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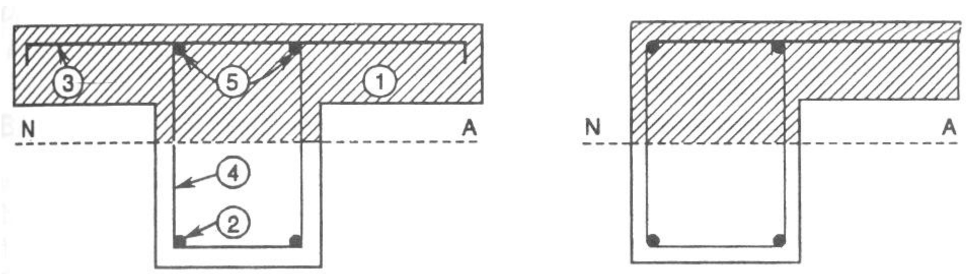
UNIT II

LIMIT DESIGN FOR FLEXURE

DESIGN OF FLANGED BEAMS

INTRODUCTION

When there is a reinforced concrete slab, over a reinforced concrete beam the slab and bean, can be designed and constructed in such a way that they act together. The concrete in the slabs, which is on the compression side of the beam (in the middle portions of continuous beams), can be made to resist the compression forces, and the tension can be carried by the steel in the tension side of the beam. These combined beam and slab units are called flanged beams. They may be T or L beams, depending on whether the slab is on both or only on one side of the beam .



Flanged beams: (a) T beam—1–Compression in concrete; 2–Tension steel; 3–Transverse steel; 4–Stirrups for shear; 5–Anchorage of stirrups; (b) L beam.

One should be aware that continuous T or L beams act as flanged beams only between the supports where the bending moments are conventionally taken as positive and the slabs are on the compression side of the beam. Over the supports, where the bending moments are negative, the slabs are on the tension side and here the beam acts only as a rectangular beam, with the tension steel placed in the slab portion of the beam. Thus at places of negative moments these beams have to be designed as singly or doubly reinforced rectangular beams as shown in figure.

In order to make the slab and beam act together, transverse steel should be placed at the top of the slab with sufficient cover for the full effective width of slab. This steel is also useful to resist the shear stresses produced by the variation of compressive stress across the width of the slab.

The four main components that constitute these flanged beams (in addition to the hanger bars) for which strength calculations are necessary:

- **The compression flange**
- **Tension steel**
- **Transverse steel in the slab for integral action**
- **Stirrups for shear.**

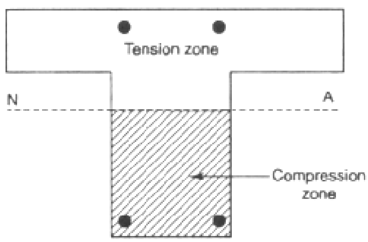


Fig. 8.2 Flanged beams over supports with negative moments.

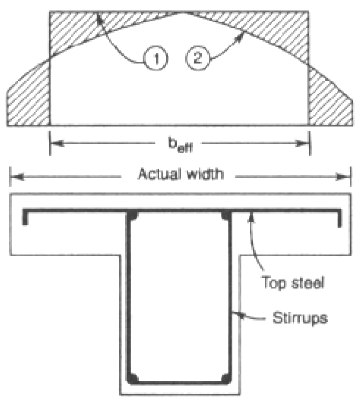


Fig. 8.3 Effective width of T beams: 1. Actual stress distribution in compression flange; 2. Assumed stress distribution in compression flange.

EFFECTIVE FLANGE WIDTH

An equivalent width of the slab with uniform stress distribution that can be assumed to act along with the beam for strength calculation is called the effective width of the flanged beam. The compressive stress in the flange just above the rib is higher than that at some distance away from the rib as in figure.

The nature of this variation is very indeterminate, and the effective width concept that enables the use of an imaginary width of beam over which a uniform compressive stress is assumed to act is very useful.

