



Civinnovate

Discover, Learn, and Innovate in Civil Engineering

Chapter-2 Working stress Method

Assumptions

1. Plane section before bending remains plane after bending.
2. All tensile forces are taken by reinforcement alone unless otherwise stated.
3. Stress-strain curve is always linear.
4. Bond between steel and concrete is uniform.
5. Modular ratio (m) is taken as E_s/E_c .

where, σ_{cbc} = permissible compressive bending stress in concrete

Permissible stress or allowable stress

It is a fraction of yield stress which is allowed in material.

$$\sigma_{bc} = \frac{\sigma_{yc}}{F_s} \quad \sigma_{cs} = \frac{3}{4} \text{ for concrete}$$

$$\frac{F_s}{F_c} = \frac{f_y}{f_{cs}} \quad F_{cs} = 2.25$$

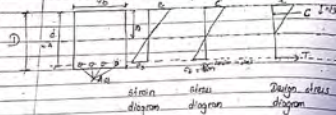
$$\frac{F_c}{F_s} = \frac{f_c}{f_{cs}}$$

permissible stress in steel in compression

Type of steel

1. Mild steel (Fe 250) $f_y = 250 \text{ N/mm}^2$
 2. Torsal (Tatacon) (Fe 250) $f_y = 250 \text{ N/mm}^2$
 3. TMT (Fe 250) $f_y = 250 \text{ N/mm}^2$
- Thermo mechanically treated

Moment of resistance of a section



Stress in steel $\sigma_s = \sigma_{st}$ (1)

Stress in concrete around steel

$$\begin{aligned} \sigma_c &= \sigma_c + \sigma_s \\ &= \sigma_c + \frac{1}{m} \sigma_s \\ &= \frac{1}{m} \sigma_s \end{aligned}$$

$$\frac{F_s}{F_c} = m$$

Depth of N.A. (N) = $n \cdot d$

$$\begin{aligned} n &= \frac{m}{m_r} \\ &= \frac{r}{c} \end{aligned}$$

$$\begin{aligned} \text{MOR}_{\text{work}} &= (b \cdot n) \cdot \left(\frac{1}{2} \cdot c\right) \cdot (d - n \cdot d) \\ &= \frac{1}{2} \cdot b \cdot n \cdot c \cdot (d - n \cdot d) \\ &= \frac{1}{2} \cdot b \cdot n \cdot d \cdot c \cdot (1 - n) \\ &= \frac{1}{2} \cdot c \cdot n \cdot (1 - n) \cdot b \cdot d^2 \\ \text{MOR} &= Q b d^2 \\ Q &= \frac{1}{2} \cdot c \cdot n \cdot (1 - n) \end{aligned}$$

$$\text{MOR}_{\text{stat}} = T \cdot \left(\frac{d - n \cdot d}{3}\right)$$

$$\text{MOR} = T \cdot A_{st} \cdot \left(\frac{d - n \cdot d}{3}\right)$$

Type of section

1. Balanced or critical section
If steel and concrete at a section fails at the same time, such section is known as balanced section.
If max stresses in steel and concrete reached simultaneously

such section is balanced section. In such case, moment of resistance is calculated as:

$$\begin{aligned} \text{MOR} &= \frac{1}{2} \cdot b \cdot n \cdot c \cdot (d - n \cdot d) \quad \text{or} \\ \text{MOR} &= T \cdot \left(\frac{d - n \cdot d}{3}\right) \end{aligned}$$

2. Under reinforced section

If steel at a section fails before concrete, such section is known as under reinforced section.

In such case MOR is calculated as:

$$\text{MOR} = T \cdot \left(\frac{d - n \cdot d}{3}\right)$$

Depth of neutral axis in such case is less than that of balanced case.

3. Over reinforced section

concrete fails before steel at a section.

In such case, MOR is calculated as

$$\text{MOR} = \frac{1}{2} \cdot b \cdot n \cdot c \cdot (d - n \cdot d)$$

Depth of neutral axis in such case is more than that of balanced case.

Chapter-2 Working stress Method

Assumptions

1. Plane section before bending remains plane after bending.
2. All tensile forces are taken by reinforcement alone unless otherwise stated.
3. Stress-strain curve is always linear.
4. Bond between steel and concrete is uniform.
5. Modular ratio (m) is taken as 280

where, $\sigma_{cbc} = \frac{3 \sigma_{cbc}}{3 \sigma_{cbc}}$ permissible compressive bending stress in concrete

Permissible stress or allowable stress

It is a fraction of yield stress which is allowed in material.

$$\sigma_{cbc} = \frac{f_{ck}}{FOS}, \quad FOS = 3 \text{ for concrete}$$

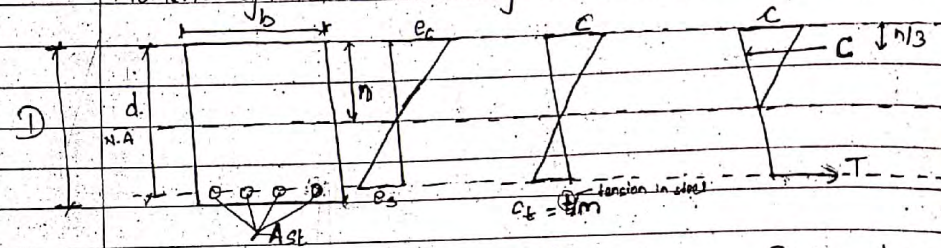
$$\sigma_{st} = \frac{f_y}{FOS} \text{ in steel in tension, } FOS = 1.78$$

$$\sigma_{sc} = \frac{f_y}{FOS} \text{ in steel in compression}$$

Types of steel

1. Mild steel ($F_e 250$) $f_y = 250 \text{ N/mm}^2$
2. Torsteel (Torkori) ($F_e 415$); $f_y = 415 \text{ N/mm}^2$
3. TMT ($F_e 500$) $f_y = 500 \text{ N/mm}^2$
Thermo mechanically treated

Moment of resistance of a section



strain diagram stress diagram Design stress diagram

$$\text{Stress in steel } (t) = E_s \times e_s \quad \text{--- (i)}$$

Stress in concrete around steel

$$\begin{aligned} c_t &= E_c \times e_s \\ &= E_c \times \frac{t}{E_s} \end{aligned} \quad \left| \frac{E_s}{E_c} = m \right.$$

$$= \frac{t}{m}$$

Depth of N.A (n) = $n_1 \cdot d$

$$n_1 = \frac{m}{m+r}$$

$$r = \frac{t}{c}$$

$$\text{MOR}_{\text{concrete}} = (b \cdot n) \cdot \left(\frac{1}{2} \cdot c\right) \cdot \left(d - \frac{n}{3}\right)$$

$$= \frac{1}{2} \cdot b \cdot n \cdot c \cdot \left(d - \frac{n}{3}\right)$$

$$= \frac{1}{2} \cdot b \cdot n_1 \cdot d \cdot c \cdot \left(d - \frac{n_1 d}{3}\right)$$

$$= \frac{1}{2} \cdot c \cdot n_1 \cdot \left(1 - \frac{n_1}{3}\right) \cdot b d^2$$

$$\text{MOR} = Q b d^2$$

$$Q = \frac{1}{2} \cdot c \cdot n_1 \cdot \left(1 - \frac{n_1}{3}\right)$$

$$\text{MOR}_{\text{steel}} = T \cdot \left(d - \frac{n}{3}\right)$$

$$\text{MOR} = t \cdot A_{st} \cdot \left(d - \frac{n}{3}\right)$$

Types of section

1. Balanced or critical section

If steel and concrete at a section fails at the same time, such section is known as balanced section.

If max. stresses in steel and concrete ~~reaches~~ simultaneously reaches

such section is balanced section. In such case, moment of resistance is calculated as:

$$\text{MOR} = \frac{1}{2} \cdot b \cdot n \cdot c \cdot \left(d - \frac{n}{3}\right) \quad \text{or}$$

$$\text{MOR} = T \cdot \left(d - \frac{n}{3}\right)$$

2. Under reinforced section

If steel at a section fails before concrete, such section is known as under reinforced section.

In such case, MOR is calculated as:

$$\text{MOR} = T \cdot \left(d - \frac{n}{3}\right)$$

Depth of neutral axis in such case is less than that of balanced case.

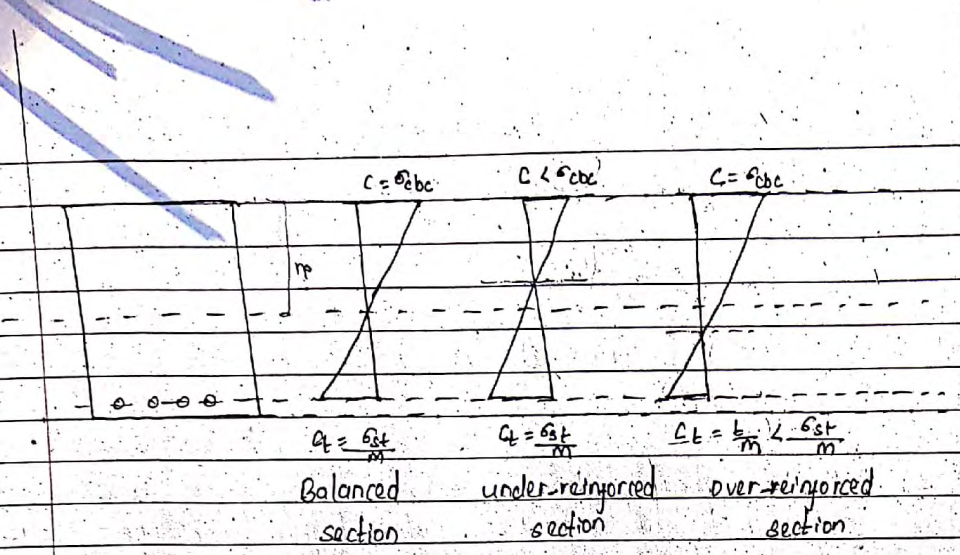
3. Over reinforced section

concrete fails before steel at a section

In such case, MOR is calculated as

$$\text{MOR} = \frac{1}{2} \cdot b \cdot n \cdot c \cdot \left(d - \frac{n}{3}\right)$$

Depth of neutral axis in such case is more than that of balanced case.



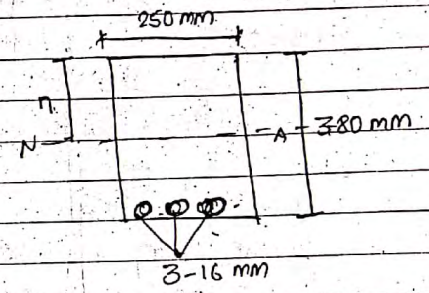
Which section would you prefer and why?

- We prefer under-reinforced section because
- i. Steel being expensive component.
 - ii. Under-reinforced section are stiffer than other section.
 - iii. It minimizes long term and short term deflection.
 - iv) Over-reinforced → brittle failure
 under-reinforced → ductile failure

Numericals

Q1. Calculate depth of Neutral axis for a singly reinforced b 250 x 380 mm to the centre of reinforcement. Also, calculate maximum stress in steel and concrete if it is reinforced with 3-16 mm dia bar with permissible stress in steel 140 N/mm^2 is achieved. Take $m = 13.33$

Solution:



$b = 250 \text{ mm}$
 $d = 380 \text{ mm}$ (eff. depth)

$A_{st} = 3 * \frac{\pi}{4} * 16^2$
 $= 603.18 \text{ mm}^2$

$m = 13.33$
 $\sigma_{st} = 140 \text{ N/mm}^2$

Taking moment about N.A

$250 * n * \frac{n}{2} = (m A_{st}) * (380 - n)$

$250 * \frac{n^2}{2} = 13.33 * 603.18 * (380 - n)$
 $\therefore n = 127.45 \text{ mm}$

$$\frac{c}{C_t} = \frac{n}{d-n}$$

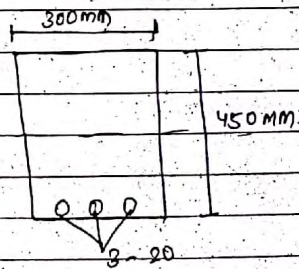
$$\text{or, } \frac{c}{(t/m)} = \frac{n}{(d-n)}$$

$$\text{or, } \frac{c}{(6_{st}/m)} = \frac{127.45}{380-127.45}$$

$$c = 5.8 \text{ N/mm}^2$$

$$t = 6_{st} = 140 \text{ N/mm}^2$$

Q.2. Calculate moment of resistance of a singly reinforced RC beam having dimension $300 \times 450 \text{ mm}$ (effective) and it is reinforced with 320 mm dia bar. Max. permissible stresses in concrete and steel are 7 N/mm^2 and 140 N/mm^2 .



$$b = 300 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.47 \text{ mm}^2$$

$$6_{st} = 140 \text{ N/mm}^2$$

$$6_{cbc} = 7 \text{ N/mm}^2$$

$$m = \frac{280}{3 \times 6_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

Taking moment about N.A

$$300 \times n \times \frac{n}{2} = m A_{st} \times (450 - n)$$

$$\therefore n = 156.73 \text{ mm}$$

Now,

$$n_c = n_1 \times d$$

$$n_1 = \frac{m}{m+r}$$

$$r = \frac{t}{c} = \frac{6_{st}}{6_{cbc}} = \frac{140}{7} = 20$$

$$n_1 = \frac{13.33}{13.33 + 20} = 0.4$$

$$n_c = 0.4 \times 450 = 180 \text{ mm}$$

Since, $n < n_c$, it is under-reinforced section.

$r = t/c$ [for balanced case]

$$MOR = A_{st} \cdot \sigma_{st} \cdot \left(d - \frac{d}{3} \right)$$

$$= 942.47 \cdot 140 \cdot \left(450 - \frac{156.73}{3} \right)$$

$$= 52.48 \text{ kNm}$$

Q.3. Design a singly reinforced RC beam of eff. span 4m carrying an UDL of 15 kN/m over the span. Beam is 250 mm wide and the material used are M20 concrete and Fe415 bar.

Soln.

$$l = 4 \text{ m}$$

$$w = 15 \text{ kN/m}$$

$$\sigma_{st} = \frac{415}{1.98} = 209.15$$

$$= 230 \text{ N/mm}^2$$

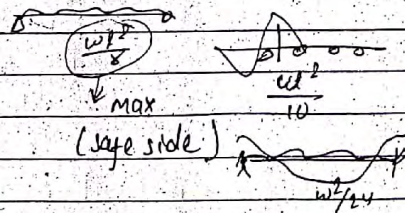
(Annex-B
Table-21, 22.)

$$\sigma_{cbc} = \frac{20}{3} = 6.67 \approx 7 \text{ N/mm}^2$$

$$b = 250 \text{ mm}$$

$$M_{max} = \frac{wL^2}{8} = \frac{15 \cdot 4^2}{8}$$

$$= 30 \text{ kN-m}$$



Now, for balanced section

$$n_c = n_s \cdot d$$

$$n_s = \frac{m}{m+r}$$

$$r = \frac{t}{c} = \frac{\sigma_{st}}{\sigma_{cbc}} = \frac{230}{7} = 32.86$$

$$n_s = \frac{m}{m+r}$$

$$m = \frac{280}{36 \text{ cbc}} = 13.33$$

$$= \frac{13.33}{13.33 + 32.86}$$

$$= 0.288$$

$$MOR = \sigma_{cbc} \cdot b \cdot n_s \cdot \left(d - \frac{d}{3} \right)$$

$$30 \times 10^6 = \frac{7}{2} \cdot 250 \cdot 0.288 \cdot d \left(d - \frac{0.288d}{3} \right)$$

$$d = 362.89 \approx 400 \text{ mm}$$

$$MOR = A_{st} \cdot \sigma_{st} \cdot \left(d - \frac{d}{3} \right)$$

$$30 \times 10^6 = \frac{\pi \cdot d^2}{4} \cdot 230 \cdot \left(\frac{400}{3} - \frac{0.288 \cdot 400}{3} \right)$$

$$n =$$

Since

$$C = T$$

$$\frac{\sigma_{cbc}}{2} \times b \times n = A_{st} \times \sigma_{st}$$

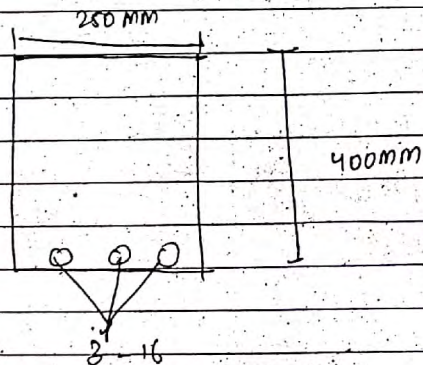
$$A_{st} = \frac{7 \times 250 \times 0.288 \times 400}{2 \times 230}$$

$$= 438.26 \text{ mm}^2$$

Adopt $\phi = 16 \text{ mm}$

$$n = A_{st} = \frac{\pi}{4} \times 16^2 = 201.06$$

$$\begin{aligned} \text{No. of bars} &= \frac{438.26}{201.06} \\ &= 2.18 \\ &\approx 3 \text{ nos.} \end{aligned}$$



$$\frac{d}{b} = 1.5$$

Moment of resistance of a RC beam 'b' mm wide and 'D' mm deep is $0.85 b d^2$. Calculate the ratio of depth of N.A to depth of beam to the centre of reinforcement if $m = 18.33$. Permissible stresses in concrete and steel are 7 N/mm^2 and 230 N/mm^2 .

