_2-y_1) =6(2.24-0.54) = 10.2m
The hydraulic jump is located at a distance of 61.9+10.2 = 72.1m from the dam.

Formulae used in the table
A= by, P = b+2y, R=A/P
V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$
$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$

14. A sluice gate in a 3m wide rectangular horizontal channel releases a discharge of 20 m^3/s. The gate opening is 0.7m and the coefficient of contraction is 0.6. Examine the type of hydraulic jump formed when the tailwater depth is (a) 3m and (b) 5m.

Solution:
Width of channel (b) = 3m
Discharge (Q) = 20 m^3/s
Depth at vena-contracta (y_a) = Coeff. of contraction x gate opening = 0.6x0.7 = 0.42m
Velocity at vena-contracta (V_a) = $\frac{Q}{by_a} = \frac{20}{3x0.42}$ = 15.9m/s
Froude number at vena-contracta $(F_{ra}) = \frac{V_a}{\sqrt{gy_a}} = \frac{15.9}{\sqrt{9.81x0.42}}$ = 7.8
y_2 = sequent depth to y_a
Finding y_2
$y_2 = \frac{y_a}{2}\left(-1 + \sqrt{1 + 8F_{ra}^2}\right)$
$= \frac{0.42}{2}\left(-1 + \sqrt{1 + 8x7.8^2}\right)$ = 4.43m

(a) Tailwater depth (y_t) = 3m
As $y_t<y_2$, a repelled jump will occur.
(b) Tailwater depth (y_t) = 5m
As $y_t>y_2$, a submerged jump will occur.

15. A wide channel with uniform rectangular section has a change of slope from 1 in 95 to 1 in 1420 and the flow is 3.75 m^3/s per m width. Determine the normal depth of flow corresponding to each slope and show that a hydraulic jump will occur in the region of the junction. Calculate the height of the jump and sketch the surface profiles between upstream and downstream regions of uniform flow. Manning's n = 0.013.

Also find the location of the jump using direct step method taking single step.

Solution:
Discharge per unit width (q) = 3.75 m^3/s per m
Slope (S_1) = 1/95
Slope (S_2) = 1/1420
n = 0.013
Normal depths at 1 (y_1) = ?
Normal depths at 1 (y_2) = ?

For wide rectangular channel, R = y
$Q = \frac{1}{n}AR^{2/3}S^{1/2}$

$Q = \frac{1}{n} b y_n (y_n)^{2/3} S^{1/2}$

$y_n = \left(\frac{nq}{S^{1/2}}\right)^{3/5}$

For channel 1

$y_{n1} = \left(\frac{nq}{S_1{}^{1/2}}\right)^{3/5} = \left(\frac{0.013x3.75}{(1/95)^{1/2}}\right)^{3/5}$ = 0.64m

For channel 2

$y_{n2} = \left(\frac{nq}{S_2{}^{1/2}}\right)^{3/5} = \left(\frac{0.013x3.75}{(1/1420)^{1/2}}\right)^{3/5}$ = 1.44m

Critical depth is given by

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.75^2}{9.81}\right)^{1/3}$ = 1.127m

$y_1 = y_{n1}$ = 0.64m, $y_2 = y_{n2}$ = 1.44m

$V_1 = q/y_1$ = 3.75/0.64 = 5.86m/s

$V_2 = q/y_2$ = 3.75/1.44 = 2.6m/s

Computing Froude number

$F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{5.86}{\sqrt{9.81x0.64}}$ = 2.34

As F_{r1}>1, the u/s flow is supercritical.

$F_{r2} = \frac{V_2}{\sqrt{g y_2}} = \frac{2.6}{\sqrt{9.81x1.44}}$ = 0.69

As F_{r1}<1, the d/s flow is subcritical.

As the flow changes from supercritical to subcritical state, the hydraulic jump occurs.

$y_c > y_{n1}$: steep slope u/s, $y_{n2} > y_c$: mild slope d/s

Sequent depth corresponding to y_1 is

$y_{1a} = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$= \frac{0.64}{2}\left(-1 + \sqrt{1 + 8x2.34^2}\right)$ = 1.82m

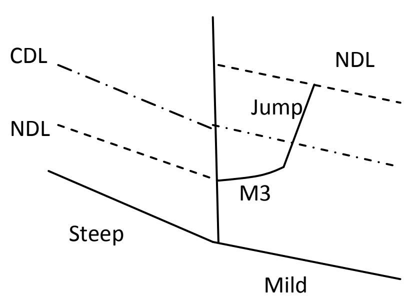
As $y_{1a} > y_2$, the jump will be formed in the mild slope channel. M3 profile will be developed from the break until the jump starts.