ACTION OF TWO-WAY SLABS

- When a slab simply supported on all the four sides is loaded, the corners tend to curl and lift up. This is to compensate for the non-uniform distribution of pressure exerted by the slab on the supporting walls.
- This behavior can be easily demonstrated by supporting a rigid cardboard with *Ly* less than *2Lx* on its four sides and pressing it down wards .

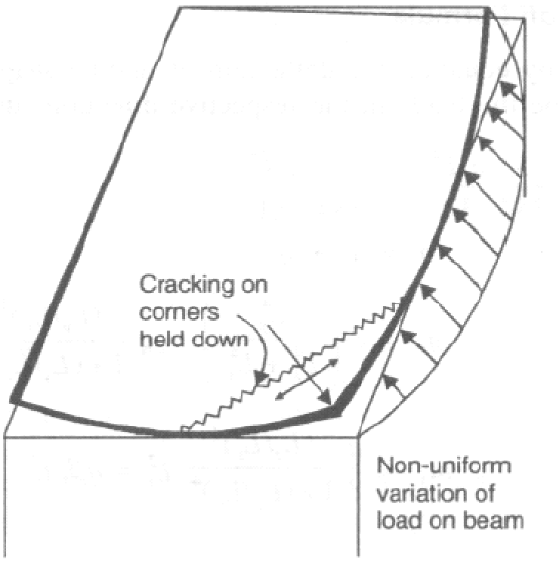


Fig. 12.2 Action of two-way slabs.

- In practice, the upward movement described above may be prevented by a wall or such other construction at the edge of the slab.
- In such circumstances it is said that the corner is 'torsionally restrained'. Unless top steel is provided for such slabs, these will crack at the corners.
- These slabs also tend to carry loads by spanning diagonally across the corners. Hence, in addition to the top steel along the diagonal, bottom steel is also needed at the corners.
    - Even though for both these purposes (Le. the curling effect and diagonal transfer of load at corners) steel is needed diagonally, it is more convenient to provide two-way steel in the *x* and *y* directions at the top and bottom surfaces of the slab. The steel is known as ***corner steel.***

MOMENTSIN TWO-WAY SLABS SIMPLY SUPPORTED ON ALL SUPPORTS

For computation of moments in simply supported cases of two-way slabs, Table 27 in Annexure D of IS 456 can be used. These are derived from the Rankine-Grashoff formula.

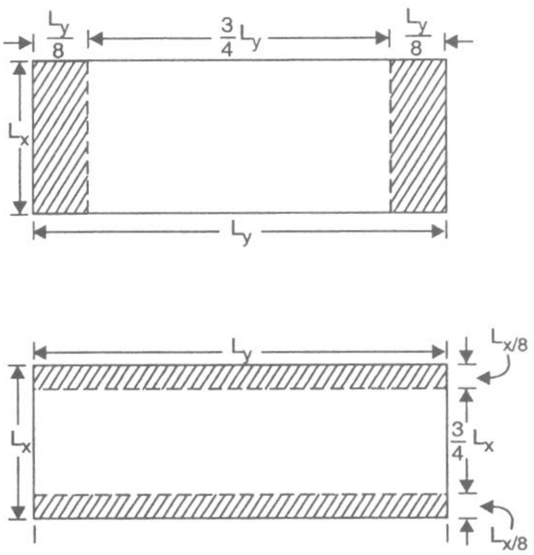


Fig. 12.4 Edge and middle strips of two-way slabs.