Solution:

$$\text{Moment of resistance} = 0.85 b d^2$$

$$m = 18.33$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

For balanced section,

$$r = \frac{t}{c} = \frac{230}{7} = 32.86$$

$$n_c = \frac{m}{m+r} = \frac{18.33}{18.33+32.86} = 0.358$$

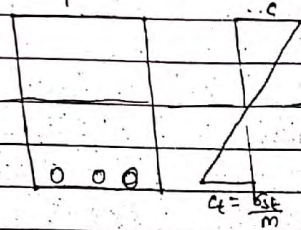
$$\begin{aligned} n_c &= n_c \times d \\ &= 0.358 \times d \end{aligned}$$

$$\text{MOR} = \frac{\sigma_{cbc}}{2} \times b \times n \times \left(d - \frac{n}{3} \right)$$

$$0.85 b d^2 = \frac{7}{2} \times b \times 0.358 \times d \times \left(d - \frac{0.358 \times d}{3} \right)$$

$$\begin{aligned} 1.103 b d^2 &= 0.2 \times 2.78 \times 0.266 \\ &= 1.103 b d^2 \end{aligned}$$

Since, $(MOR) < (MOR)_{critical}$, it is under-reinforced section.



$$C = \frac{c}{2} \cdot b \cdot n$$

$$T = t \cdot A_{st}$$

Let, $n = n_1 \cdot d$

$$\frac{C}{T} = \frac{n}{d - n}$$

$$\frac{c}{\frac{\sigma_{st}}{m}} = \frac{n_1 \cdot d}{d - n_1 \cdot d}$$

$$\therefore c = \frac{\frac{\sigma_{st}}{m} \cdot n_1 \cdot d}{d - n_1 \cdot d}$$

$$= \frac{\sigma_{st}}{m} \cdot \frac{n_1}{1 - n_1}$$

$$= \frac{280}{18.23} \cdot \left(\frac{n_1}{1 - n_1} \right) \quad \text{--- (1)}$$

$$MOR = \frac{C}{2} \cdot b \cdot n \cdot \left(d - \frac{n}{3} \right)$$

$$0.85 = \frac{1.280}{2 \cdot 18.23} \cdot \left(\frac{n_1}{1 - n_1} \right) \cdot n_1 \cdot \left(1 - \frac{n_1}{3} \right)$$

$$n_1 = 0.321$$

2.1)

$$b = 300 \text{ mm}$$

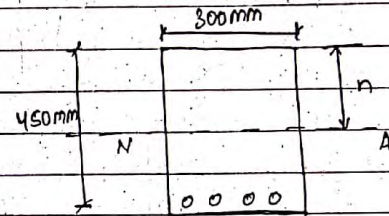
$$d = 450 \text{ mm}$$

$$\text{No. of bars} = 4$$

$$\text{dia. of bars} = 20 \text{ mm}$$

$$m = 9$$

$$\text{Permissible stress in concrete } \sigma_{cbc} = 7 \text{ N/mm}^2$$



Taking moment about N.A.,

$$300 \cdot n \cdot \frac{n}{2} = m \cdot A_{st} \cdot (450 - n)$$

$$300 \cdot \frac{n^2}{2} = 9 \cdot 4 \cdot \frac{\pi}{4} \cdot 20^2 \cdot (450 - n)$$

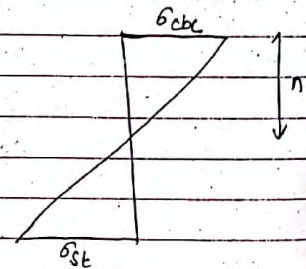
$$\therefore n = 150.32 \text{ mm}$$

From similar triangles,

$$\frac{\sigma_{cbc}}{n} = \frac{\sigma_{st}/m}{450 - n}$$

$$\text{or, } \frac{7}{150.32} = \frac{\sigma_{st}/9}{450 - 150.32}$$

$$\sigma_{st} = 125.6 \text{ N/mm}^2$$



2.2

$$b = 300 \text{ mm}$$

$$\phi = 600 \text{ mm}$$

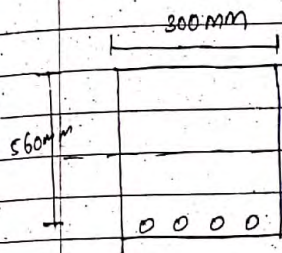
$$d = 600 - 40 = 560 \text{ mm}$$

$$m = 280 = \frac{280}{13.33}$$

$$3 f_{bc} = \frac{3 \times 7}{13.33}$$

$$f_{cbc} = \frac{20}{1.78} = 6.67 \approx 7 \text{ N/mm}^2$$

$$f_{st} = \frac{415}{1.78} = 233.15 \approx 230 \text{ N/mm}^2$$



For balanced section,

$$r = \frac{t}{c} = \frac{230}{7} = 32.85$$

$$n_1 = \frac{m}{m+r} = \frac{13.33}{13.33 + 32.85} = 0.288$$

$$n_c = n_1 \times d$$

$$= 0.288 \times 560$$

$$= 161.28 \text{ mm}$$

Taking moment about N.A,

$$\frac{300 \times n^2}{2} = m \times A_{st} \times (560 - n)$$

$$\text{or, } 150 \times n^2 = 13.33 \times \frac{\pi}{4} \times 16^2 \times (560 - n)$$

$$n = 167.49 \text{ mm}$$

Since $n > n_c$, it is over-reinforced section.

$$MOR = A_{st} \times t \times \left(\frac{d - n}{3} \right)$$

$$= 4 \times \frac{\pi}{4} \times 16^2 \times 230 \times \left(\frac{560 - 167.49}{3} \right)$$

$$= 93.25 \text{ kNm}$$

$$MOR = \frac{1}{2} \times c \times n_1 \times \left(\frac{d - n_1}{3} \right)$$

$$= \frac{1}{2} \times 7 \times 0.288 \times \left(\frac{560 - 161.28}{3} \right)$$

$$= \frac{1}{2} \times b \times n \times c \times \left(\frac{d - n}{3} \right)$$

$$= \frac{1}{2} \times 300 \times 167.49 \times 7 \times \left(\frac{560 - 167.49}{3} \right)$$

$$= 88.66 \text{ kNm}$$

2.3) $b = 250 \text{ mm}$
 $d = 500 \text{ mm}$
 $MOR = 45 \text{ kNm}$
 $\sigma_{st} = \frac{415}{1.78} = 233.15 \approx 230 \text{ N/mm}^2$
 $\sigma_{cbc} = \frac{20}{3} = 7 \text{ N/mm}^2$

$$m = \frac{280}{3 \sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$MOR = \frac{1}{2} \times b \times n \times C \times \left(\frac{d-n}{3} \right)$$

$$45 \times 10^6 = \frac{1}{2} \times 250 \times n \times 7 \times \left(\frac{500-n}{3} \right)$$

$$n = 111.08 \text{ mm}$$

$$n_c = n_1 \times d \quad r = \frac{t}{c} = \frac{230}{7} = 32.86$$

$$n_1 = \frac{m}{m+r} = \frac{13.33}{13.33+32.86} = 0.288$$

$$n_c = 0.288 \times 500 = 144.3 \text{ mm}$$

Since, $n < n_c$, it is under-reinforced section.

$$MOR = A_{st} \times t \times \left(\frac{d-n}{3} \right)$$

$$45 \times 10^6 = A_{st} \times 230 \times \left(\frac{500 - 111.08}{3} \right)$$

$$A_{st} = 422.6 \text{ mm}^2$$

$$\frac{\pi}{4} d^2$$

Adopt $\phi = 16 \text{ mm}$

$$A_{st} = 201.06$$

No. of bars ≈ 3

2.4) $l = 4 \text{ m}$

$$\text{Total load} = 30 \text{ kN/m}$$

$$\sigma_{cbc} = \frac{20}{3} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = \frac{415}{1.78} \approx 230 \text{ N/mm}^2$$

$$m = \frac{280 \sigma_{cbc}}{3 \sigma_{st}} = \frac{280}{3 \times 7} = 13.33 \quad r = \frac{t}{c} = \frac{230}{7} = 32.86$$

MOR For balanced section,

$$n = n_1 \times d$$

$$n_1 = \frac{m}{m+r} = \frac{13.33}{13.33+32.86} = 0.288$$

$$n =$$

$$\text{Moment}_{\max} = \frac{wl^2}{8}$$

$$= \frac{30 \times 4^2}{8}$$

$$= 60 \text{ KN-m}$$

$$\text{MOR} = \frac{1}{2} b \cdot n \cdot c \left(\frac{d-n}{3} \right)$$

$$60 \times 10^6 = \frac{1}{2} \cdot$$

2.4)

Dead load = 10 kN/m

Live load = 20 kN/m

Total load = 30 kN/m

clear span = 4m

As per clause 22.2(2) IS 456:2000

eff. span shall be smaller of

i) centre to centre distance between bearings

$$= 4000 + 300$$

$$= 4300 \text{ mm}$$

ii) clear span plus eff. depth

$$= 4000 + 550 \text{ (say)}$$

$$= 4550 \text{ mm}$$

∴ eff span = 4.3m

$$\text{Max. bending moment} = \frac{wl^2}{8} = \frac{30 \times 4.3^2}{8}$$

$$= 69.33 \text{ KN m}$$

$$f_{bc} = 7 \text{ N/mm}^2$$

$$f_{st} = 230 \text{ N/mm}^2$$

$$m = 13.33$$

$$r = \frac{t}{c} = 32.86$$

for balanced section,

$$n = n_1 \cdot d$$

$$n_1 = \frac{m}{m+r} = \frac{13.33}{13.33+32.86} = 0.288$$

$$\text{MOR} = \frac{1}{2} b \cdot n \cdot c \left(\frac{d-n}{3} \right)$$

Assume, $d/b = 1.5$

$$d = 1.5b$$

$$\text{or } 69.33 \times 10^6 = \frac{1}{2} \cdot b \cdot 0.288 \cdot 1.5b \cdot f \left(\frac{1.5b - 0.288 \cdot 1.5b}{3} \right)$$

$$\therefore b = 618.58 \approx 323.37 \text{ mm}$$

$$\hat{=} 350 \text{ mm}$$

$$d = 1.5 \times 350 = 525 \text{ mm}$$

$$n = 0.288 \times 525$$

$$= 151.2 \text{ mm}$$

$$\text{MOR} = A_{st} \cdot f_{st} \cdot \left(\frac{d-n}{3} \right)$$

$$69.33 \times 10^6 = A_{st} \cdot 230 \cdot \left(\frac{525 - 151.2}{3} \right)$$

$$A_{st} = 635.1 \text{ mm}^2$$

Adopt $\phi 16 \text{ mm}$ bar

$$\therefore A \text{ of 1 bar} = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

$$\text{No. of bar} = \frac{635.1}{201.06} = 3.15 \approx 4$$

Provide 4-16 mm dia bar

From clause 26.5.1.1

$$A_s = 0.85$$

$$bd \quad f_y$$

$$\text{or, } A_s = 0.85 bd$$

$$f_y$$

$$= \frac{0.85 \times 850 \times 525}{415}$$

$$= 376.33 \text{ mm}^2 < 804 \text{ mm}^2 \text{ OK}$$

$$\text{Max. area of tension reinforcement} = 0.04 \times \frac{350}{525} \times 550$$
$$= 7700 \text{ mm}^2$$
$$> 804 \text{ mm}^2 \text{ OK}$$

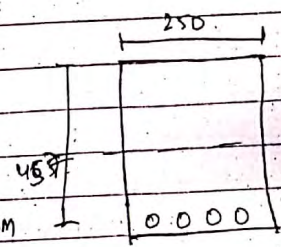
25)

$$b = 250 \text{ mm}$$

$$D = 500 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804 \text{ mm}^2$$

$$d = 500 - 35 - 8 = 457 \text{ mm}$$



$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$\sigma_{cbc} = 20 \text{ N/mm}^2$$

$$r = \frac{t}{c} = \frac{230}{7} = 32.85$$

$$m = \frac{280}{3 \sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$n = n_1 + d$$

$$n_1 = m$$

$$m + r$$

$$= 13.33$$

$$13.33 + 32.85$$

$$= 46.18$$

$$n_c = 0.288 \times 457 = 131.616 \text{ mm}$$

MOR =

Taking moment about N.A.,

$$\frac{250 \times n \times n}{2} = \frac{13.33 \times 4 \times \pi}{4} \times 16^2 \times (457 - n)$$

$$n = 159.66 \text{ mm}$$

$\therefore n > n_c$, it is over-reinforced section

$$MOR = \frac{1}{2} * b * n * c * \left(\frac{d-n}{3} \right)$$

$$= \frac{1}{2} * 250 * 159.66 * 7 * \left(\frac{457 - 159.66}{3} \right)$$

$$= 56.41 \text{ KN/m}$$

$$\frac{wL^2}{8} = 56.41$$

$$\frac{w * 4^2}{8} = 56.41$$

$$w = 28.205 \text{ KN/m}$$

$$\text{self wt of beam} = 0.25 * 0.5 * 25 = 3.125 \text{ KN/m}$$

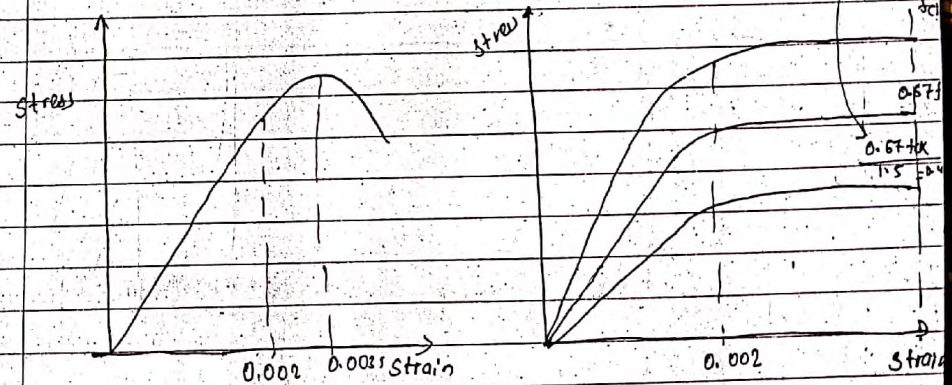
Chapter-3 Limit state Method

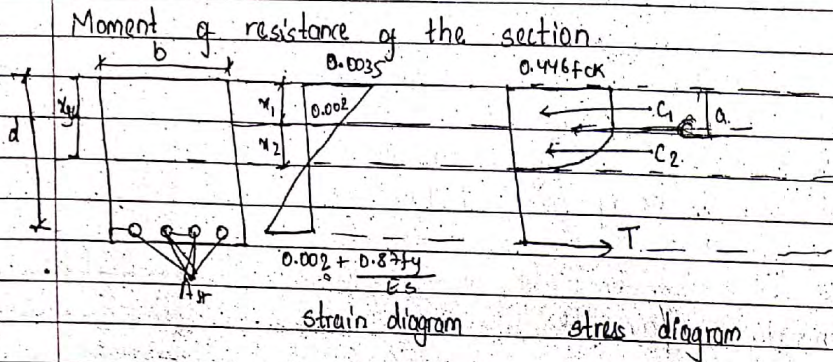
Limit state

beyond which structure is considered unusable.