Sequent depth corresponding to y_2 is

$y_{2a} = \frac{y_{n2}}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

$= \frac{1.44}{2}\left(-1 + \sqrt{1 + 8x0.69^2}\right)$ = 0.86m

Height of jump = $y_2 - y_{2a}$ = 1.44-0.86 = 0.58m



Flow profile

The location of the jump is computed by direct step method taking 0.64m as the starting depth and 0.86m as the ending depth.

Formulae used in the table

R=y, V= q/y, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\bar{S_f} = \frac{1}{2}\left[(S_f)_{i-1} + (S_f)_i\right]$, $\Delta x = \frac{E_i - E_{i-1}}{S_2 - \bar{S_f}}$

y	R	V	E	Sf	sfb	dx	x
0.64	0.64	5.86	2.39	0.0105			0
0.86	0.86	4.36	1.83	0.0039	0.0072	86	86

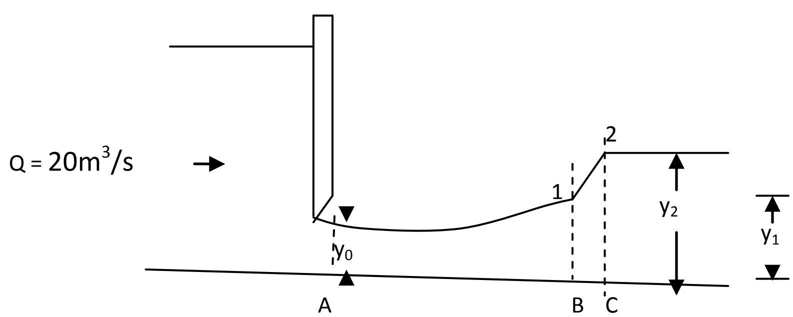
The jump starts at a distance of 86m d/s from the break in slope.

16. A vertical sluice gate with an opening of 0.67m produces a downstream jet depth of 0.4m when installed in a long rectangular channel 5m wide conveying a steady discharge of 20m^3/s. Assuming that the flow downstream of the gate eventually returns to the uniform flow depth of 2.5m,

a) Verify that a hydraulic jump occurs. Assume $\alpha = \beta = 1$.

b) If the downstream depth is increased to 3m, analyze the flow condition at the gate.

Solution:



Discharge (Q) = 20m^3/s

Depth at vena-contracta (y_0) = 0.4m

Width of channel (b) =5m

D/s depth (y_2) = 2.5m

a)

d/s Velocity (V_2) = $\frac{Q}{by_2} = \frac{20}{5x2.5}$ = 1.6 m/s

d/s Froude no. $(F_{r2}) = \frac{V_2}{\sqrt{gy_2}} = \frac{1.6}{\sqrt{9.81x2.5}}$ = 0.32

As $F_{r2}<1$, d/s flow is sub-critical.

u/s depth (y_1) is computed from the following equation.

$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

$= \frac{2.5}{2}\left(-1 + \sqrt{1 + 8x0.32^2}\right)$ = 0.436m

u/s Velocity (V_1) = $\frac{Q}{by_1} = \frac{20}{5x0.436}$ = 9.17 m/s u/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{9.17}{\sqrt{9.81x0.436}}$ = 4.43

As F_{r1}>1, d/s flow is supercritical.

As the flow changes from supercritical to sub-critical state, hydraulic jump occurs.

b) d/s depth (y_2) =3m

Flow condition at gate =?

d/s Velocity (V_2) = $\frac{Q}{by_2} = \frac{20}{5x3}$ = 1.33 m/s

d/s Froude no. $(F_{r2}) = \frac{V_2}{\sqrt{gy_2}} = \frac{1.33}{\sqrt{9.81x3}}$ = 0.245

u/s depth (y_1) in this case is

$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

$= \frac{3}{2}\left(-1 + \sqrt{1 + 8x0.245^2}\right)$ = 0.325m

Since the depth at vena-contracta (0.4m) is greater than y_1, the jump is submerged.

17. A wide rectangular channel with Manning's n = 0.025 is laid with a change in slope from 0.01 to 0.0025. The depth of uniform flow in the upstream and downstream reach is 1m and 2m respectively. Determine the location of hydraulic jump using two step of the direct step method.