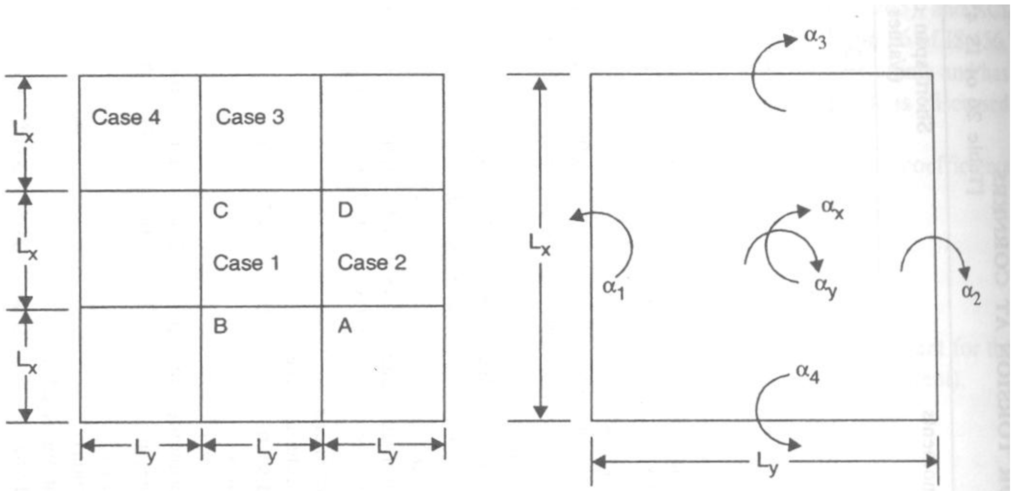


Fig. 12.5 Coefficients for moments in one-way slabs.

2. Determination of a α_x

Having determined a the value of a can also be determined from the relations

$$r = \frac{2}{9}\left[3 - \sqrt{18}\frac{L_x}{L_y}\left(\sqrt{\alpha_y + \alpha_1} + \sqrt{\alpha_y + \alpha_2}\right)\right]$$

$$\sqrt{r} = \sqrt{\alpha_x + \alpha_3} + \sqrt{\alpha_x + \alpha_4}$$

The values of α_3 and α_4 are obtained from the assumptions

$\alpha_3, \alpha_4 = 0$ for discontinuous edges

$\alpha_3, \alpha_4 = 4/3\ \alpha_x$ for continuous edges

The solution of these equations can be further simplified by putting

$\alpha_1 = K_1\alpha_y$, $\alpha_2 = K_2\alpha_y$, $\alpha_3 = K_3\alpha_x$, $\alpha_4 = K_4\alpha_x$

Substituting these values, Eq. (12.4) becomes

$$r = \frac{2}{9}\left[3 - \sqrt{18}\frac{L_x}{L_y}\left(\sqrt{\alpha_y + K_1\alpha_y} + \sqrt{\alpha_y + K_2\alpha_y}\right)\right]$$

Similarly, squaring Eq. (12.5), we obtain

$$r = \alpha_x\left(\sqrt{1 + K_3} + \sqrt{1 + K_4}\right)^2$$

Equating the two and solving for α_x, we get

$$\alpha_x = \frac{r}{\left(\sqrt{1 + K_3} + \sqrt{1 + K_4}\right)^2}$$

- **NEGATIVE MOMENTS AT DISCONTINUOUS EDGES**

Negative moments may be experienced at discontinuous edges since, in practice, they are not supported on rollers but partially restrained at their ends. The magnitude of this moment depends on the degree of fixity at the edge of the slab and is indeterminate.

The usual practice is to provide at these edges top reinforcement for negative moment equal to (0.042 w1 i.e. w However, IS 456, Clause D 1.6 of Annexure D recommends provision of 50 per cent of steel provided at mid span along these edges, and this negative steel has to extend into the span 0.1 times the span length, as indicated in Figure.

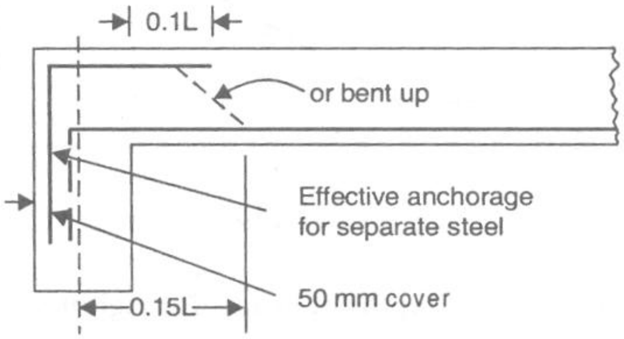


Fig. 12.6 Details of steel at discontinuous edge.

- **CHOOSING SLAB THICKNESS**

The following three equally important conditions regarding the minimum thickness should be satisfied by the slab.