- i) Limit state of safety/strength
- ii) Limit state of serviceability

Property of concrete





$$\epsilon_2 = 0.002$$

$$\epsilon_u = 0.0035$$

$$T = 0.87 f_y A_{st}$$

$$\epsilon_2 = \frac{4}{7} \epsilon_u$$

$$\epsilon_1 = \frac{3}{7} \epsilon_u$$

$$C_1 = b \times \epsilon_1 \times 0.446 f_{ck} = b \times \frac{3}{7} \times \epsilon_u \times 0.446 f_{ck} = 0.191 f_{ck} b \times \epsilon_u$$

$$C_2 = b \times \epsilon_2 \times \frac{2}{3} \times (0.446 f_{ck})$$

$$= b \times \frac{4}{7} \times \epsilon_u \times \frac{2}{3} \times (0.446 f_{ck})$$

$$= 0.169 f_{ck} b \times \epsilon_u$$

$$\text{Total compressive force (C)} = C_1 + C_2$$

$$= 0.191 f_{ck} b \times \epsilon_u + 0.169 f_{ck} b \times \epsilon_u$$

$$= 0.36 f_{ck} b \times \epsilon_u$$

Here,

$$C = T$$

$$0.36 f_{ck} b \times \epsilon_u = 0.87 f_y A_{st}$$

$$\epsilon_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$C_1 \times \frac{\epsilon_1}{2} + C_2 \times \left(\epsilon_1 + \frac{2}{3} \epsilon_2 \right) = C \times a$$

$$0.191 f_{ck} b \times \epsilon_u \times \frac{3}{2} \times \frac{\epsilon_u}{7} + 0.169 f_{ck} b \times \epsilon_u \times \left(\frac{3}{7} \epsilon_u + \frac{2}{3} \times \frac{4}{7} \epsilon_u \right)$$

$$= 0.36 f_{ck} b \times \epsilon_u \times a$$

$$0.0818 \epsilon_u + 0.1086 \frac{\epsilon_u}{7} = 0.36 a$$

$$a = \frac{0.529 \epsilon_u}{7}$$

$$a = 0.416 \epsilon_u \approx 0.42 \epsilon_u$$

$$\text{Lever arm (Z)} = (d - 0.42 \epsilon_u)$$

$$\text{(MOR)}_{\text{concrete}} = C \times Z$$

$$\text{MOR} = 0.36 f_{ck} b \times \epsilon_u (d - 0.42 \epsilon_u)$$

$$\text{(MOR)}_{\text{rebar}} = T \times Z$$

$$\text{MOR} = 0.87 f_y A_{st} (d - 0.42 \epsilon_u)$$

$$(MOR)_{concrete} = C * Z$$

Limiting depth of N.A

$$\frac{x_{u,lim}}{d - x_{u,lim}} = \frac{0.0035}{0.002 + \frac{0.87 f_y}{E_s}}$$

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

i) for mild-steel
 $f_y = 250 \text{ N/mm}^2$

Page-70

$$\frac{x_{u,lim}}{d - x_{u,lim}} = 1.1336$$

$$x_{u,lim} = 0.53 d$$

ii) Torsteel (Fe 415)
 $f_y = 415$

$$\frac{x_{u,lim}}{d} = 0.48$$

iii) TMT (Fe 500)
 $f_y = 500 \text{ N/mm}^2$

$$\frac{x_{u,lim}}{d} = 0.46$$

For Fe-250

$$MOR = 0.36 f_{ck} * b * (0.53d) * (d - 0.42 * 0.53 * d) = 0.148 f_{ck} * b d^2$$

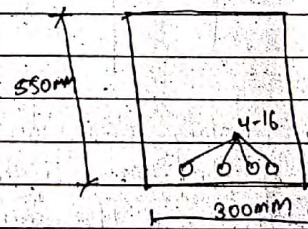
For Fe 415

$$MOR = 0.138 f_{ck} b d^2$$

For Fe 500

$$MOR = 0.133 f_{ck} b d^2$$

Q. Calculate MOR of a singly reinforced RC beam $300 \times 550 \text{ mm}$ (eff. depth) reinforced with 4-16 mm dia. bar. material used are M20 concrete and Torsteel bar.



Then,

Depth of N.A

$$T = C$$

$$0.87 f_y A_{st} = 0.36 f_{ck} * b * x_u$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 * 415 * 804.25}{0.36 * 20 * 300}$$

$$= 134.4 \text{ mm}$$

$$b = 300 \text{ mm}$$

$$d = 550 \text{ mm}$$

$$A_{st} = 4 * \pi * 16^2 = 804.25 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

load calculation

i) Imposed load = 20 kN/m

ii) Floor finish = 1 kN/m

iii) self-weight = $\frac{\gamma \cdot b \cdot d \cdot x}{k}$

$$= 25 \cdot 0.3 \cdot 0.5$$

$$= 3.75 \text{ kN/m}$$

Total load = 24.75 kN/m

Factored load = $1.5 \cdot 24.75$

$$= 37.125 \text{ kN/m}$$

$$M_{max} = \frac{wL^2}{8}$$

$$= \frac{37.125 \cdot 4.23^2}{8}$$

$$= 83.03 \text{ kN-m}$$

$$MOR = 0.133 f_{ck} b d^2$$

$$= 0.133 \cdot 20 \cdot 300 \cdot 450^2$$

$$= 167.6 \text{ kN-m}$$

Since, $M_{max} \leq MOR$, it is designed as singly reinforced section.

$A_{st} = ?$ Assume Under-reinforced sec

Annex-C 1.10 page - 96

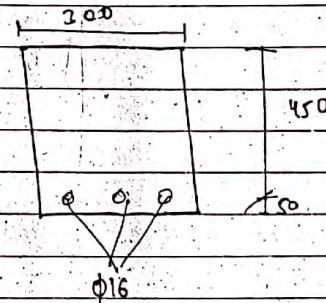
$$M_{max} = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bd f_{ck}} \right)$$

$$83.03 \times 10^6 = 0.87 \cdot 500 \cdot A_{st} \cdot 450 \cdot \left(1 - \frac{500 \cdot A_{st}}{20 \cdot 300 \cdot 450} \right)$$

$$A_{st} = 464.04 \text{ mm}^2$$

Adopt $\phi 16 \text{ mm}$

$$n = \frac{464.04}{201} \approx 3$$



Cl 26.5.1.1

$$A_{st} = 0.85$$

$$(A_{st})_{min} = \frac{0.85 \cdot 300 \cdot 500}{900} = \frac{308 \text{ mm}^2}{280.5 \text{ mm}^2}$$

$$(A_{st})_{max} = 0.04 \text{ of } bD$$

$$= 0.04 \cdot 300 \cdot 500$$

$$= 7200 \text{ mm}^2$$

Since, $(A_{st})_{min} < 603.18 < (A_{st})_{max}$ OK

2. Calculate MOR of a RC beam 300×500 mm overall depth with $\phi 5$ - 25mm dia bar with having 50mm effective cover. Use M20 concrete and Torsteel bar.

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

over-reinforced - brittle failure
under-reinforced - ductile

active cover = 50mm

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b \cdot d}$$

$$= \frac{0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 25^2}{0.36 \times 20 \times 300 \times 450}$$

$$\approx 0.912$$

$$x_u = 0.912 \cdot d$$

$$= 401.25$$

$$x_{u,lim} = 0.48 \cdot d$$

$$\Rightarrow x_{u,lim} = 0.48 \cdot d$$

$$= 216 \text{ mm}$$

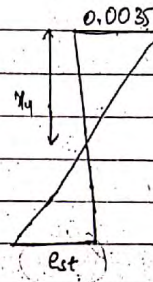
Since $x_u > x_{u,lim}$, it is over-reinforced

Trial-I

let us consider,

$$x_u = \frac{216 + 410.25}{2}$$

$$= 313.125 \text{ mm}$$



$$e_{st} = \frac{(d - x_u)}{x_u} \times 0.0035$$

$$= 0.00153$$

By interpolation (Table A SP-16 Page-6)

$$f_{st} = 297.23 \text{ N/mm}^2$$

$$x_u = \frac{0.87 (f_y) A_{st}}{0.36 f_{ck} b}$$

$$= 337.73 \text{ mm}$$

Trial-II

$$x_u = \frac{313.125 + 337.73}{2} = 325.427 \text{ mm}$$

$$e_{st} = \frac{(450 - 325.427)}{325.427} \times 0.0035$$

$$= 0.00134$$

$$f_{st} = 288.7 \text{ N/mm}^2$$

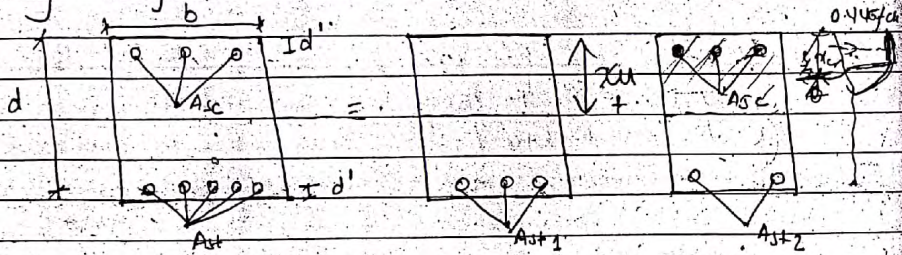
$$x_u = 328.04 \text{ mm}$$

$$MOR = 0.36 \frac{x_u}{d} \left(1 - 0.42 \frac{x_u}{d} \right) b d^2 f_{ck}$$

$$= 0.36 \times \frac{328.04}{450} \left(1 - 0.42 \times \frac{328.04}{450} \right) 300 \times 450^2 \times 20$$

$$= 221.28 \text{ KNm}$$

Doubly-Reinforced section



$$T = 0.87 f_y A_{st}$$

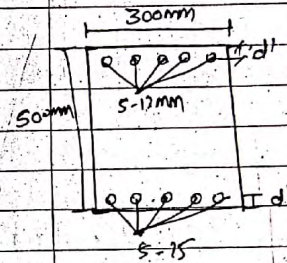
$$C = 0.36 f_{ck} b x_u \left(1 - 0.42 \frac{x_u}{d} \right) + (f_{sc} - f_{cc}) A_{sc}$$

f_{sc} = compressive strength of rebar
 f_{cc} = " " " " concrete = $0.446 f_{ck}$

$$MOR = 0.36 f_{ck} b x_u \left(1 - 0.42 \frac{x_u}{d} \right) + (f_{sc} - f_{cc}) A_{sc} (d - d')$$

$$p_{sc} = 0.0035 = \left(\frac{x_u - d'}{x_u} \right)$$

Q. Calculate MOR of a RC beam reinforced with 5-25mm ϕ bar as tensile reinforcement and 5-12mm ϕ bar as compression reinforcement. The beam is 300x500mm with 25mm clear cover on both sides. Use M20 concrete and Fe415 steel.



$$b = 300 \text{ mm}$$

$$D = 500 \text{ mm}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$$

$$A_{sc} = 5 \times \frac{\pi}{4} \times 12^2 = 565.49 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$d' = \frac{25 + 12}{2} = 31 \text{ mm}$$

$$d'' = \frac{25 + 25}{2} = 37.5 \text{ mm}$$

$$d = D - d''$$

$$= 500 - 37.5$$

$$= 462.5 \text{ mm}$$

$$x_{u,lim} = 0.48 \times d$$

$$= 222 \text{ mm}$$

$$x_u = \frac{(0.87 \times f_y) A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 2454.37 - (0.87 \times 415 - 0.446 \times 20) \times 565.49}{0.36 \times 20 \times 300}$$

$$= 318.06 \text{ mm}$$

Trial-I

$$x_u = \frac{2227 + 318.06}{2} = 270 \text{ mm}$$

Now,

$$e_{tc} = 0.0035 (x_u - d')$$

$$= 0.0031$$

$$e_{st} = 0.0035 \left(\frac{d - x_u}{x_u} \right)$$

$$= 0.0025$$

Now, Table-A

$$f_{sc} = \frac{860.9 - 351.8}{0.00380 - 0.00276} (0.0031 - 0.00276) + 351.8$$

$$= 354.77 \text{ N/mm}^2$$

$$f_{st} = \frac{851.8 - 342.8}{0.00276 - 0.00241} (0.0025 - 0.00241) + 342.8$$

$$= 245.11 \text{ N/mm}^2$$

$$x_u = \frac{(f_{st} A_{st} - (f_{sc} - f_{cc}) A_{sc})}{0.36 f_{ck} b}$$

$$= \frac{345.11 \times 2454.37 - (354.77 - 0.446 \times 20) \times 565.49}{0.36 \times 20 \times 300}$$

$$= 301.6 \text{ mm}$$

Trial-II

$$x_u = \frac{301.6 + 270}{2}$$

$$= 285.8 \text{ mm}$$

$$e_{tc} = 0.00312$$

$$e_{st} = 0.0022$$

$$f_{sc} = 354.95 \text{ N/mm}^2$$

$$f_{st} = \frac{335.08 \text{ N/mm}^2 - 337.4 \text{ N/mm}^2}{0.0022 - 0.0022} \dots$$

$$x_u = \frac{335.98 \times 2454.37 - (354.95 - 0.446 \times 20) \times 565.49}{0.36 \times 20 \times 300}$$

$$= 290.18 \text{ mm}$$

$$MOR = 0.36 f_{ck} b \times x_u (d - 0.42 x_u) + (f_{sc} - f_{cc}) A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 300 \times 290.15 (462.5 - 0.42 \times 290.15) + (354.95 - 0.446 \times 20) \times 565.49$$

$$= 297.91 \text{ kNm} \quad (462.5 - 31)$$

Q. Calculate MOR of RC beam 300×600 mm reinforced with 4-20mm dia bar as tensile reinforcement and 3-16mm dia bar as compression reinforcement. Use effective cover of 50mm on both reinforcement. Use M25 concrete, Fe 415.

$$b = 300 \text{ mm}$$

$$D = 600 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63 \text{ mm}^2$$

$$A_{sc} = 3 \times \frac{\pi}{4} \times 16^2 = 603.185 \text{ mm}^2$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$d' = 50 + \frac{\phi}{2} = 58 \text{ mm}$$

$$d'' = 50 + \frac{\phi}{2} = 58 \text{ mm}$$

$$d = D - d'' = 600 - 50 = 550 \text{ mm}$$

$$\begin{aligned} x_{u,lim} &= 0.48 \times d \\ &= 0.48 \times 550 \\ &= 264 \text{ mm} \end{aligned}$$

$$\begin{aligned} x_u &= \frac{0.87 f_y A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b} \\ &= \frac{0.87 \times 415 \times 1256.63 - (603.185 - 0.446 \times 20) \times 603.185}{0.36 \times 25 \times 300} \\ &= 90 \text{ mm} \end{aligned}$$

$$\begin{aligned} e_{sc} &= 0.0035 \frac{(90 - 50)}{90} \\ &= 0.00155 \end{aligned}$$

From table-A

$$f_{sc} = \frac{306.7 - 288.7}{0.00163 - 0.00144} (0.00155 - 0.00144) + 288.7 = 299.12 \text{ N/mm}^2$$

$$\begin{aligned} x_u &= \frac{f_{st} \cdot 0.87 f_y A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b} \\ &= \frac{0.87 \times 415 \times 1256.63 - (299.12 - 0.446 \times 20) \times 603.185}{0.36 \times 25 \times 300} \\ &= 103.708 \text{ mm} \end{aligned}$$

$$e_{sc} = 0.0035 \frac{(90 - 50)}{103.708 - 50} = 0.0018$$

$$f_{sc} = \frac{304.8 - 306.7}{0.00192 - 0.00163} (0.0018 - 0.00163) + 306.7 = 317.31 \text{ N/mm}^2$$

$$\begin{aligned} x_u &= \frac{0.87 \times 415 \times 1256.63 - (317.31 - 0.446 \times 20) \times 603.185}{0.36 \times 25 \times 300} \\ &= 101.037 \text{ mm} \end{aligned}$$

MOR =

Design an RC beam of clear span 5m carrying UDL of 50 kN/m. Use effective cover of 50 mm on either side and material used are M20 concrete and TMT bar.
Solⁿ

Take $L = 12$

d

$$d = \frac{5}{12} = 0.416 \approx 450 \text{ mm}$$

Take

$$\frac{d}{b} = 1.5$$

$$b = \frac{450}{1.5} = 300 \text{ mm}$$

Given $d' = 50$

$$D = d + d' = 450 + 50 = 500 \text{ mm}$$

Effective length (assume 230 mm support width)

$$i) \text{ Clear span} + \text{eff. depth} = 5 + 0.45 = 5.45 \text{ m}$$

$$ii) \text{ Clc} = 5 + \frac{0.23}{2} + \frac{0.23}{2} = 5.23 \text{ m}$$

$$\therefore l_{\text{eff}} = 5.23 \text{ m}$$

Load calculation

$$i) \text{ UDL} = 50 \text{ kN/m}$$

$$ii) \text{ Self wt} = 25 \times 0.3 \times 0.5 = 3.75 \text{ kN/m}$$

$$\text{Total load} = 53.75 \text{ kN/m}$$

$$\text{Factored load} = 80.625 \text{ kN/m}$$

Now,

$$M_{\text{max}} = \frac{wl^2}{8} = \frac{80.625 \times 5.23^2}{8} = 275.67 \text{ kNm}$$

For TMT

$$MOR = 0.133 f_{ck} b d^2$$

$$= 0.133 \times 20 \times 300 \times 450^2$$

$$= 161.595 \text{ kNm}$$

Since, $M_{\text{max}} > MOR$, it is designed as doubly reinforced section

First calculate area of steel (A_{st1}) to balance the concrete section.

$$MOR = 0.87 f_y A_{st1} \times d \times \left(1 - \frac{f_y A_{st1}}{f_{ck} b d}\right)$$

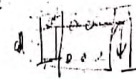
$$\text{or, } 161.595 \times 10^6 = 0.87 \times 500 \times A_{st1} \times 450 \left(1 - \frac{500 \times A_{st1}}{20 \times 300 \times 450}\right)$$

$$A_{st1} = 1017.08 \text{ mm}^2$$

Now, remaining moment

$$M_r = M_{\text{max}} - MOR$$

$$= 275.67 - 161.595 = 114.075 \text{ kNm}$$

(ii) $0.87 f_y A_{st2}$ 

Now, $M_R = 0.87 f_y A_{st2} \cdot (d - d')$
 $114.075 \times 10^6 = 0.87 \times 500 \times A_{st2} (450 - 50)$
 $\therefore A_{st2} = 655.6 \text{ mm}^2$

Similarly,

$M_R = (f_{sc} - f_{cc}) A_{sc} (d - d')$

for $\frac{d'}{d} = \frac{50}{450} = 0.11 \approx 0.1$

$f_{sc} = 412 \text{ N/mm}^2$ (Page 13 sp 16)

Then,

$114.075 \times 10^6 = (412 - 0.646 \times 20) \cdot A_{sc} (450 - 50)$
 $A_{sc} = 707.52 \text{ mm}^2$