Solution:

Manning's n = 0.025

Slope 1 = 0.01

Slope 2 = 0.0025

$y_1 = y_{n1}$ = 2m

$y_2 = y_{n2}$ = 2m

For wide rectangular channel, R = y

$Q = \frac{1}{n}AR^{2/3}S^{1/2} = \frac{1}{n}by_1{y_1}^{2/3}S^{1/2}$

$q_1 = \frac{1}{n}y_1^{5/3}S_1^{1/2} = \frac{1}{0.025}1^{5/3}0.01^{1/2}$ = 4m^3/s/m

$q_2 = \frac{1}{n}y_2^{5/3}S_2^{1/2} = \frac{1}{0.025}2^{5/3}0.0025^{1/2}$ = 6.35m^3/s/m

$V_1 = q/y_1$ = 4/1 = 4 m/s

U/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{4}{\sqrt{9.81x1}}$ = 1.27

Depth sequent to y_1 is

$y_{1a} = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$= \frac{1}{2}\left(-1 + \sqrt{1 + 8x1.27^2}\right)$ = 1.36m

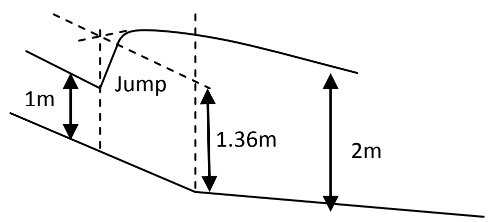
As y_{1a}<y_2, the jump will be formed on the steep channel. The location of the jump is computed by direct step method taking 2m as the starting depth and 1.36m as the ending depth.

Formulae used in the table

R=y, V= q_1/y, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$, $\Delta x = \frac{E_i - E_{i-1}}{S_{01} - \overline{S_f}}$

y	R	V	E	Sf	$\overline{S_f}$	Δx	x
2	2	2	2.204	0.000992			0
1.8	1.8	2.22	1.965	0.001407	0.001199	27	27
1.36	1.36	2.94	1.454	0.003585	0.002496	69	96

The jump will start at a distance of 96m u/s of the break in slope.



18. Find the pre jump and post jump heights of the hydraulic jump formed at the toe of the spillway. Neglect energy loss due to flow over spillway.

Height of the crest above D/S bed level = 3m

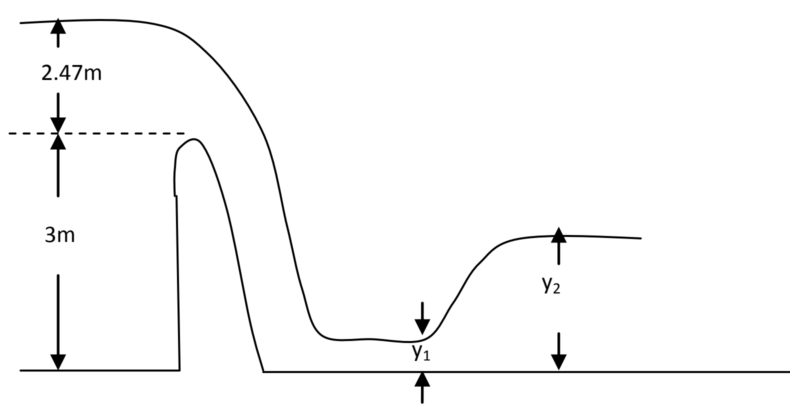
Discharge = 80 m^3/s

Width of canal = 10m

Head over the crest level = 2.47m

Explain the formation condition of repelled and submerged jump for the above flow condition.

Solution:



Solution:

Specific energy at u/s of spillway (E_u) = 5.47m (neglecting velocity head)

Discharge (Q) = 80 m^3/s
Width of canal (b) = 10m
y_1 = ?
y_2 = ?
u/s specific energy = specific energy at 1
$E_u = E_1$

$$5.47 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2g(by_1)^2}$$

$$5.47 = y_1 + \frac{80^2}{2x9.81(10y_1)^2}$$

$$5.47 = y_1 + \frac{3.26}{y_1^2}$$

Solving
y_1 = 0.839 m

$V_1 = Q/A_1$ = 80/(10x0.839) = 9.5m/s

$$F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{9.5}{\sqrt{9.81x0.839}} = 3.31$$

$$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$$

$$= \frac{0.839}{2}\left(-1 + \sqrt{1 + 8x3.31^2}\right) = 3.53\text{m}$$

Let y_t = tailwater depth. If $y_t < y_2$, the jump is repelled downstream of the vena-contracta, i.e. a repelled jump is formed. If $y_t > y_2$, the jump gets drowned out, i.e. a submerged jump is formed.