Condition 1: The minimum should be such that it is safe in compression. Thus the depth is calculated for the greater value of the negative moment on the short span denoted M Hence

$$M_u = K f_{ck} b d^2$$

$$d = \sqrt{\frac{M_u}{K f_{ck} b}}$$

Total thickness = d (short) + 0.5φ + cover. The steel for the short span is placed at the lowest layer so that for the long strip the total thickness = d (long) + φ + φ + cover. An average of (d + φ) may also be used for practical design purposes.

Condition 2: The slab should satisfy the span/effective depth ratio to control deflection. For this purpose the short span is considered in the calculation of L/d ratios. The value of span/effective depth ratios of 28 for simply supported slabs and 32 for continuous slabs may be adopted for initial trials.

Condition .3: The slab should be safe in shear without shear reinforcements as in the case of one-way slabs.

- **SELECTING DEPTH AND BREADTH OF SUPPORTING BEAMS**

The depth of beams used for supporting the slab should be sufficient to justify the assumptions of unyielding supports. The empirical relation used in Swedish Regulations between depth of beams and depth of slabs as given in figure may be used for arriving at the preliminary depth of these beams. It can be seen that the depth of beam necessary from this consideration lies between 2.5 to 5.0 times the depth of slab, depending on the ratio of Ly to Lx. It can be represented by the equation

$$\frac{D_b}{D_s} = 1.67\frac{L_y}{L_x} \quad \text{but} \not< 2.5$$

where

D_b = depth of the beam

D_s = depth of the slab

Again the breadth b of the beam is to be selected to ensure adequate torsional rigidity. An empirical relationship giving breadth as a function of the length of short span can be derived as $b = 3.24\,(L_x)^{1/3}$

Having selected these preliminary values for depth and breadth, they can be later checked by other methods for deflection control and torsional strength.

- **CALCULATION OF AREAS OF STEEL**

The depth of slab selected from deflection criterion will be generally greater than the minimum required from strength considerations. The areas of steel are calculated on the assumption that the short span steel will be placed below the long span steel.

At corners like C where the slab is discontinuous only on one side, half of the above area is to be provided as corner steel in each of the layers. Corner steel need not be provided in corners such as C which is continuous on all sides.

- **LOADS ON SUPPORTING BEAMS**

According to IS 456, Clause 24.5, the total loads that act on the support beams for two-way slabs may be assumed as the load within the respective area of the slab bounded by the intersection of 450 line from the corners with the median line of the panel parallel to the long side. As is well known from yield line theory of the slabs, this is a good approximation if all the sides are similarly supported, either as discontinuous or as continuous.

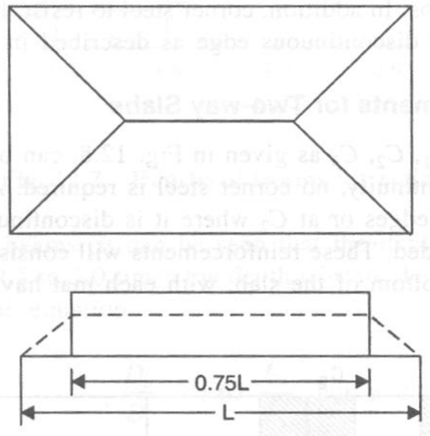


Fig. 12.9 Transfer of load from slab to beam.

The total load so obtained can be converted to an equivalent distributed load for design of these beams. As the intensity of the loads on the beam is non-uniform along its length, being higher at the central portion than at the ends some codes like BS 8110 make a further assumption and calculate the equivalent load as if it is acting only on 3/4 of the length Lx and Ly.

- **PROCEDURE FOR SAFETY AGAINST EXCESSIVE DEFLECTION**

As already explained, deflection of slabs is controlled by span/effective depth ratio similar to the case of beams and one-way slabs. For two-way slabs, the shorter span and the percentage of steel in that direction have to be considered for this purpose. The correction factors to be used in basic span depth ratios for slabs for deflection control have already been explained. Methods for computation of deflection of slabs are generally used only under special circumstances.