$A_{st} = A_{st1} + A_{st2}$
 $= 1672.68 \text{ mm}^2$

$(A_{st})_{min} = 0.85 \cdot b d$
 $= \frac{f_y}{500} \times 300 \times 450$
 $= 229.5 \text{ mm}^2$

Available bars = 3, 4, 5, 6, 7, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 36, 40
 * order Jiffy

$(A_{st})_{max} = 0.004 \text{ of } b D$
 $= 0.004 \times 300 \times 550$
 $= 660 \text{ mm}^2$

For tension rebar,
 Adopt $\phi = 25 \text{ mm}$

$(A_{st})_{w} = \frac{\pi}{4} 25^2 = 490.87 \text{ mm}^2$

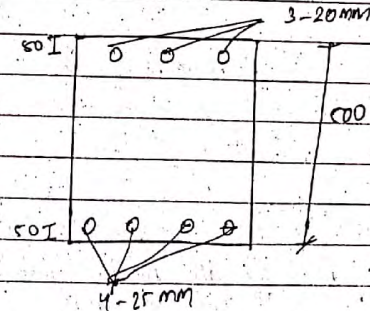
No. of bar = $\frac{1672.68}{490.87} = 3.41 \approx 4$

For comp. rebar
 Adopt $\phi = 20 \text{ mm}$

Adopt $\phi = 20 \text{ mm}$

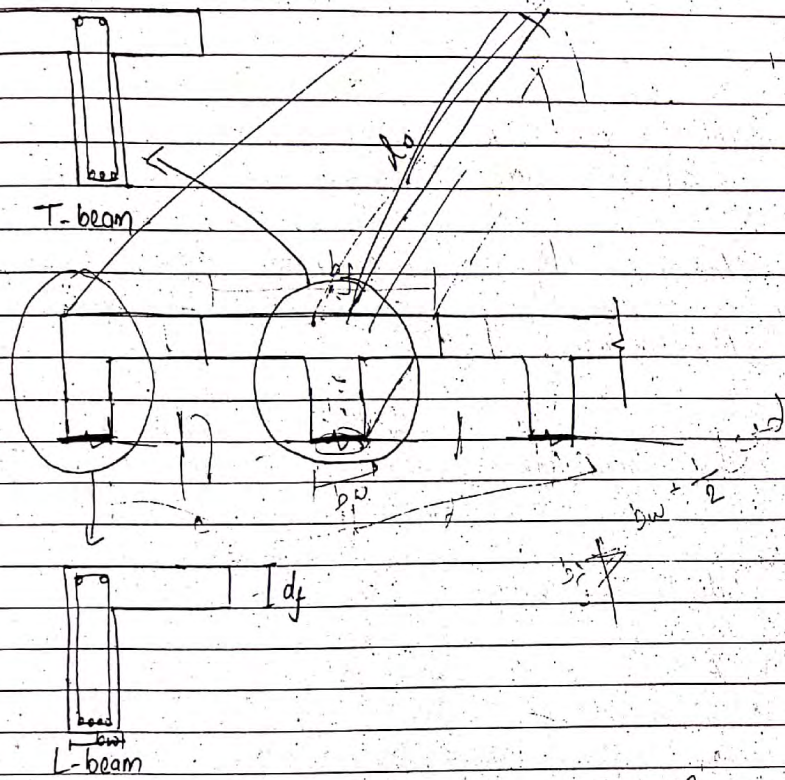
$(A_{sc})_{nd} = \frac{\pi}{4} 20^2 = 314.16 \text{ mm}^2$

No. of bars = $\frac{707.52}{314.16} = 2.25 \approx 3$



(refer 26.32 pg 45)

Flanged beam



- For isolated,

$$b_f = \frac{l_0}{b} + 4 + b_w \quad (\text{T-beam})$$

$$b_f = \frac{0.5 l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w \quad (\text{L-beam})$$

l_0 = distance between two points of contraflexure
 Taken as centre to centre distance l_0 or effective in case
 of simply supported beam.
 In case of continuous beam, $l_0 = 0.7$ times eff length
 of beam.

page-37

For continuous

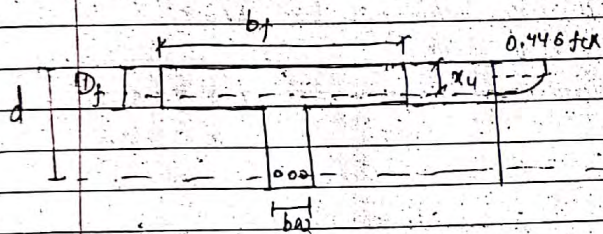
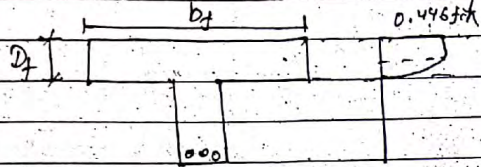
$$b_f = \frac{l_0}{6} + b_w + 6 D_f \quad (\text{T-beam})$$

$$b_f = \frac{l_0}{12} + b_w + 3 D_f \quad (\text{L-beam})$$

b_f = effective width of flange
 b_w = breadth of the web
 D_f = Thickness of flange

Moment of Resistance (MOR)

Case-I when N.A lies on the flange



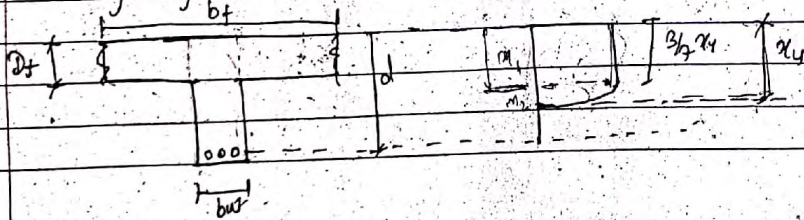
$$\text{Compressive force (C)} = 0.36 f_{ck} b_f x_u$$

$$\text{Tensile force (T)} = 0.87 f_y A_{st}$$

$$\begin{aligned} \text{MOR} &= 0.36 f_{ck} b_f x_u (d - 0.42 x_u) \\ &= 0.87 f_y A_{st} (d - 0.42 x_u) \end{aligned}$$

Case II: when N.A lies outside of flange

① If $D_f \leq \frac{3}{7} x_u$

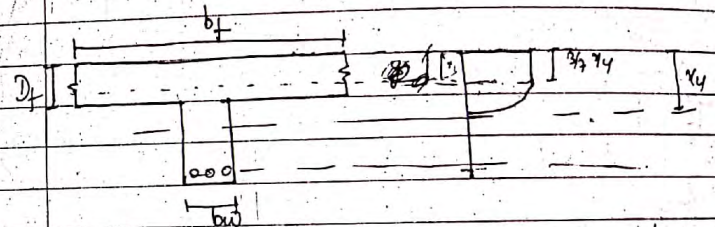


Whitney's stress block

$$\begin{aligned} \text{Compressive force (C)} &= 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f \\ \text{Tensile force (T)} &= 0.87 f_y A_{st} \end{aligned}$$

$$\begin{aligned} \text{MOR} &= 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) y_f (d - \frac{D_f}{2}) \\ &= 0.87 f_y A_{st} \end{aligned}$$

② If $D_f > \frac{3}{7} x_u$

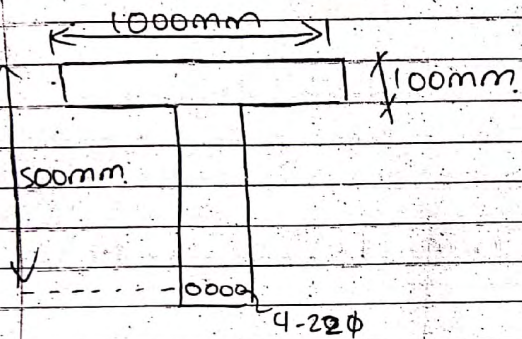


Depth of rectangular stress block is assumed to be $y_f = 0.15 x_u + 0.65 D_f$

$$\begin{aligned} \text{Compressive force} &= 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f \\ T &= 0.87 f_y A_{st} \end{aligned}$$

$$\text{MOR} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) y_f (d - \frac{D_f}{2})$$

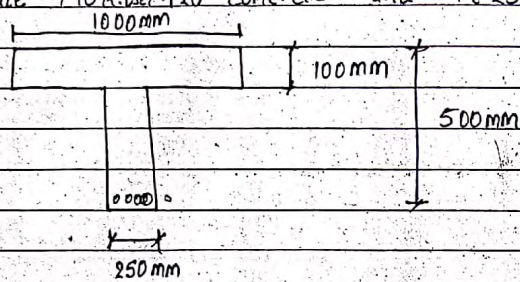
2. A T-beam of flange width 1000mm, flange thickness 100mm, rib width 250mm has an effective depth of 500mm. The beam is reinforced with 4 bars of 22mm diameter. Find the ultimate m_{or}. Use M20 concrete and Fe 250 steel.



here,

$$\begin{aligned}
 b_f &= 1000 \text{ mm} & d &= 500 \text{ mm} \\
 D_f &= 100 \text{ mm} & f_{ck} &= 20 \text{ N/mm}^2 \\
 b_w &= 250 \text{ mm} & f_y &= 250 \text{ N/mm}^2 \\
 A_{st} &= \frac{4 \times \pi \times 22^2}{4} = 1520.54 \text{ mm}^2
 \end{aligned}$$

Q. A T-beam of flange width $b_f = 1000\text{mm}$, $t_f = 100\text{mm}$ ribbed with of 250mm has an effective depth of 500mm . The beam is reinforced with 4 bars of 22mm ϕ . Find ultimate MOR. Use M20 concrete and Fe 250 steel.



$$\begin{aligned}
 b_f &= 1000\text{mm} \\
 D_f &= 100\text{mm} \\
 b_w &= 250\text{mm} \\
 d &= 500\text{mm} \\
 f_{ck} &= 20\text{N/mm}^2 \\
 f_y &= 250\text{N/mm}^2 \\
 A_{st} &= 4 \times \pi \times 22^2 = 1520.53\text{mm}^2
 \end{aligned}$$

Let us assume $x_u < D_f$
(i.e. NA lies on flange)

$$\begin{aligned}
 \text{Compressive force, } C &= 0.36 f_{ck} b_f x_u \\
 \text{Tensile force, } T &= 0.87 f_y A_{st}
 \end{aligned}$$

$$C = T$$

$$\text{or, } 0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$\begin{aligned}
 \text{or, } x_u &= \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 250 \times 1520.53}{0.36 \times 20 \times 1000} = 45.93\text{mm} \\
 &< D_f (100\text{mm})
 \end{aligned}$$

\therefore Our assumption is true

Now,

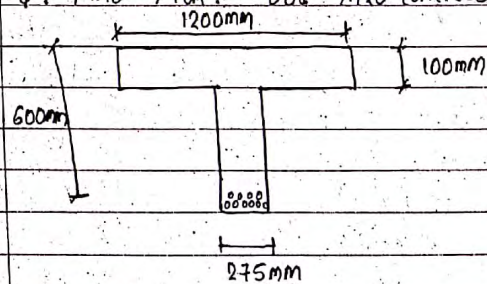
$$\begin{aligned}
 \text{MOR} &= 0.36 f_{ck} b_f x_u (d - 0.42 x_u) \\
 &= 0.36 \times 20 \times 1000 \times 45.93 \times (500 - 0.42 \times 45.93) \\
 &= 158.87\text{KNm} \approx 159\text{KNm}
 \end{aligned}$$

For Fe 250

$$\begin{aligned}
 x_{u\text{lim}} &= 0.53d \\
 &= 0.53 \times 500 \\
 &= 265\text{mm}
 \end{aligned}$$

Since $x_u < x_{u\text{lim}}$, it is under-reinforced section.

Q. A T-beam with flange width 1200mm , flange thickness 100mm ribbed with of 275mm has an effective depth of 600mm and is reinforced with 6 bars with 25mm ϕ and 4 bars of 16mm ϕ . Find MOR. Use M20 concrete.



Here,

$$b_f = 1200 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$d = 600 \text{ mm}$$

$$b_w = 275 \text{ mm}$$

$$A_{st} = \frac{6 \times \pi \times 25^2}{4} + \frac{4 \times \pi \times 16^2}{4} = 3749.5 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Let us assume N.A lies on flange (i.e. case I)

$$x_u < D_f$$

$$\text{Compressive force } C = 0.36 f_{ck} b_f \times x_u$$

$$\text{Tensile force } T = 0.87 f_y A_{st}$$

$$T = C$$

$$0.87 f_y A_{st} = 0.36 f_{ck} b_f \times x_u$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 3749.5}{0.36 \times 20 \times 1200}$$

$$= 156.68 \text{ mm} > 100 \text{ mm}$$

∴ Our assumption is wrong.

Let us assume NA lies on flange and $\frac{3}{7} x_u \geq D_f$

$$C = 0.36 f_{ck} b_w \times x_u + 0.446 f_{ck} (b_f - b_w) D_f$$

$$T = 0.87 f_y A_{st}$$

$$C = T$$

$$\text{or, } 0.36 f_{ck} b_w \times x_u + 0.446 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$\text{or, } 0.36 \times 20 \times 275 \times x_u + 0.446 \times 20 (1200 - 275) \times 100 = 0.87 \times 415 \times 3749.5$$

$$\therefore x_u = 267 \text{ mm}$$

$$\frac{3}{7} x_u = 114.43 \text{ mm} > D_f$$

∴ Our assumption is correct.

$$x_{u,lim} = 0.48 d$$

$$= 0.48 \times 600$$

$$= 288 \text{ mm} > x_u$$

∴ Section is under-reinforced.

$$MOR = 0.36 f_{ck} b_w \times x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) \times D_f (d - \frac{D_f}{2})$$

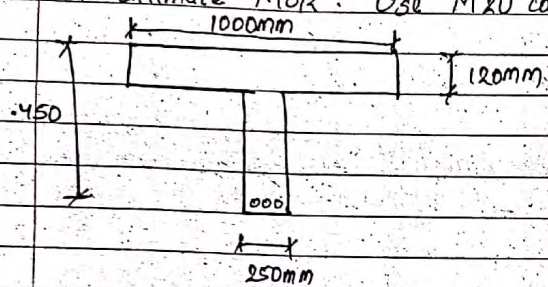
$$= 711.7 \text{ kNm}$$

How to check case a and case b)

$$\frac{D_f}{d} = 0.167 \quad \left(\frac{D_f}{d} < 0.2 \right)$$

(How)

- Q. A T-beam of flange width 1000mm, flange thickness 120mm, ribbed width of 250mm and an effective depth of 450mm. It is provided with a tension reinforcement of 4909 mm². Find ultimate MOR. Use M20 concrete and Fe 250 steel.



$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$b_f = 1000 \text{ mm}$$

$$b_w = 250 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$D_f = 120 \text{ mm}$$

let us assume N.A lies on flange i.e. $x_u < D_f$

$$C = 0.36 f_{ck} b_f x_u$$

$$T = 0.87 f_y A_{st}$$

$$C = T$$

$$\text{or, } 0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$\text{or, } x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 250 \times 4909}{0.36 \times 20 \times 1000}$$

$$= 148.3 \text{ mm} > D_f$$

\therefore Our assumption is wrong.

let us assume N.A lies below flange and $D_f < x_u$

$$C = 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) D_f$$

$$T = 0.87 f_y A_{st}$$

$$C = T$$

$$\text{or, } 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$x_u = 147.17 \text{ mm}$$

$$\frac{3}{7} x_u = 63.07 \text{ mm} < D_f$$

(assumption is wrong)

\therefore N.A lies below flange and $D_f < 3/7 x_u$

$$y_f = 0.15 x_u + 0.65 D_f$$

$$C = 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f$$

$$T = 0.87 f_y A_{st}$$

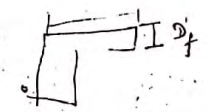
$$C = T$$

$$\text{or, } 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$\therefore x_u = 194.716 \text{ mm}$$

$$\frac{3}{7} x_u = 83 \text{ mm} < D_f$$

$$y_f = 107.207 \text{ mm}$$



$$MOR = 0.36 f_{ck} b_w x_u (\frac{d}{2} - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) y_f (\frac{d}{2} - x_u)$$

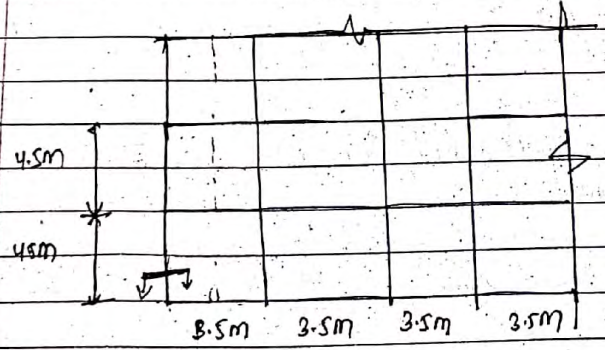
$$= 413.36 \text{ kNm}$$

$$\frac{D_f}{d} = 0.267 > 0.2 \quad \therefore \text{Case b}$$

i.e. $D_f > \frac{3}{7} x_u$

$\frac{D_f}{d} < 0.2$ (Case II a)
$\frac{D_f}{d} > 0.2$ (Case II b)

Design a corner beam of span 4.5m carrying on UDL of 5 kN/m^2 with adjacent beam spaced at 3.5m c/c. The depth of slab is 150mm. Use M20 concrete and Fe 500 rod. Assume both ends are fixed.