19. Water flows through a 2m wide rectangular channel at the rate of $5m^3/s$. The channel has a change of slope from 0.01 to 0.00023. Verify whether hydraulic jump occurs. If so, find the location of the jump using direct step method taking only 2 steps. Take n = 0.014.

Solution:
Width (b) = 2m
Discharge (Q) = $5m^3/s$
Slope1 (S_1) = 0.01
Slope2 (S_2) = 0.0001
y_1 = normal depth at 1 and y_2 = normal depth at 2
$A_1 = by_1 = 2\,y_1$
$P_1 = b+2y_1 = 2+2\,y_1$

$$R_1 = \frac{A_1}{P_1} = \frac{2y_1}{2+2y_1}$$

$A_2 = by_2 = 2\,y_2$
$P_2 = b+2y_2 = 2+2\,y_2$

$$R_2 = \frac{A_2}{P_2} = \frac{2y_2}{2+2y_2}$$

From Manning's equation

$Q = \frac{1}{n} AR^{2/3} S^{1/2}$

$AR^{2/3} = \frac{nQ}{\sqrt{S}}$

For 1

$2y_1 \left(\frac{2y_1}{2+2y_1}\right)^{2/3} = \frac{0.014x5}{\sqrt{0.01}}$

$\frac{y_1^{5/3}}{(1+y_1)^{2/3}} = 0.35$

Solving

y_1 = 0.65m

For 2

$2y_2 \left(\frac{2y_2}{2+2y_2}\right)^{2/3} = \frac{0.014x5}{\sqrt{0.00023}}$

$\frac{y_2^{5/3}}{(1+y_2)^{2/3}} = 2.3078$

Solving

y_2 = 2.82m

Critical depth

$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{5^2}{2^2 x 9.81}\right)^{1/3}$ = 0.86m

$y_c > y_1$: steep slope, supercritical flow

$y_2 > y_c$: mild slope, subcritical flow

As the flow changes from supercritical to subcritical state, hydraulic jump occurs.

$V_1 = \frac{Q}{A_1} = \frac{5}{2x0.65}$ = 3.84m/s

u/s Froude no. $(F_{r1}) = \frac{V_1}{\sqrt{gy_1}} = \frac{3.84}{\sqrt{9.81x0.65}}$ = 1.52

$V_2 = \frac{Q}{A_2} = \frac{5}{2x2.82}$ = 0.88m/s

d/s Froude no. $(F_{r2}) = \frac{V_2}{\sqrt{gy_2}} = \frac{0.88}{\sqrt{9.81x2.82}}$ = 0.17

Depth sequent to y_1 is

$y_{1a} = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$

$= \frac{0.65}{2}\left(-1 + \sqrt{1 + 8x1.52^2}\right)$ = 1.11m

As $y_{1a} < y_2$, the jump will be formed on the steep slope. The location of the jump is computed by direct step method taking 2.82m as the starting depth and 1.11m as the ending depth.

y	A	P	R	V	E	S_f	$\overline{S_f}$	Δx	x
2.82	5.84	7.84	0.745	0.856	2.957	0.000213			0
2	4	6	0.666	1.25	2.079	0.000527	0.00037	91	91
1.11	2.22	4.22	0.526	2.252	1.368	0.002341	0.001434	83	174

The hydraulic jump starts at a distance of 174m u/s of the break in slope.

Formulae used in the table

$A = by,\ P = b + 2y$, R =A/P

V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$, $\overline{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \overline{S_f}}$, x = cumulative sum of Δx

Mobile boundary channel

1. In a wide rectangular channel, average velocity is 0.75m/s, depth of flow is 1.5m, sand grain size is 0.95mm with specific gravity of 2.65. The bed slope of channel is 1/6000, kinematic viscosity of water is 10^{-6} m^2/s. Estimation the possibility of motion of bed load by using Shield's Curve.

Solution:
Average velocity (V) = 0.75m/s
Depth of flow (y) = 1.5m
Grain size (d) = 0.95mm = $0.95x10^{-3}$ m
Bed slope (S) = 1/6000
kinematic viscosity of water (ν) = 10^{-6} m^2/s
In a wide rectangular channel, Hydraulic radius (R) = y
Boundary shear $(\tau_0) = \gamma RS$ = 9810x1.5x1/6000 = 2.4525 N/m^2
Shield's entrainment function $= \frac{\tau_0}{(\gamma_s - \gamma)d} = \frac{2.4525}{(2.65X9810 - 9810)0.95x10^{-3}}$ = 0.16 >0.056
Reynolds no. $(R_*) = \frac{V_* d}{\nu} = \frac{\sqrt{gyS}\, d}{\nu} = \frac{(\sqrt{gx1.5x1/6000})0.95x10^{-3}}{10^{-6}}$ = 47
The point (47 , 0.16) lies above the Shield's curve. Hence, there is possibility of bed load motion.

2. A wide rectangular channel in alluvium of 5mm median size (relative density = 2.65) has a longitudinal slope of 0.0004. Estimate the depth of flow in this channel which will cause incipient motion.

Solution:
Slope (S) = 0.0004
Size of sediment (d) = 5mm
Depth of flow (y) = ?
For incipient motion to begin

$$\frac{\tau_0}{(\gamma_s - \gamma)d} \geq 0.056$$

$$\tau_0 \geq 0.056(\gamma_s - \gamma)d$$

$= 0.056(2.65x9810 - 9810)5x10^{-3}$ = 4.53Pa

For wide rectangular channel

$$\tau_0 = \gamma y S$$

$$4.53 = 9810xyx0.0004$$

y =1.15m

3. A wide open channel has a depth of flow 1.7m and mean velocity 2.5m/s. Find minimum size of bed material needed to obtain a stable bed.

Solution:
Depth of flow (y) = 1.7m
Velocity (V) = 2.5m/s
Minimum size of bed material (d_c) = ?
For d≥6mm

$$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = 0.056$$
$$\tau_c = 0.056(\gamma_s - \gamma_w)d_c$$
$$\gamma_w RS = 0.056(\gamma_s - \gamma_w)d_c$$
$$9810xRS = 0.056(2.65x9810 - 9810)d_c$$

d_c = 10RS (a)
From Manning's equation
$V = \frac{1}{n}R^{2/3}S^{1/2}$
$S = \frac{n^2V^2}{R^{4/3}}$ (b)
$n = \frac{d_c^{1/6}}{21.1}$ (c)
From a, b and c
$d_c^{2/3} = 0.0247\frac{V^2}{R^{1/3}}$
$d_c^{2/3} = 0.0247\frac{2.5^2}{1.7^{1/3}}$
d_c = 0.046m

4. Design a channel (trapezoidal shape) to carry a discharge of 6.9m^3/s using permissible velocity method. Take S = 0.003, n = 0.015, Z:1 = 1:1. (The maximum permissible velocity for the channel = 1.8 m/s).

Solution:
Discharge (Q) =6.9m^3/s
Maximum permissible velocity (V) = 1.8m/s
S = 0.003, n = 0.015, z:1 = 1:1
Channel width (b) = ?
Channel depth (y) = ?

C/s area (A) = Q/V = 6.9/1.8 = 3.83 m^2

$A = (b + zy)y$
$3.83 = (b + y)y$ (a)
$P = b + 2y\sqrt{1 + z^2} = b + 2y\sqrt{1 + 1} = b + 2.828y$
$R = \frac{A}{P} = \frac{3.83}{b+2.828y}$

$V = \frac{1}{n} R^{2/3} S^{1/2}$

$1.8 = \frac{1}{0.015} \left(\frac{3.83}{b+2.828y}\right)^{2/3} 0.003^{1/2}$

b+2.828y = 11.06 (b)

Solving a and b

y^2-6.05y+2.1 =0

Solving for y

y = 5.7m, 0.37m

For y = 5.7m, b = -5.05 which is impossible

For y = 0.37m, b = 10m

The width of channel is 10m. Provide freeboard of 0.25y = 0.09m

5. Design a straight trapezoidal channel for Q = 6.5 m^3/s. The bottom slope is 0.0007 and the channel is excavated through fine gravel having particle size of 8mm. Use tractive force method. (Take Z:1 = 3:1 and angle of repose = 30^0 for 8mm gravel)

Solution:

Discharge (Q) =6.5 m^3/s

Bed slope (S_0) =0.0007

Z:1 = 3:1

Angle of repose (ϕ) = 30^0

Particle size (d) = 8mm

Channel depth (y) = ?

Channel width (b) = ?