Assume $\frac{l}{d} = 12 \Rightarrow d = \frac{4.5 \text{ m}}{12} = 375 \text{ mm}$

Take $d = 450 \text{ mm}$
 $d' = 50 \text{ mm}$
 $D = d + d' = 500 \text{ mm}$

Take $d/b_w = 1.5$
 $b_w = \frac{450}{1.5} = 300 \text{ mm}$

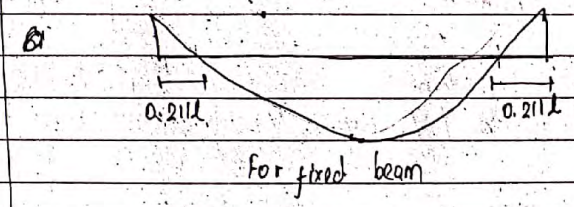
$D_f = 150 \text{ mm}$

$$l_0 = 4.5 - (0.211 \times 2) \times 2$$

$$= 4.5 - 0.211 \times 4.5 \times 2$$

$$= 2.601 \text{ m}$$

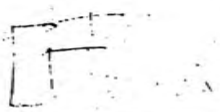
$l_0 =$ distance between points of zero moments (Page 37)



(Since it is at edge, we use formula for L beam page-37)

$$b_f = \frac{l_0}{12} + b_w + 3D_f$$

$$= \frac{2601}{12} + 300 + 3 \times 150 = 966.75 \text{ mm} = 967 \text{ mm}$$



load calculation

- i) Imposed load = $\frac{5 \times 3.5}{2} = 8.75 \text{ kN/m}$
- ii) Self wt. of slab = $\frac{25 \times 0.15 \times 3.5}{2} = 6.56 \text{ kN/m}$
- iii) self wt of beam = $0.5 \times 0.3 \times (0.25 - 0.15) = 2.625 \text{ kN/m}$

Total load = 17.935 kN/m
 Factored load = $1.5 \times 17.935 \text{ kN/m} = 26.9 \text{ kN/m}$

$M_{max} = \frac{wL^2}{12}$
 $= \frac{26.9 \times 4.5^2}{12}$
 $= 45.4 \text{ kN-m}$

Assume N.A lies at flange

$M_{max} = 0.87 f_y A_{st} (d - 0.42 x_u)$

$M_{max} = 0.36 f_{ck} b x_u (d - 0.42 x_u)$
 $45.4 \times 10^6 = 0.36 \times 20 \times 967 \times x_u (450 - 0.42 x_u)$
 $x_u = 14.69 \text{ mm} < D_f$

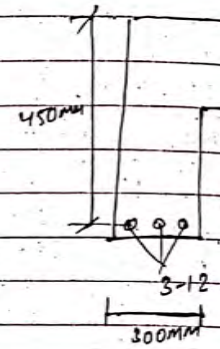
$45.4 \times 10^6 = 0.87 \times 500 \times A_{st} (450 - 0.42 \times 14.69)$
 $A_{st} = 235.15 \text{ mm}^2$

$\frac{\pi}{4} d^2$
 $d = 17.3 \approx 18$

$(A_{st})_{min} = \frac{0.85 b_w d}{f_y}$
 $= \frac{0.85 \times 300 \times 450}{5000}$
 $= 229.5 \text{ mm}^2$

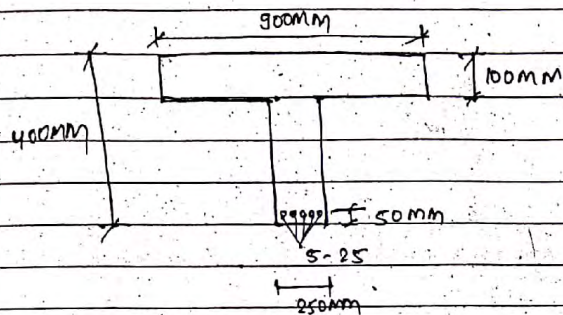
$(A_{st})_{max} = 0.04 \text{ of } b D$
 $= \frac{0.04 \times 300 \times 500}{100}$
 $= 6000 \text{ mm}^2$

Adopt $\phi = 12 \text{ mm}$
 No. of bar = $\frac{235.15}{113.09}$
 $= 2.1$
 $\approx 3 \text{ nos.}$



NOTE: In case of over-reinforced section, calculate MOR assuming critical depth

Q. Calculate MOR of the flanged beam as shown



$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$D = 400 \text{ mm}$$

$$d = 350 \text{ mm}$$

$$b_w = 250 \text{ mm}$$

$$b_f = 900 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$$

$$x_{u, \max} = 0.48 d$$

$$= 0.48 \times 350$$

$$= 168 \text{ mm}$$

Assuming N.A lies at flange, $x_u < D_f$

$$C = 0.36 f_{ck} b_f x_u$$

$$T = 0.87 f_y A_{st}$$

$$C = T$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 2454.37}{0.36 \times 20 \times 900}$$

$$= 136.25 \text{ mm}$$

Assumption is wrong.

$$\frac{D_f}{d} = \frac{100}{350} = 0.28 > 0.2$$

(Case II b)

$$y_f = 0.15 x_u + 0.65 D_f$$

$$= 0.15 x_u$$

$$C = T$$

$$0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 x_u + 0.446 \times 20 (900 - 250) (0.15 x_u + 0.65 \times 100)$$

$$= 0.87 \times 415 \times 2454.37$$

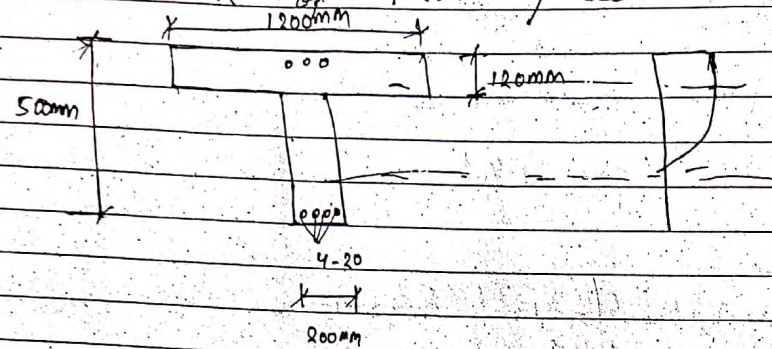
$$x_u = 190.76 \text{ mm} > \frac{3}{4} x_u < D_f$$

$$MOR = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right)$$

$$= 0.36 \times 20 \times 250 \times 190.76 (350 - 0.42 \times 190.76) + 0.446 \times 20 (900 - 250) \times (0.15 \times 190.76 + 0.65 \times 100) \left(350 - \frac{0.15 \times 190.76 + 0.65 \times 100}{2} \right)$$

$$= 243.86 \text{ kNm}$$

Q. Calculate MOR of following section.



Use $d' = 50\text{mm}$ on both sides

$$f_{ck} = 20 \text{ N/mm}^2$$

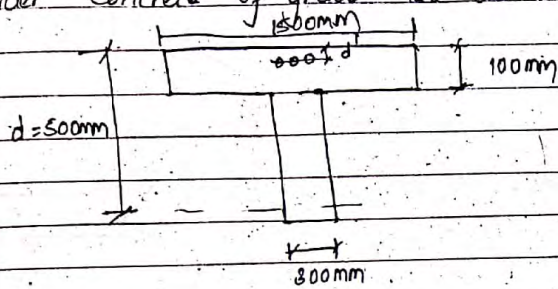
$$f_y = 415 \text{ N/mm}^2$$

Calculate the depth of line of action of compressive force.

$$x_m = 0.48d$$

$$a = 0.36 f_{ck} b_w \cdot x_m + 0.446 f_{ck} (b_f - b_w) \cdot \frac{d'}{2}$$

Q. Design a T-beam section of 1500 mm width of flange, 100 mm depth of flange, 300 mm width of web & 500 mm eff. depth which is subjected to a moment of 750 kNm. Consider concrete of grade M20 and steel of grade Fe 415.



$$\begin{aligned}
 b_f &= 1500 \text{ mm} & f_{ck} &= 20 \text{ N/mm}^2 \\
 b_w &= 300 \text{ mm} & f_y &= 415 \text{ N/mm}^2 \\
 d &= 500 \text{ mm} \\
 d_f &= 100 \text{ mm}
 \end{aligned}$$

For Fe 415

$$\begin{aligned}
 x_{u,lim} &= 0.48 \cdot d \\
 &= 0.48 \cdot 500 \\
 &= 240 \text{ mm} > D_f
 \end{aligned}$$

Also, $\frac{3}{7} x_{u,lim} = \frac{3}{7} \cdot 240$

$$= 102.85 \text{ mm} > D_f$$

Then,

$$MOR = 0.36 f_{ck} \cdot b_f \cdot x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) \cdot D_f \cdot (d - \frac{D_f}{2})$$

$$\text{or, } \frac{300}{1500} \cdot 240 = 0.36 \cdot 20 \cdot 1500 \cdot \frac{240}{100} (500 - 0.42 \cdot 240) + 0.446 \cdot 20 (1500 - 300) \cdot 100 \cdot (500 - \frac{100}{2})$$

$$688.63 \text{ kN}\cdot\text{m}$$

Since, $MOR < M_{max}$, it is designed as doubly reinforced section.

Then,

$$\begin{aligned}
 MOR &= 0.87 f_y A_{st1} \cdot (d - 0.42 x_u) \\
 688.63 \times 10^6 &= 0.87 \cdot 415 \cdot A_{st1} \cdot (500 - 0.42 \cdot 240) \\
 A_{st1} &= 4777.77 \text{ mm}^2
 \end{aligned}$$

Remaining moment (M_R)

$$\begin{aligned}
 &= M_{max} - MOR \\
 &= 750 - 688.63 \\
 &= 61.37 \text{ kN}\cdot\text{m}
 \end{aligned}$$

Let, $d' = 50 \text{ mm}$

$$\begin{aligned}
 M_R &= 0.87 f_y A_{st2} (d - d') \\
 61.37 \times 10^6 &= 0.87 \cdot 415 \cdot A_{st2} \cdot (500 - 50) \\
 A_{st2} &= 377.7 \text{ mm}^2
 \end{aligned}$$

Now

$$M_R = (f_{sc} - f_{cc}) A_{sc} (d - d') \rightarrow 61.37 \times 10^6 = (351.8 - 0.446 \cdot 20) \cdot A_{sc} \cdot (500 - 50)$$

$$\begin{aligned}
 e_{sc} &= 0.0035 \cdot \left(\frac{x_{u,lim} - d'}{x_{u,lim}} \right) & A_{sc} &= 297.7 \text{ mm}^2 \\
 &= 0.0035 \cdot \left(\frac{240 - 50}{240} \right) & (A_{st})_{min} &= 0.85 \cdot \frac{b_w \cdot d}{f_y} \\
 &= 0.00277 & &= \frac{0.85 \cdot 300 \cdot 500}{415} \\
 & & &= 309.2 \text{ mm}^2
 \end{aligned}$$

From table - 1 (sp-16)

$$e_{sc} = 0.00277 \approx 0.00276$$

$$f_{sc} = 351.8 \text{ N/mm}^2$$

$$(A_{st})_{max} = (A_{sc})_{max} = 0.04 \cdot \frac{500 \cdot 300}{2} = 6000 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 4777.77 + 377.77$$

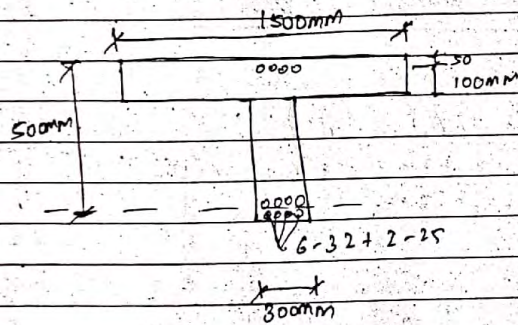
$$= 5155.54 \text{ mm}^2$$

Adopt $\phi = 32 \text{ mm}$ and 25 mm

Provide 6-32 + 2-25 mm on tension zone
for compression

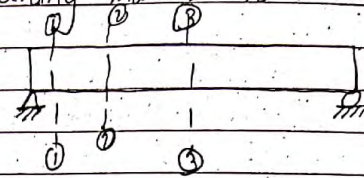
Adopt $\phi = 12 \text{ mm}$

$$\text{No. of bar} = \frac{397.7}{113.09} = 3.51 \approx 4 \text{ nos.}$$



Shear and Bond

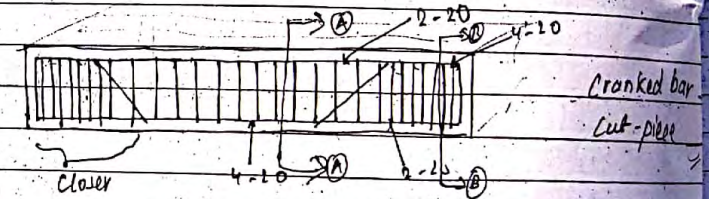
→ At a member, generally shear force is maximum at support and Bending moment is maximum at the mid-span



At position (1), shear force is maximum & B.M. is negligible

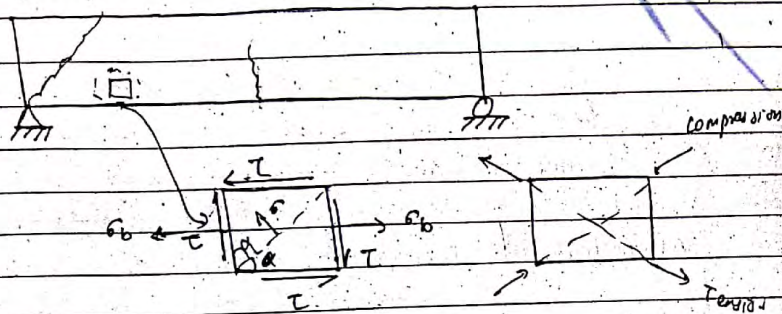
At position (2), SF & B.M. co-exist

At position (3), SF is almost negligible & B.M. is max.



0	0	00.00
0	00.00	0
A-A		B-B

Shear failure due to tension



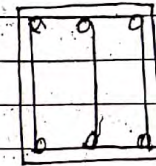
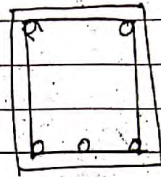
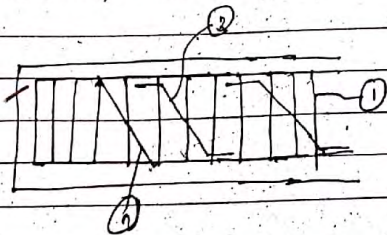
$$\tan 2\alpha = \frac{2\tau}{\sigma_b}$$

At support, $\sigma_b = 0$
 $\alpha = 45^\circ$

At mid span, $\tau = 0$
 $\alpha = 0^\circ$

Shear reinforcement

- i) Vertical shear bar
- ii) Bent-up bar
- iii) Inclined stirrups.



2 legged stirrup

2 legged + 1 legged

Nominal shear capacity: (τ_c) (40.1) pg. 72

$$\tau = \frac{V}{bd}$$

τ_c = Nominal design shear strength calculated as per $\sqrt{A_t}$ % and concrete grade

τ_c is due to

- i) Interlocking of aggregate
- ii) Dowel action of main reinforcement
- iii) Vertical shear carried by section

i) If $\tau \leq \tau_c$, provide nominal shear bar.

26.5.1.5 Nominal shear bar - represents spacing not less than ^{more} 1

i) $0.75d$ for vertical stirrup and d for inclined stirrup.

ii) 300mm

& minimum area of steel is given by

$$A_{sv} \geq \frac{0.85 f_y A_{sv}}{0.4 k} \quad 26.5.1.6$$

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

$$\tau_s \leq \frac{0.87 f_y A_{sv}}{0.4 b} \quad \tau_s \leq \tau_c$$

ii) ~~ii)~~ If $\tau_c < \tau \leq \tau_{max}$, Design shear bar for

$$V_{us} = (\tau_v - \tau_c) b d$$

and spacing is calculated as

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} \quad (40.4 @ \text{page-73})$$

\leq Nominal spacing as (i).

iii) If $\tau > \tau_{max}$, redesign the section

A_{sv} = shear reinforcement area.

Q. Design shear reinforcement for a section of a beam dimension 300×450 mm effective depth to carry factored shear of 250 kN. Use M20 concrete and Fe415 provided with 3-20mm bar as tensile reinforcement.

Solⁿ

Given

$$b = 300 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$V_u = 250 \text{ kN}$$

$$A_{st} = 3 \times \pi \times 20^2 = 942.48 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_{yk} = 415 \text{ N/mm}^2$$



$$\text{Shear stress at section } (\tau) = \frac{V_u}{b d} = \frac{250 \times 10^3}{300 \times 450}$$

Now,

$$\frac{A_{st}}{b d} \% = \frac{942.48}{300 \times 450} \times 100 = 0.698$$

For M20.

$$\tau_c = \frac{0.56 - 0.48}{0.75 - 0.5} (0.698 - 0.5) + 0.48 = 0.543 \text{ N/mm}^2$$

For M20 (Table-20)

$$\tau_{max} = 2.8 \text{ N/mm}^2$$

$$\tau_c < \tau \leq \tau_{max}$$

$$V_{uw} = (\tau - \tau_c) b d$$

$$= (1.85 - 0.543) \times 300 \times 450$$

$$= 176.445 \text{ kN}$$

Adopt 8mm dia bar - 2 legged stirrup.