Tanθ = 1/3

$cos\theta = 3/\sqrt{10}$

$K_1 = cos\theta \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2} = \frac{3}{\sqrt{10}}\left[1 - \frac{1/9}{tan^2 30}\right]^{1/2}$ =0.632

Consider straight channel, K_2 = 0.9

For d>6mm,

$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = 0.056$

$\frac{\tau_c}{(2.65x9810-9810)x8x10^{-3}} = 0.056$

τ_c =7.25 N/m^2

Manning's n = $\frac{d^{1/6}}{21.1} = \frac{(0.008)^{1/6}}{21.1}$ = 0.021

Tractive shear on the bed $(\tau_b) = K_2\tau_c$ = 0.9x7.25 = 6.525 N/m^2

Tractive shear on the side $(\tau_w) = K_1\tau_b$ = 0.632x6.516 = 4.1238 N/m^2

$(\tau_b)_{max} = \gamma S_0 y$ = 9810x0.0007y = 6.867y
$(\tau_w)_{max} = 0.75\gamma S_0 y$ =0.75x9810x0.0007y= 5.15y

Making $\tau_b = (\tau_b)_{max}$
$6.525 = 6.867y$
y = 0.95m

Making $\tau_w = (\tau_w)_{max}$
$4.1238 = 5.15y$
y = 0.8m

Take smaller value of y.
y = 0.8m

Finding b using Manning's equation (considering trapezoidal channel)

$A = (b + Zy)y = (b + 3x0.8)0.8 = 0.8b + 1.92$
$P = b + 2y\sqrt{1 + Z^2} = b + 2x0.8\sqrt{1 + 3^2} = b + 5.06$
$R = \frac{A}{P} = \frac{0.8b+1.92}{b+5.06}$

$Q = \frac{1}{n} AR^{2/3} S^{1/2}$
$6.5 = \frac{1}{0.021}(0.8b + 1.92)\left(\frac{0.8b+1.92}{b+5.06}\right)^{2/3} 0.0007^{1/2}$
$\frac{(0.8b+1.92)^{5/3}}{(b+5.06)^{2/3}} = 5.159$
Solving
b = 6.5m

The width of channel is 6.5m. Provide freeboard of 0.25y = 0.2m.
The channel depth is 0.8+0.2 = 1.0m.

6. Design a channel by Lacey's theory to carry 50 m^3/s of water. The channel is to be cut in an alluvial soil of median size 0.9mm.

Solution:
Discharge (Q) = 50 m^3/s
Median size of particle (d_{mm}) = 0.9mm

Silt factor $(f_s) = 1.76\sqrt{d_{mm}} = 1.76\sqrt{0.9}$ = 1.67
Longitudinal slope $(S) = \frac{0.0003 f_s^{5/3}}{Q^{1/6}} = \frac{0.0003x1.67^{5/3}}{50^{1/6}}$ = 0.000367

Hydraulic radius $(R) = 0.48\left(\frac{Q}{f_s}\right)^{1/3} = 0.48\left(\frac{50}{1.67}\right)^{1/3}$ = 1.49m

Wetted perimeter $(P) = 4.75\sqrt{Q} = 4.75\sqrt{50}$ = 33.58m

Cross sectional area (A) = PR = 33.58x1.49 = 50.03 m^2

Side slopes (Z:1) = 0.5:1 for final regime channel

b = width of channel, y = depth of flow

$A = (b + Zy)y = (b + 0.5y)y$

$50.03 = (b + 0.5y)y$ (I)

$P = b + 2y\sqrt{1 + Z^2} = b + 2xy\sqrt{1 + 0.5^2} = b + 2.236y$

$33.58 = b + 2.236y$ (II)

b = 33.58-2.236y

Substituting b in I

$50.03 = (33.58 - 2.236y + 0.5y)y$

$y^2 - 19.34y + 28.819 = 0$

Solving for y

y = 17.71m, 1.63m

Neglecting higher value of y which is impracticable,

Depth (y) = 1.63m

Width (b) = 33.58-2.236x1.63 = 29.9m

7. Design the geometric size of a trapezoidal channel to carry a discharge equal to 55 m^3/s in mobile boundary of coarse in alluvium with longitudinal slope 1:1000 and side slopes of 1.75:1. Take critical dimensionless tractive shear stress as 0.05, diameter of particle 5.08cm, angle of internal friction 37^0, Manning's n 0.023 and sp gr of sediment particle as 2.65.

Solution:

Discharge (Q) =55 m^3/s

Bed slope (S_0) =1/1000

Z:1 = 1.75:1

Angle of repose (ϕ) = 37^0

Particle size (d) = 5.08cm

n = 0.023

Channel depth (y) = ?

Channel width (b) = ?

Tanθ = 1/1.75

$cos\theta = 1.75/\sqrt{1 + 1.75^2}$ = 0.868

$K_1 = cos\theta \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2} = 0.868\left[1 - \frac{(1/1.75)^2}{tan^2 37}\right]^{1/2}$ =0.566

Consider straight channel, K_2 = 0.9

$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = 0.05$

$\frac{\tau_c}{(2.65x9810 - 9810)x5.08x10^{-2}} = 0.05$

τ_c =41.1 N/m^2

Tractive shear on the bed $(\tau_b) = K_2\tau_c$ = 0.9x41.1 = 36.99 N/m^2

Tractive shear on the side $(\tau_w) = K_1\tau_b$ = 0.566x36.99 = 20.94 N/m^2

$(\tau_b)_{max} = \gamma S_0 y$ = 9810x(1/1000)y = 9.81y

$(\tau_w)_{max} = 0.75\gamma S_0 y$ =0.75x9810x(1/1000)y= 7.5375y

Making $\tau_b = (\tau_b)_{max}$

$36.99 = 9.81y$

y = 3.77m

Making $\tau_w = (\tau_w)_{max}$

$20.94 = 7.5375y$

y = 2.85m

Take smaller value of y.

y = 2.85m

Finding b using Manning's equation (considering trapezoidal channel)

$A = (b + Zy)y = (b + 1.75x2.85)2.85 = 2.85b + 14.214$

$P = b + 2y\sqrt{1 + Z^2} = b + 2x2.85\sqrt{1 + 1.75^2} = b + 11.488$

$R = \frac{A}{P} = \frac{2.85b + 14.214}{b + 11.488}$

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$55 = \frac{1}{0.023}(2.85b + 14.214)\left(\frac{2.85b + 14.214}{b + 11.488}\right)^{2/3} (1/1000)^{1/2}$

$\frac{2.85b + 14.214^{\ 5/3}}{(b + 11.488)^{2/3}} = 40$

Solving for b

b = 4.8m

Channel width = 4.8m, Channel depth = 2.85m

Provide freeboard of 0.25y

8. A channel 2.0 m deep, 15 m bed width, with 2:1 side slopes is excavated in gravel of d= 50mm. What is the maximum permissible channel slope, and what discharge can the channel carry without disturbing its stability? Take $\phi = 37^0$ and K$_2$ = 0.9.

Solution:
Depth (y) = 2m
Bed width (b) = 15m
Z:1 = 2:1
Angle of repose (ϕ) = 37^0
Particle size (d) = 50mm
Maximum permissible bed slope (S_0) =?
Discharge (Q) =?

Tanθ = 1/2
$cos\theta = 2/\sqrt{5}$
$K_1 = cos\theta\ \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2} = \frac{2}{\sqrt{5}}\left[1 - \frac{1/4}{tan^2 37}\right]^{1/2}$ =0.73
For d>6mm,
$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = 0.056$
$\frac{\tau_c}{(2.65x9810 - 9810)x50x10^{-3}} = 0.056$
τ_c =45.32 N/m^2

Manning's n = $\frac{d^{1/6}}{21.1} = \frac{(0.05)^{1/6}}{21.1}$ = 0.028

Tractive shear on the bed $(\tau_b) = K_2\tau_c$ = 0.9x45.32 = 40.788 N/m^2
Tractive shear on the side $(\tau_w) = K_1\tau_b$ = 0.73x40.788 = 29.775 N/m^2

$(\tau_b)_{max} = \gamma S_0 y$ = 9810xS_0x2 = 18360 S_0
$(\tau_w)_{max} = 0.75\gamma S_0 y$ =0.75x9810x S_0x2= 13770 S_0

Making $\tau_b = (\tau_b)_{max}$
40.788=18360 S_0
S_0 = 0.0022

Making $\tau_w = (\tau_w)_{max}$
29.775=13770 S_0
S_0 = 0.0021
Take greater of two values
S_0 = 0.0022

$A = (b + Zy)y = (15 + 2x2)2 = 38$
$P = b + 2y\sqrt{1 + Z^2} = 15 + 2x2\sqrt{1 + 2^2} = 23.94$
$R = \frac{A}{P} = 1.587$

$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$

$= \frac{1}{0.028} 38x(1.587)^{2/3} x0.0022^{1/2}$ = 86.6 m^3/s

9. A channel which is to carry 10 m^3/s through moderately rolling topography on a slope of 0.0016 is to be excavated in coarse alluvium with 50% of particles being 3cm or more in diameter. Assume that channel is to be unlined and of trapezoidal section. Find suitable value of base width and side slope. Take φ = 34^0 and K_2 (ratio between bed shear stress and critical shear stress) = 0.75. Use tractive force method.

Solution:

Discharge (Q) = 10 m^3/s

Bed slope (S_0) =0.0016

Angle of repose (ϕ) = 34^0

Particle size (d) = 3cm = 30mm

K_2 = 0.75

side slope (θ) = ?