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

Then,

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{uw}}$$

$$= \frac{0.87 \times 415 \times 100.53 \times 450}{176.445 \times 10^3}$$

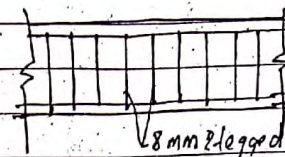
$$= 92.57 \text{ mm}$$

Nominal shear is given by spacing not greater than $\leq 0.35 \times d = 0.35 \times 450 = 157.5 \text{ mm}$

$$\leq \frac{0.87 f_y A_{sv}}{0.46} = \frac{0.87 \times 415 \times 100.53}{0.46 \times 300}$$

$$= 302.47 \text{ mm}$$

Provide 8mm 2 legged stirrup @ 90mm c/c



Q1. A RC beam has an effective depth of 500mm and a breadth of 350mm. It consists of 4-25mm dia-bars. Calculate the shear reinforcement needed for a factored shear force of 350kN. Adopt $\sigma_{ck} = 20 \text{ N/mm}^2$ and $\sigma_{sv} = 415 \text{ N/mm}^2$.

$$d = 500 \text{ mm}$$

$$b = 350 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5 \text{ mm}^2$$

$$V_u = 350 \text{ kN}$$

$$\sigma_{ck} = 20 \text{ N/mm}^2$$

$$\sigma_{sv} = 415 \text{ N/mm}^2$$

$$\text{Nominal shear stress } (\tau) = \frac{V_u}{b d} = \frac{350 \times 10^3}{350 \times 500} = 2 \text{ N/mm}^2$$

Now,

$$A_{st} \rho = \frac{1963.5}{500 \times 350} \times 100 = 1.122 \%$$

From Table 19,

$$\tau_c = \frac{0.67 - 0.62}{1.25 - 1} \times (1.122 - 1) + 0.62 = 0.644 \text{ N/mm}^2$$

Maximum shear stress in M20, $\tau_{max} = 2.8 \text{ N/mm}^2$

$$\tau_c < \tau < \tau_{max} \quad V_{us} = (T - T_c) b d$$
$$= (2 - 0.649) \times 500 \times 350$$
$$= 238.0 \text{ kN}$$

$$S_v = \frac{0.87 f_y A_{vd}}{V_{us}}$$
$$= \frac{0.87 \times 415 \times 1963.5 \times 350}{238.0}$$

Adopt 8mm dia bar - 2 legged stirrups,
 $A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.5 \text{ mm}^2$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$
$$= \frac{0.87 \times 415 \times 100.5 \times 500}{238 \times 10^3}$$
$$= 76.23 \text{ mm}$$
$$= 76 \text{ mm}$$

The code requires that $S_v < 800 \text{ mm}$
 $< 0.75 d$ i.e. 375 mm

Minimum shear reinforcement

$$A_{sv} \geq \frac{0.4 b S_v}{0.87 f_y} = \frac{0.4 \times 350 \times 76}{0.87 \times 415} = 29.47 \text{ mm}^2$$

$100.51 > 29.47 \text{ mm}^2$ (OK)

Although all requirement of the code are satisfied by 8mm - 2 legged Fe415 grade bars @ 76mm c/c but it is suggested that minimum spacing of stirrups be limited to 100mm in order to permit space for compaction of the concrete.

∴ Revised area of stirrups

$$A_{sv} = \frac{V_{us} S_v}{0.87 f_y d} = \frac{238 \times 10^3 \times 100}{0.87 \times 415 \times 500} = 131.83$$

$$\text{Area of one leg} = \frac{131.83}{2} = 65.9 \text{ mm}^2$$

$$\text{Area of 10mm dia bar} = \frac{\pi}{4} \times 10^2 = 78.5 \text{ mm}^2 > 65.9 \text{ mm}^2$$

∴ Use 10mm ϕ - 2 legged vertical stirrups @ 100mm

check:

Min shear reinforcement

$$A_{sv} \geq \frac{0.4 b S_v}{0.87 f_y} = \frac{0.4 \times 350 \times 100}{0.87 \times 415} = 88.97 \text{ mm}^2 < 2 \times 78.5$$

(Page 73) 40.4 @ @

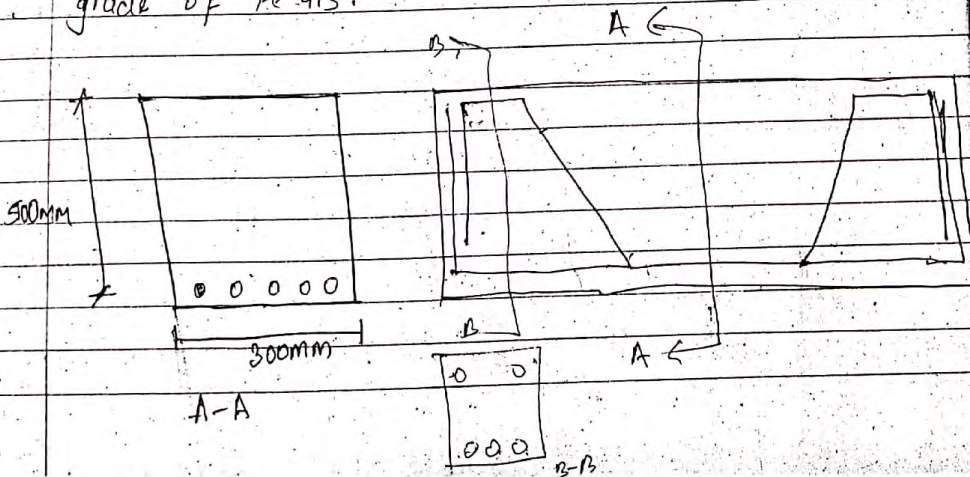
Shear resistance due to bent-up bar & inclined stirrups

If bars are bent-up at one section
 $V_{us} = 0.87 f_y A_{sv} \sin \alpha$

ii) If bars are bent-up at different section
 $V_{us} = 0.87 f_y A_{sv} d (\sin \alpha + \cos \alpha)$

A_{sv} = area of bent-up bar or inclined stirrup

Q. A rectangular beam section 300 mm wide and 500 mm depth is reinforced with 5 bars of 20 mm out of which 2 bars have been bent at 45° . Determine the shear resistance of the bent-up bar and the additional shear reinforcement required if it is subjected to ultimate shear force of 300 kN. Concrete grade of M20 and steel grade of Fe 415.



$$V_u = 300 \text{ kN}$$
$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2$$
$$= 942.47 \text{ mm}^2$$

Then,

$$\frac{A_{st}}{bd} \times 100\% = 0.628\%$$

$$\tau_c = \frac{0.56 - 0.48}{0.25 - 0.5} (0.628 - 0.5) + 0.48$$
$$= 0.521 \text{ N/mm}^2$$

$$\tau_c < \tau < \tau_{max}$$

$$\tau = \frac{V}{bd} = \frac{300 \times 10^3}{300 \times 500} = 2 \text{ N/mm}^2$$

From Table - 20

$$\tau_{max} = 2.8 \text{ N/mm}^2$$

$$\tau - \tau_c = 2 - 0.521 = 1.479 \text{ N/mm}^2$$

For bent-up bar

$$\text{Shear capacity of bent-up bar} = 0.87 f_y A_{sv} \sin \alpha$$
$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 20^2 \times \sin 45^\circ$$
$$= 160.41 \text{ kN}$$

Also,

$$V_{us} = 1.479 \times 300 \times 500 = 221.85 \text{ kN}$$

Page 73 - @ bent up bar does not carry more than $\frac{1}{2}$ of V_{us}

$$\text{Hence, Design shear force} = \frac{1}{2} V_{us} = \frac{1}{2} \times 221.85 = 110.925 \text{ kN}$$

Adopt 8mm dia bar - 2 legged stirrup

$$A_{sv} = 2 * \frac{\pi}{4} * 8^2 = 100.53 \text{ mm}^2$$

Then:

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 * 415 * 100.53 * 500}{110.925 * 10^3}$$

$$= 163.6 \text{ mm}$$

Nominal shear is given by spacing not greater than

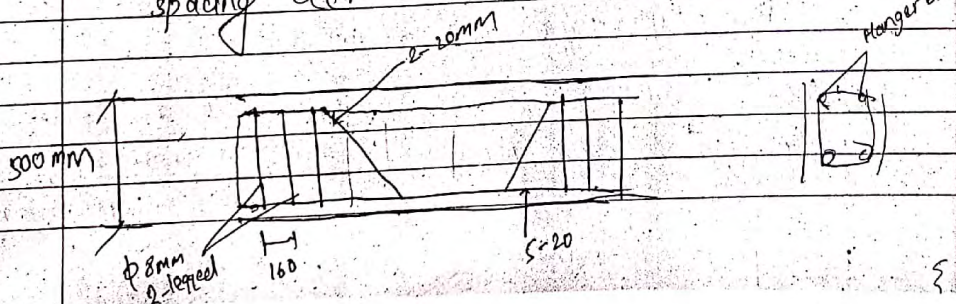
$$\leq \frac{0.75 * 500}{0.48}$$

$$\leq \frac{0.87 f_y A_{sv}}{0.48}$$

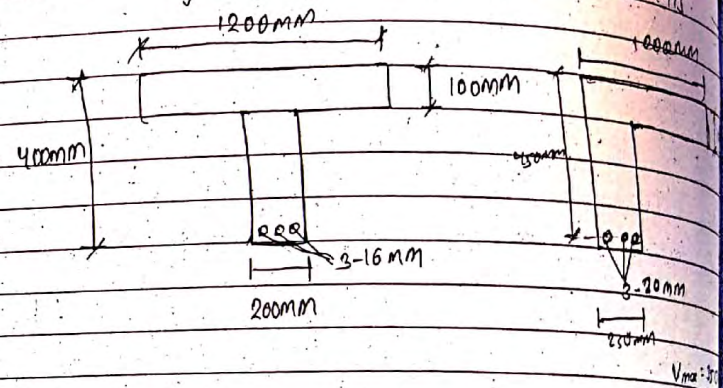
$$= \frac{0.87 * 415 * 100.53}{0.48 * 500}$$

$$= 302.47$$

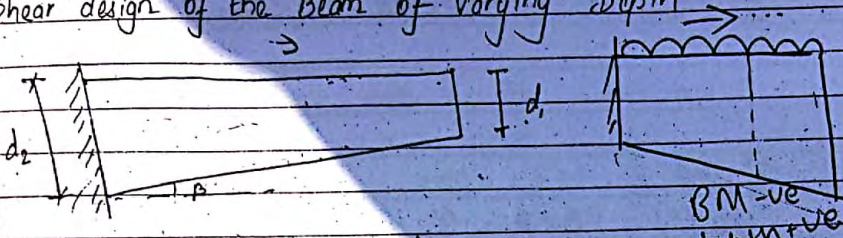
∴ Provide 8mm dia bar - 2 legged stirrup @ 160mm spacing etc.



Q. Design shear reinforcement for following T-beam for factored shear force of 220 kN. Double reinforcement → Tension



Shear design of the Beam of Varying Depth



$$\text{Design SF at section } (V_u) = V \pm \frac{M}{d} \tan \alpha$$

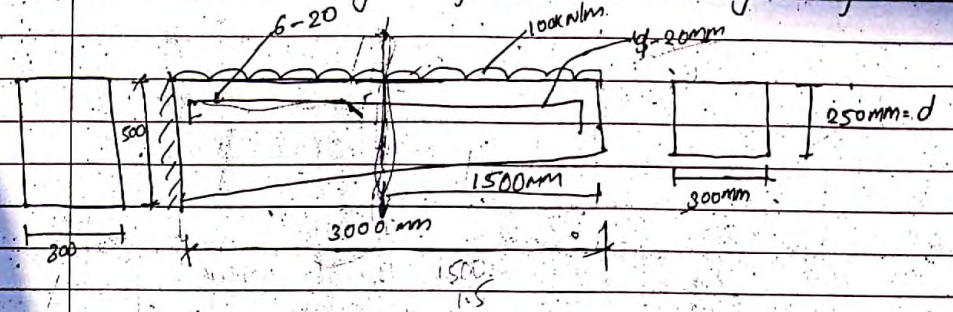
-ve if BM increases along increase in depth and ~~increase~~
 +ve if BM decrease along increase in depth

V = actual SF at section

M = actual BM at section

d = eff depth

Q. Design shear reinforcement for the RCC beam as shown in fig. Consider concrete grade of 120 and steel grade of Fe 415.



Design shear reinforcement for midspan and fixed support.

~~Q.~~

At midspan

$$\text{Eff. depth} = \frac{500 + 250}{2} = 375 \text{ mm}$$

$$b = 300 \text{ mm}$$

$$A_{st} = 4 * \frac{\pi}{4} * 20^2 = 1256.64 \text{ mm}^2$$

$$V = 100 * 1.5 = 150 \text{ kN}$$

$$\therefore \text{Factored } V = 150 * 1.5 = 225 \text{ kN}$$

$$M = \left(100 * 1.5 * \frac{1.5}{2} \right) * 1.5 = 168.75 \text{ kNm}$$

Design sf at mid-span = $\frac{V_u M}{V_{u1} d}$

$$= \frac{168.75}{225} = 0.75$$

$$= \frac{168.75}{0.375} + \frac{250}{3000}$$

$$= 224.96 \text{ KN} - 187.50 \text{ KN}$$

Adopt 8mm dia bar 2 legged stirrup

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

Then,

$\frac{f_y}{f_c}$

$$\frac{A_{sv}}{bd} \times 100\% = \frac{1256.64}{300 \times 375} \times 100\% = 1.12\%$$

$$T_c = \frac{0.67 - 0.62}{1.25 - 1} (1.12 - 1) + 0.62 = 0.64 \text{ N/mm}^2$$

For M20

$$T_{c \text{ max}} = 2.8 \text{ N/mm}^2$$

$$T = \frac{V_u}{bd} = \frac{187.50 \times 10^3}{300 \times 375} = 1.67 \text{ N/mm}^2$$

$$T_c < T < T_{c \text{ max}}$$

→ A

$$V_{us} = (T - T_c) b d$$

$$= (1.67 - 0.64) \times 300 \times 375$$

$$= 115875 \text{ KN}$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 100.53 \times 375}{115875 \times 10^3}$$

$$= 117.46 \text{ mm}$$

Nominal shear is given by spacing not greater than

$$L = 0.75 \times d = 0.75 \times 375 = 281.25 \text{ mm} \leq 300 \text{ mm}$$

$$\leq 0.87 f_y A_{sv}$$

$$\leq 0.4 b$$

$$\leq 0.87 f_y A_{sv}$$

$$\leq 0.4 b$$

$$= 302.47$$

Provide 8mm dia bar 2 legged stirrup @ 100 mm c/c.

$$V = 100 \times 3 \times 1.5 = 450 \text{ KN}$$

$$M = 100 \times 3 \times \frac{2}{2} \times 1.5 = 675 \text{ KNm}$$

At fixed support,

$$V_{us} = \frac{V - M}{d} + \alpha \beta$$

$$= \frac{450 - 675}{0.500} + \left(\frac{250}{3000} \right)$$

$$= 337.5 \text{ KN}$$

$$\frac{A_{it}}{bd} = \frac{0.7 \times 20^2}{500 \times 300} \times 100 = 1.26 \%$$

From Table - 19

$$T_c = \frac{0.72 - 0.67}{1.5 - 1.25} \times (1.26 - 1.25) + 0.67$$

$$= 0.67 \text{ N/mm}^2$$

$$T_{max} = 2.8 \text{ N/mm}^2$$

$$T = \frac{V_u}{bd} = \frac{337.5 \times 10^3}{500 \times 300} = 2.25 \text{ N/mm}^2$$

$$T_c < T < T_{max}$$

Adopt 8mm dia bar 2 legged stirrup
 $A_{sv} = 100.53 \text{ mm}^2$

$$V_{us} = (T - T_c) b d$$

$$= (2.25 - 0.67) \times 500 \times 300 = 237 \text{ KN}$$

$$s_v = 0.87 \frac{f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 100.53 \times 500}{237 \times 1000}$$

$$= 76.57 \text{ mm}$$

$$\leq 0.95 d = 0.95 \times 500 = 375 \text{ mm} < 500 \text{ mm}$$

$$\leq 0.87 \frac{f_y A_v}{0.7 b}$$

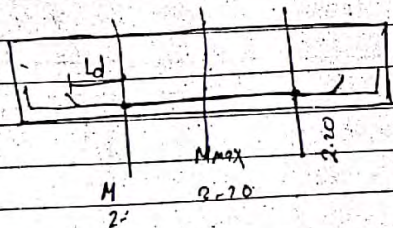
$$= \frac{0.87 \times 415 \times 100.53}{0.7 \times 300}$$

$$= 302.47 \text{ mm}$$

75mm spacing

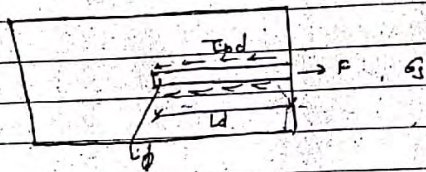
Bond and Development length

When load is applied at the structure, there should be effective transfer of load between used material (such as concrete and steel) in RCC. It is done by ensuring proper bond between concrete and steel. Bond between steel and concrete is due to aggregate inter-locking friction between member and grooves on bars.