Channel width (b) = ?

$$K_1 = cos\theta \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2} = cos\theta \left[1 - \frac{tan^2\theta}{tan^2 34}\right]^{1/2} = cos\theta(1 - 2.2tan^2\theta)^{1/2}$$

For d>6mm,

For d>6mm,

$$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = 0.056$$

$$\frac{\tau_c}{(2.65x9810 - 9810)x3x10^{-2}} = 0.056$$

τ_c = 27.19 N/m^2

Manning's n = $\frac{d_{50}^{1/6}}{21.1} = \frac{(0.03)^{1/6}}{21.1}$ = 0.026

Tractive shear on the bed $(\tau_b) = K_2\tau_c$ = 0.75x27.15 = 20.3925 N/m^2

$$(\tau_b)_{max} = \gamma S_0 y$$

Making $\tau_b = (\tau_b)_{max}$

$$\gamma S_0 y = 20.3925$$

9810x0.0016y = 20.3925

y = 1.3m

Tractive shear on the side $(\tau_w) = K_1\tau_b = 20.3925cos\theta(1 - 2.2tan^2\theta)^{1/2}$

$$(\tau_w)_{max} = 0.75\gamma S_0 y$$

Making $\tau_w = (\tau_w)_{max}$

$$20.3925cos\theta(1 - 2.2tan^2\theta)^{1/2} = 0.75\gamma S_0 y$$

$20.3925(cos^2\theta - 2.2sin^2\theta)^{1/2} = 0.75x9810x0.0016x1.3$
$cos^2\theta - 2.2sin^2\theta = 0.563$
$1 - sin^2\theta - 2.2sin^2\theta = 0.563$
$sin\theta = 0.369$
θ = 21.65^0
$Z = 1/tan21.64 = 2.5$
Z:1 = 2.5:1

Finding b using Manning's equation

$A = (b + Zy)y = (b + 2.5x1.3)1.3 = 1.3b + 4.225$
$P = b + 2y\sqrt{1 + Z^2} = b + 2x1.3\sqrt{1 + 2.5^2} = b + 7$
$R = \frac{A}{P} = \frac{1.3b+4.225}{b+7}$

$Q = \frac{1}{n}AR^{2/3}S_0^{1/2}$
$10 = \frac{1}{0.026}(1.3b + 4.225)\left(\frac{1.3b+4.225}{b+7}\right)^{2/3} 0.0016^{1/2}$
$\frac{(1.3b+4.225)^{5/3}}{(b+7)^{2/3}} = 6.5$
Solving
b = 2.6m

10. A stream has a sediment bed of median size 0.35mm. The slope of the channel is $1.5x10^{-4}$. Stream is considered as trapezoidal with base width 3m and side slope 1.5H:1V.
a) If the depth of flow in the channel is 0.25m, examine whether the bed particles will be in motion or not.
b) Calculate minimum size of gravel that will not move in the bed of channel. Use empirical equation of critical shear stress as: $\tau_c(N/m^2) = 0.155 + \frac{0.409d_{mm}^2}{(1+0.177d_{mm}^2)^{1/2}}$

Solution:
Median size of sediment (d) = 0.35mm
Sloe of channel (S) = $1.5x10^{-4}$
base width (b) = 3m
Z:1 = 1.5:1
Depth of flow (y) = 0.25m
a)
$A = (b + Zy)y = (3 + 1.5x0.25)0.25$ = 0.84375 m^2
$P = b + 2y\sqrt{1 + Z^2} = 3 + 2x0.25\sqrt{1 + 1.5^2}$ = 3.9
R = A/P = 0.2163
Bed shear $(\tau_0) = \gamma RS$ = 9810x0.2163x $1.5x10^{-4}$= 0.32 N/m^2
Critical shear stress

$\tau_c = 0.155 + \frac{0.409d_{mm}^2}{(1+0.177d_{mm}^2)^{1/2}} = 0.155 + \frac{0.409x0.35^2}{(1+0.177x0.35^2)^{1/2}}$ = 0.2045 N/m^2

As $\tau_0 > \tau_c$, the bed particles will be in motion.

b) For the minimum size of the gravel (d_c) that will not move in the bed,

$\tau_c = \tau_0$

$\tau_c = 0.32$

$0.155 + \frac{0.409d_c^2}{(1+0.177d_c^2)^{1/2}} = 0.32$

$\frac{d_c^2}{(1+0.177d_c^2)^{1/2}} = 0.4034$

d_c = 0.65mm