Development length

It is a length required to be embedded in concrete so that maximum stress of $0.87f_y$ can be achieved at desired section.

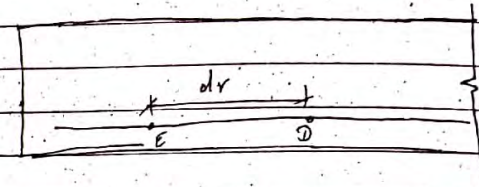


Here, let τ_{bd} be the bond stress between concrete and steel.

Then,

$$\pi \phi * L_d * \tau_{bd} = \frac{\pi}{4} * \phi^2 * \sigma_s$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$



Here, if T_D and T_E are the tension at D & E then,

$$M_D = T_D * Z \quad \text{where, } Z = \text{lever arm at the section}$$

$$M_E = T_E * Z$$

Now,

$$M_D - M_E = (T_D - T_E) * Z$$

$$\frac{dM}{dx} * Z = T_D - T_E$$

$$\frac{dM}{dx} = \pi \phi * dx * \tau_{bd}$$

$$\frac{dM}{dx} * \frac{1}{\pi \phi Z} = \tau_{bd}$$

$$\tau_{bd} = \frac{V}{\pi \phi Z} \quad \text{--- (i)}$$

If there are 'N' no. of bars.

$$T_{bd} = \frac{V}{\pi(N\phi)z}$$

Also,

$$0.87 f_y A_{st} = (\pi * \phi * l_d * T_{bd}) * N$$

$$0.87 f_y A_{st} = \pi * \phi * l_d * \frac{V}{\pi(N\phi)z} * N$$

$$l_d = \frac{(0.87 f_y A_{st}) z}{V}$$

$$\therefore l_d = \frac{M}{V}$$

it is increased by 30% for compression

Hence,

$$l_d = 1.3 \frac{M_c}{V}$$

finally

$$l_d = \frac{1.3 M_c}{V} + l_0$$

where, l_0 = anchorage length beyond mid of support

26.9.3.3 ©

Chapter 5 Design in Torsion

There are basically two types of Torsion on the section.

i) Primary Torsion :- Designed for this torsion at the section.

ii) secondary Torsion: Balanced by continuity of structure

IS-code (cl 41) suggest equivalent shear and bending for given torsion rather than torsion itself.

cl 41.3 suggest equivalent shear due to torsion

$$v_t = 1.6 \left(\frac{T_u}{b} \right), \quad T_u = \text{factored torsion}$$

b = width of section

and, effective shear force $(V_e) = V_{ut} + V_t$

V_{ut} = factored shear force

cl 41.4 suggest equivalent B.M for torsion as

$$M_t = T_u \left(\frac{1 + D/b}{1.7} \right)$$

and effective B.M is given by

$$M_e = M_u \pm M_t$$

where,

$$M_{e1} = M_u + M_t$$

$$\text{and } M_{e2} = M_u - M_t$$

If $M_t < M_u$, equivalent longitudinal reinforcement is provided for M_{e1} in tension side.

If $M_t > M_u$, in addition to tension bar for M_{e1} , additional compression bar should be provided for M_{e2} .

low,

$$V_d = 1.6 \left(\frac{T_u}{b} \right)$$

$$= 1.6 \left(\frac{20}{0.3} \right)$$

$$= 106.66 \text{ kN}$$

$$M_E = T_u \left(\frac{1 + D/b}{1.7} \right)$$

$$= 20 \left(\frac{1 + \frac{0.540}{0.3}}{1.7} \right)$$

$$= \frac{30.94}{32.715} \text{ kN-m}$$

equivalent shear force (V_e) = $V_u + V_d$

$$= 50 + 106.66$$

$$= 156.66 \text{ kN}$$

and,

$$\tau_e = \frac{V_e}{bd}$$

$$= \frac{156.66}{300 \times 500}$$

$$= 1.044 \text{ N/mm}^2$$

Equivalent moment in the given section

$$M_{e1} = M_E + M_u \quad (\because M_u > M_E)$$

$$= 50 + 32.94$$

$$= 82.94 \text{ kN-m}$$

Since, $M_u > M_E$, provide tension bar for equivalent moment

$$M_{e1} = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Here, for Fe 415

$$x_u = x_{u,lim} = 0.48d$$

$$82.94 = 0.87 \times 415 \times A_{st} (500 - 0.42 \times 0.48 \times 500)$$

$$A_{st} = 575.45 \text{ mm}^2$$

Adopt $\phi = 16 \text{ mm}$

$$\text{No. of bars} = \frac{575.45}{201.06} = 2.86 \approx 3 \text{ nos.}$$

Also, provide 2-16mm bar as hanger bar.

$$\frac{A_{st}}{bd} \% = \frac{3 \times 201.06}{300 \times 500} \times 100 = 0.402 \%$$

From table-19

$$\tau_{e1} = \frac{0.48 - 0.36}{0.5 - 0.25} (0.402 - 0.25) + 0.36$$

$$\tau_{e1} = 0.43 \text{ N/mm}^2$$

Since, $\tau_{e1} > \tau_e$

(Let us assume 8mm 2-legged stirrup)

$$A_{sv} = 100.53 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} b_1 d_1}{T_u + 0.4 V_u b_1}$$

Let clear cover $\phi = 30$ mm
 effective cover $(d') = 30 + 8 + 16$
 stirrup / longitudinal bar

Effective depth $(d') = 40$ mm

Clear cover for main bar $= 40 - 16 - 8 = 24$ mm

$b_1 = 300 - 2 \times 40 = 220$ mm

$d_1 = 540 - 2 \times 40 = 460$ mm

$x_1 = 300 - 2 \times 24 = 252$ mm

$y_1 = 540 - 2 \times 24 = 492$ mm

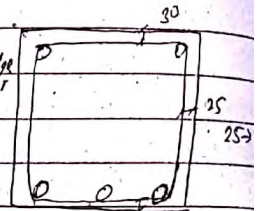
$S_v = \frac{0.87 \times 415 \times 100.53 \times 220 \times 460}{20 + 0.4 \times 50 \times 220}$

Chapter-6 Reinforcement Detailing

Types of cover

1. Clear cover

It is a minimum value of cover provided at all section of member.



2. Nominal cover

It is a minimum value of cover to the edge of rebar for major bar (stirrup)

3. Effective cover

It is a cover provided to the centre of reinforcement.

Detailment of bar (26.2.3 & 26.2.4)

Spllicing of bar (26.2.5)

i) Lapping

ii) Connector

iii) Welding

$l_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$

$= \frac{\phi (0.87 \times 415)}{4 \times (1.2 \times 1.6)} = 47.01 \phi$ (Fe415)

$= \frac{\phi (0.87 \times 500)}{4 \times 1.2 \times 1.6} = 56.64 \phi$

Chapter-7 (cl 35.3)

There are basically two aspects of limit state of serviceability.

- i) Deflection control
- ii) Crack control

Deflection control

Factors affecting deflection

Magnitude of load and their distribution

Span and types of support

Cross-sectional characteristics of structural member

Type of concrete

Stress in steel reinforcement

Amount and extent of cracking

Deflection control is done by two methods:

- i) Theoretical method

$$\delta \leq \delta_{span}$$

- ii) Empirical method (Method of sufficient stiffness) (23.2.1, pg-37)

$$l \leq \alpha \beta \gamma \lambda \delta$$

l = effective length

d = " depth

α = Basic value (Boundary Condition)

β = depends upon length of member

γ = depends upon tensile reinforcement %

λ = depends upon compression "

δ = " " shape of member

- ii) Crack control

Causes of cracking

- i) Due to settlement of plastic concrete
- ii) Due to uneven volumetric change of concrete
- iii) due to external load

- i) Theoretical method (Annex-F)

$$\Delta_{cr} \leq \Delta_{per}$$

Δ_{cr} = calculated crack width

Δ_{per} = permissible crack

Allowable crack as per ACI

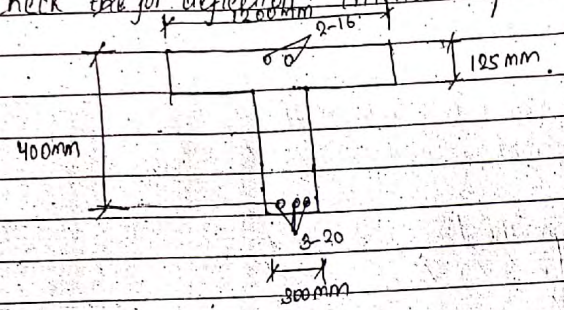
Exposure condition	Permissible crack (mm)
Dry air	0.41
Humid & moist	0.3
Chemically affected	0.18
Sea water	0.15
Water retaining structure	0.1

- ii) Empirical method (Rule of proper Detailing)

Crack controlling is done by proper detailing

slab

Q. Check the deflection criteria of the following beam.



Length of beam = 6m
 $f_{ck} = 20 \text{ N/mm}^2$
 $f_y = 500 \text{ N/mm}^2$ It is a continuous beam.

Here,
 $\frac{l}{d} = \frac{5000}{400} = 12.5$

CL 23.2.1 pg-37
 $\alpha = 26$
 $\beta = 1$

for ϕ
 $A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.47 \text{ mm}^2$

$\frac{A_{st}}{bd} \% = \frac{942.47}{300 \times 450} = 0.00698 \times 100 = 0.698 \%$

$f_t = 0.58 f_y \times \frac{\text{Area required}}{\text{Area provided}} = 0.58 \times 500 \times 1 = 290 \text{ N/mm}^2$

$\therefore \rho = 0.9$ (From fig-4) pg-38

for λ ,
 $A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$

$\frac{A_{sc}}{bd} \times 100 = \frac{402.12}{300 \times 450} \times 100 = 0.298 \%$

fig-5 $\lambda = 1.1$

For δ ,
 $\frac{b_w}{b_f} = \frac{300}{1200} = 0.25$
 $\delta = 0.8$

$\alpha \beta \gamma \lambda \delta = 26 \times 1 \times 0.9 \times 1.1 \times 0.8 = 20.6$
 $\therefore \frac{l}{d} \leq \alpha \beta \gamma \lambda \delta$ OK

Chapter 8

slab

It is a thin sheet/membrane that carries load on its surface and bends on its own plane. It is critical under bending moment and depth of slab is governed by shear criteria.

Types of slab

A) As per section

i) Solid slab

ii) Hollow slab

iii) Ribbed slab

iv) Grid slab

B) As per Boundary condition

i) Simply supported

ii) Continuous

iii) Fixed

c) As per load distribution

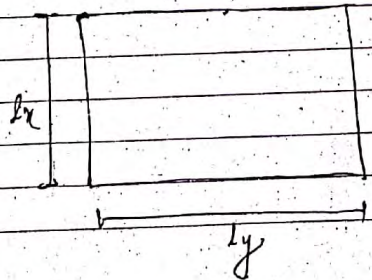
i) One-way slab ($\frac{l_y}{l_x} > 2$)

ii) Two-way slab ($\frac{l_y}{l_x} \leq 2$)

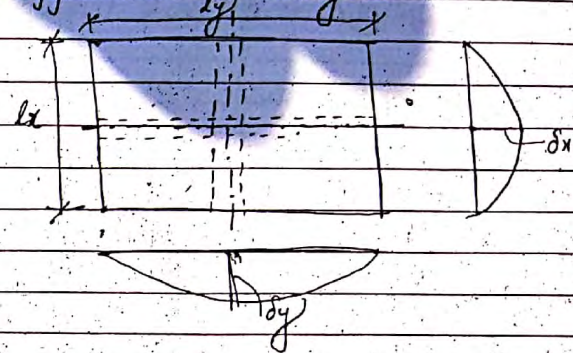
where,

l_y = longer span

l_x = shorter span



Grasshoff Rankine's Theory



Here,

$$\delta_x = \frac{5}{384} \frac{w_x l_x^4}{EI}$$

$$\delta_y = \frac{5}{384} \frac{w_y l_y^4}{EI}$$

Since,

$$\delta_x = \delta_y$$

$$\frac{5}{384} \frac{w_x l_x^4}{EI} = \frac{5}{384} \frac{w_y l_y^4}{EI}$$

$$w_x = w_y \left(\frac{l_y}{l_x} \right)^4 \quad \text{--- (i)}$$

Since, $w = w_x + w_y$ --- (ii)

Using eqⁿ (i) on eqⁿ (ii)

$$w = w_y \left(\frac{l_y}{l_x} \right)^4 + w_y$$

If $r = \frac{l_y}{l_x}$

$$w_y = w \left(\frac{1}{1+r^4} \right) \quad \text{--- (iii)}$$

Also, using eqⁿ (iii) in eqⁿ (ii)
we get,

$$w = w_x + w \left(\frac{1}{1+r^4} \right)$$

$$w \left(\frac{r^4}{1+r^4} \right) = w_x$$

$$w_x = w \left(\frac{r^4}{1+r^4} \right) \quad (iv)$$

Case-I $l_y = l_x, r = 1$
 $w_x = 0.5w$
 $w_y = 0.5w$

Case-II If $l_y = 1.5 l_x$
 $w_x = 0.835 w$
 $w_y = 0.165 w$

Case-III If $l_y = 2 l_x$
 $w_x = 0.941 w$
 $w_y = 0.059 w$

Case-IV: If $l_y = 3 l_x$
 $w_x = 0.988 w$
 $w_y = 0.012 w$

For $l_y < 2 l_x$, load carried by both axis is comparable. Hence, it is designed as shell section and is termed two-way slab.

For $l_y > 2 l_x$, load carried by longer axis is almost negligible. Hence, such slab are designed as one beam and are termed as one way slabs.

Design of one-way slab

Design steps

1. Calculate effective depth
 $\frac{l}{d} = 25$ (simply supported)
 $= 30$ (continuous slab)

$$\text{Overall depth} = d + \phi/2 + \text{clear cover}$$

2. Calculate effective length (cl. 22.2 pg- 34)
3. Calculate total load
4. Calculate maximum bending moment and shear force.
5. Check for depth against bending
6. Calculate area of steel and appropriate spacing
Annex-6, 1-1(b) pg- 96

Check for shear force (Cl-40 (40.2) Table-19)

Check for deflection (Cl-~~23~~ 23.2.1, pg-37)

Check for development length (cl 26.2.1, pg-42
cl 26.2.3.3 @, pg-44)

Provide distribution bar and reinforcement detailing.

Design a RC slab over a room $2.5\text{ m} \times 5.4\text{ m}$ to carry imposed load of 6 kN/m^2 and floor finish of 1 kN/m^2 . Use M20 concrete and torsteel.

$l =$ (always shorter span)
(critical)

Step 1:

$$\frac{l}{d} = 25$$

$$\text{or } d = \frac{l}{25} \\ = \frac{2.5}{25} \\ = 0.1\text{ m}$$

$$d \approx 110\text{ mm}$$

Adopt $\phi = 10\text{ mm}$

clear cover = 15 mm (From Table-16)

$$\text{Overall depth (D)} = 110 + \frac{\phi}{2} + 15 \\ = 130\text{ mm}$$

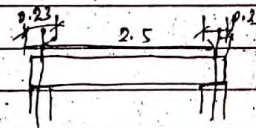
Step 2: Along x-axis

(let support width of 230 mm)

$$\text{i) } c/c = 2.5 + \frac{0.23}{2} + \frac{0.23}{2} = 2.73\text{ m}$$

$$\text{ii) } \text{clear span} + \text{eff. depth} = 2.5 + 0.11 \\ = 2.61\text{ m}$$

$$l_{ex} = 2.61\text{ m}$$



Along y-axis

$$\text{i) } c/c = 0.5 + \frac{0.23}{2} + \frac{0.23}{2} = 0.63\text{ m}$$

$$\text{ii) } \text{Clear span} + \text{eff. depth} = 5.4 + 0.11 \\ = 5.5\text{ m}$$

$$l_{ey} = 5.5\text{ m}$$

$$\frac{l_{ey}}{l_{ex}} = \frac{5.5}{2.61} = 2.11 \geq 2 \quad \text{one-way slab}$$



Step-3: load calculation

i) Imposed load = $5 \times 1 = 5 \text{ kN/m}$

ii) Floor finish = $1 \times 1 = 1 \text{ kN/m}$

iii) Self-wt = $25 \times 1 \times 0.13 = 3.25 \text{ kN/m}$

Total load = 9.25 kN/m

Factored load = $1.5 \times 9.25 = 13.875 \text{ kN/m}$

Step-4:

$$M_{max} = \frac{wl^2}{8} = \frac{13.875 \times (2.61)^2}{8} = 11.815 \text{ kN-m}$$

$$V_{max} = \frac{wl}{2} = \frac{13.875 \times 2.61}{2} = 18.106 \text{ kN}$$

Step-5:-

$$M_{max} = 11.815 \times 10^6 = 0.138 \text{ fck } bd^2 \quad \left(\text{for } 1\text{m } b=1000 \right)$$
$$d_{req} = 65.43 \text{ mm} \leq 110 \text{ mm}, \text{ OK}$$

Step-6: Annex - G, 1.1(b) pg-96

$$M_{max} = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} bd} \right)$$

$$\text{or } 11.82 \times 10^6 = 0.87 \times 415 \times (A_{st}) \times 110 \times \left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 110} \right)$$

$$\therefore A_{st} = 316.5 \text{ mm}^2$$

$$(A_{st})_{min} = 0.12\% \text{ of } bD \quad (\text{Cl. 26.5.2.1 pg-48})$$
$$= \frac{0.12}{100} \times 1000 \times 130$$

$$= 156 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000}{\left(\frac{316.5}{78.53} \right)} = 248.1 \text{ mm} \quad \left(\text{Area of } 10 \text{ mm bar} = 78.53 \right)$$

$\left. \begin{array}{l} \leq 300 \text{ mm} \\ \leq 3 \times d = 3 \times 110 = 330 \text{ mm} \end{array} \right\} 26-6-3$

Provide 10 mm dia bar @ 150 mm c/c.

$$(A_{st})_{provided} = \frac{1000}{150} \times 78.53 = 523.53 \text{ mm}^2 \quad \left(\text{or } 1.0 \times 10^3 \right)$$

Step-7:

$$V_{max} = \frac{wl}{2} = \frac{13.875 \times 2.61}{2} = 18.11 \text{ kN}$$

always use max

$$\tau = \frac{V}{bd} = \frac{18.11 \times 10^3}{1000 \times 110} = 0.164 \text{ N/mm}^2$$

$$\text{Design shear capacity} = kT_c \quad (40.2.1)$$

$$= 1.3 \times 0.28$$

$$= 0.36 \text{ N/mm}^2 > 0.164 \text{ N/mm}^2 (\text{OK})$$

Step 8:

$$\frac{l}{d} = \frac{2.61}{0.11} = 23.73$$

From cl 23.2.1 pg-37

$$\alpha = 20$$

$$\beta = 1$$

$$\lambda = 1$$

$$\delta = 1$$

For 8:

$$\frac{A_{st}}{bD} \% = \frac{523.53}{1000 \times 130} \times 100 \%$$

$$= 0.403 \%$$

$$f_s = 0.58 \times f_y \times \frac{(A_{st})_{\text{required}}}{(A_{st})_{\text{provided}}} = 0.58 \times 415 \times \frac{316.5}{523.53}$$

$$= 145.52 \text{ N/mm}^2$$

From fig-4

$$\gamma = 2$$

$$\alpha \beta \gamma \lambda \delta = 20 \times 1 \times 2 \times 1 \times 1 = 40$$

$$\therefore \frac{l}{d} \leq \alpha \beta \gamma \lambda \delta, \text{ OK}$$

Step 9: cl 26.2.1 pg-42

$$L_d = \frac{\phi \sigma_s}{4 T_{bd}}$$

$$= \frac{10 \times (0.87 \times 415)}{4 \times (1.2 \times 1.6)}$$

$$= 470.11 \text{ mm}$$

cl 26.2.3.3 (c)

$$M_2 = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{cr} b d} \right)$$

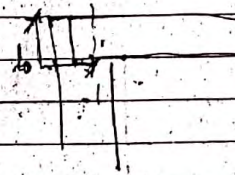
$$= 0.87 \times 415 \times 523.53 \times 110 \times \left(1 - \frac{415 \times 523.53}{20 \times 1000 \times 110} \right)$$

$$= 9.88 \text{ kN-m}$$

$$1.3 \frac{M_2}{V} = \frac{1.3 \times 9.88 \times 10^6}{18.11 \times 10^3} = 709.22 \text{ mm}$$

let $l_0 = 0$ (assume)

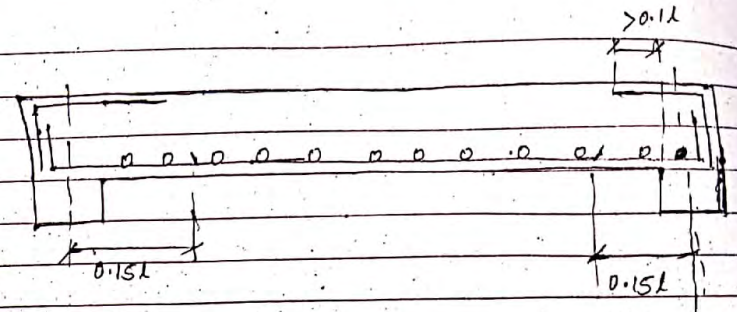
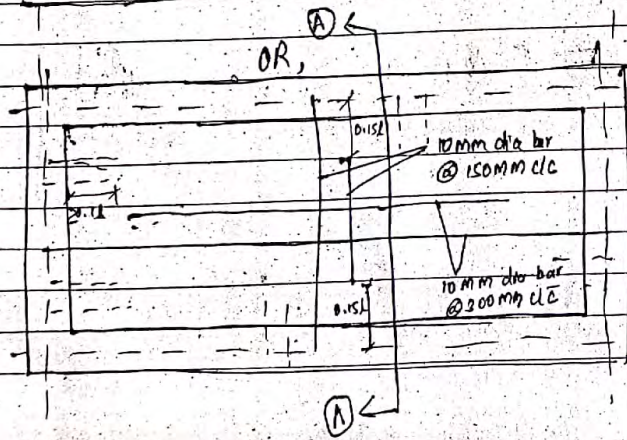
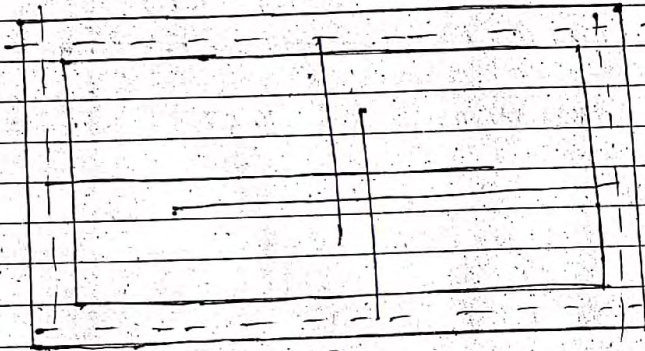
$$L_d \leq 1.3 \frac{M_2}{V} + l_0 \quad \text{OK}$$



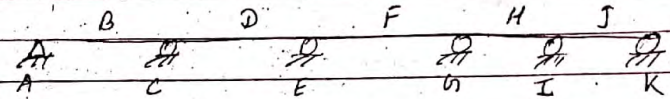
Step 10: Provide minimum (A_{st}) for distribution bar.

Spacing = $\frac{1000}{\left(\frac{156}{78.53}\right)} = 603.4 \text{ mm}$ shall not be more than 450mm or 5 times d
(Cl. 26.3.3 b-2)

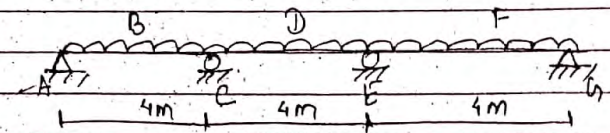
Provide 10 mm dia bar @ 300mm c/c along y-axis.



Continuous slab



Q. Design an RC slab for a continuous span of 4m as shown in fig below. It carries ^{imposed load} UDL of 3 kN/m^2 and floor finish of 1 kN/m^2 . Use M20 concrete and Torsteel.



Soln,

$$\text{Step 1: } \frac{l}{d} = 25$$
$$d = \frac{4}{25} = 0.16 \text{ m}$$

Adopt $\phi = 10 \text{ mm}$
Clear cover = 15 mm
 $D = 160 + \frac{10}{2} + 15 = 180 \text{ mm}$

Step 2: effective length (l_{eff}) = 4m (use 22.2)

Step 3: load calculation

- i) Imposed load = $3 \times 1 \text{ kN/m}$
- ii) Floor finish = $1 \times 1 = 1 \text{ kN/m}$
- iii) Self-wt = $25 \times 0.18 \times 1 = 4.5 \text{ kN}$

Total load = 8.5 kN/m
Factored load = $1.5 \times 8.5 = 12.75 \text{ kN/m}$

Step 4: $M_D = M_F = \frac{1}{12} \times (w \times l) \times l$ (Table-12)

$$= \frac{1}{12} \times (12.75 \times 4) \times 4$$
$$= 1.7 \text{ kN-m}$$

$$M_D = \frac{1}{16} \times (w \times l) \times l$$
$$= \frac{1}{16} \times (12.75 \times 4) \times 4 = 12.75 \text{ kN-m}$$

$$M_C = M_E = -\frac{1}{10} \times (w \times l) \times l$$
$$= -\frac{1}{10} \times (12.75 \times 4) \times 4$$
$$= -20.4 \text{ kN-m}$$

Shear force

$$V_{\text{max}} = 0.6 \times (w \times l)$$
$$= 0.6 \times (12.75 \times 4)$$
$$= 30.6 \text{ kN}$$

Step 5: (for -ve rebar)

$$M_{\text{max}} = 0.138 f_{\text{ck}} b d^2$$
$$20.4 \times 10^6 = 0.138 \times 20 \times 1000 \times d_{\text{req}}^2$$
$$d_{\text{req}} = 85.97 \text{ mm} \leq 160 \text{ mm} \quad \text{OK}$$

Step 6: for +ve rebar (tension bar)

$$M_{\text{max}} = 0.87 f_y A_{\text{st}} d \times \left(1 - \frac{f_y A_{\text{st}}}{f_{\text{ck}} b d}\right)$$
$$17 \times 10^6 = 0.87 \times 415 \times A_{\text{st}} \times 160 \left(1 - \frac{415 \times A_{\text{st}}}{20 \times 1000 \times 160}\right)$$
$$A_{\text{st}} = 306.46 \text{ mm}^2$$

For -ve bar

$$20.4 \times 10^6 = 0.87 \times 415 \times A_{st} \times 160 \times \left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 180}\right)$$

$$(A_{st})^- = 370.99 \text{ mm}^2$$

$$\begin{aligned} (A_{st})_{\min} &= 0.12 \% \text{ of } bD \\ &= \frac{0.12}{100} \times 1000 \times 180 \\ &= 216 \text{ mm}^2 \end{aligned}$$

$$(\text{Spacing})^+ = \frac{1000}{\frac{306.46}{78.53}} = 256.25 \text{ mm}$$

$$(\text{Spacing})^- = \frac{1000}{\frac{370.99}{78.53}} = 211.68 \text{ mm}$$

Provide 10mm dia bar @ 125 mm c/c at support
Provide " " @ 150 " " mid span

$$(A_{st})^{\text{provided}} = \frac{1000}{150} \times 78.53 = 523.53 \text{ mm}^2$$

Step 7:

$$V_{\max} = 30.6 \text{ kN}$$

$$\begin{aligned} \tau_{\max} &= \frac{V_{\max}}{bd} \\ &= \frac{30.6 \times 10^3}{1000 \times 160} \\ &= 0.19 \text{ N/mm}^2 \leq 0.28 \text{ N/mm}^2 \text{ (OK)} \end{aligned}$$

Step 8:

$$\frac{l}{d} = \frac{4000}{160} = 25$$

Step 9:

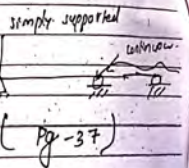
$$\alpha = \frac{20+26}{2}$$

$$\begin{aligned} \beta &= 1 \\ \lambda &= 1 \\ \delta &= 1 \end{aligned}$$

for χ

$$\frac{A_{st}}{bD} \chi = \frac{523.53}{1000 \times 180} \times 100 = 0.291 \%$$

$$f_c = 0.58 \times f_y \times \frac{(A_{st})_{\text{req}}}{(A_{st})_{\text{provided}}} = 0.58 \times 415 \times \frac{370.99}{523.53} = 140.87 \text{ N/mm}^2$$



From fig: 4
 $\gamma = 2$

$$\alpha_{BY\lambda\delta} = 23 \times 1 \times 2 \times 1 \times 1$$
$$= 46$$

$$\frac{l}{d} \leq \alpha_{BY\lambda\delta} \quad (\text{OK})$$

Step 9:

$$l_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{10 \times (0.87 \times 415)}{4 \times (1.2 \times 1.6)}$$
$$= 470.11 \text{ mm}$$

$$M_l = 0.87 f_y A_{st} \times d \times \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right)$$

$$= 0.87 \times 415 \times \left(\frac{523.53}{2} \right) \times 160 \times \left(1 - \frac{415 \times 523.53}{20 \times 1000 \times 160} \right)$$
$$= 14.61$$
$$= 10.04 \text{ KN-m}$$

$$\frac{1.3 M_l}{V} = \frac{1.3 \times 14.61 \times 10^5}{30.6 \times 10^3}$$
$$= 620.7 \text{ mm}$$

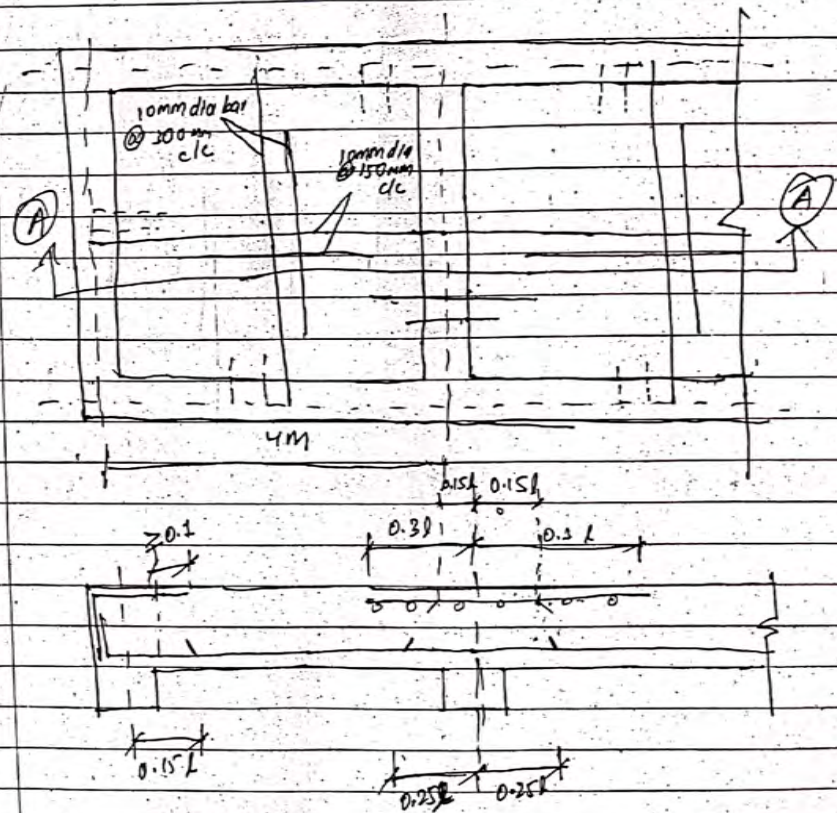
Assume, $\lambda_0 = 0$

$$l_d \leq \frac{1.3 M_l}{V} + \lambda_0 \quad (\text{OK})$$

Step 10 for distribution bar

$$\text{Spacing} = \frac{1000}{\frac{216}{78.52}} = 363.56 \text{ mm}$$

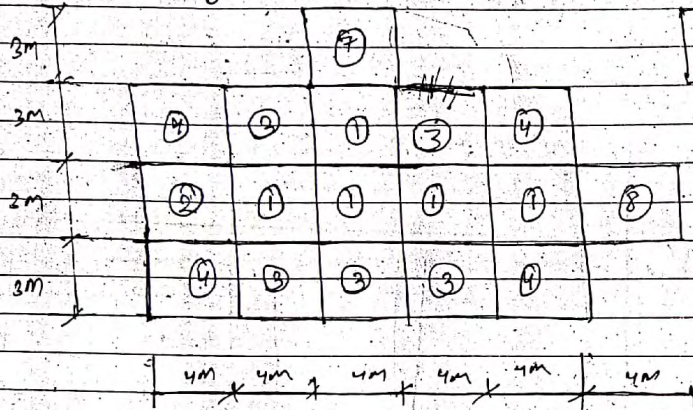
Provide 10mm dia bar @ 300mm as distribution bar



Two-way slab

If $\frac{l_y}{l_x} \leq 2$, it is known as two-way slabs.

Type of two-way slab



$$M_x = \alpha_x w l_x^2$$
$$M_y = \alpha_y w l_y^2$$

Design steps

1. Calculate eff depth
 $\frac{l}{d} = 28$ (simply supported)
 $\frac{l}{d} = 32$ (continuous)
2. Calculate eff length
(cl. 22.2, pg. 34)
3. Calculate total load
4. Calculate M_{max}
 $M_x = \alpha_x w l_x^2$
 $M_y = \alpha_y w l_y^2$
5. Check for depth against bending
6. Calculate area of steel
7. Check for shear force
8. Check for deflection
9. Check for development length
10. Provide torsion bar at discontinuous corner
11. Reinforcement detailing



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