- **PROCEDURE FOR DESIGN OF TWO-WAY SIMPLY-SUPPORTED SLABS**

Step 1: Assume a slab thickness with proper cover to steel. This is to be based on (spanl effective depth) ratio of shorter span.

The minimum practical depth of slab is 90 to 100 mm. As the percentage of steel in slabs is low compared to that in beams, a larger value of L/d ratio than that of beams will be found to be acceptable for slabs. (The correction factor is larger in slabs due to low percentage of steel used in them.)

Usually, the following span/effective depth ratios may be assumed for preliminary design of two-way slabs (15 456, Clause 24.1):

- **Simply supported slabs = 28**
- **Continuous slabs = 32**

Assume suitable concrete cover of at least 15 mm depending on the environmental conditions.

STEP 2: Calculate the design load and the value of dead load and live load for the slab:

$$w=(l.5LL+1.5DL)$$

Step 3: Calculate design moment. Determine l_y/l_x where l is the shorter span, find the moment coefficients from Table 23 of IS 456, and calculate the moments M_x and M_y.

Step 4: Calculate maximum shear and check shear stress in the slab. For calculating the shear, assume shear based on load distribution of the beams or use Table 12.3. For refined analysis, critical section may be assumed .

Step5: Calculate steel required in both directions. Check the value of M_x/bd^2 It should not be greater than that allowed for compression failure in concrete. (d is the centre of steel in the x-direction.) Calculate the area of reinforcement required. Choose diameter and find the spacing. Check for maximum spacing allowed: 3d or 450 mm. For slabs less than 300 mm, limit spacing to 200 mm. These steps should be carried out for both directions.

Step 6: Check for deflection. The span depth ratio for deflection is based on l_x and $A_{s1.}$

Step 7: Check for cracking minimum steel in both directions and spacing.

Step 8: Detail the steel preferably as given in simplified detailing of reinforcement in slabs.

- **PROCEDURE FOR DESIGN OF TWO-WAY RESTRAINED SLABS (WITH TORSION AT CORNERS)**

Step 1: Assume a slab thickness with proper cover to steel.

Step 2: Calculate the design load from dead and live loads:

$$w = (1.5\ DL + 1.5\ LL)$$

Step 3: Draw the slab pattern of each slab in plan and show which case in Table 22 of JS 456 each of the slabs refers to. Find Ly/Lx for the slab.

Step 4: Write down the shear force coefficients on the assumptions based on distribution of slab loads to supporting beams. Make approximate check for shear.

Step 4(a): Determine the maximum moment for the middle strips in the long and short directions. Make adjustment for the negative moments at continuous edges of common spans if necessary.

(b): Take each panel, check depth for bending moment. ensure that $MI(bd^2)$ is not greater than that allowed for both Ly and Lx, directions, and calculate the steel required in each direction at mid-span and supports of the middle strip. One may assume the average effective depth for both the directions for calculation of steel area.

d = (h — cover — diameter of reinforcement)

Find the required spacings of steel.

(c): Calculate the nominal steel for the edge strips.

(d): Identify the corners to be provided with corner steel and calculate the corner steel required. It is equal to 0.75 times the area of the maximum positive steel and is to be provided over 0.2L the width at the corners.

(e): If the shear check in step 4 is critical, make a final check for shear, taking the value of design shear strength as that allowed for the area of steel provided at the edges and using the enhancement factor.

Step 5: Check the deflection (the span/depth ratio in short direction with the corresponding percentage of steel).

Step 6: Check for cracking (the minimum steel in both directions). Check the rule for spacing of steel.

Step 7: Detail the main steels (edge strip steels and corner steels) preferably according to standard practice.

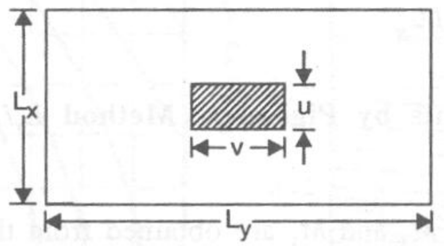
- **CONCENTRATED LOAD ON TWO-WAY SLABS**

As slabs are two-dimensional structures, concentrated load produces saucer-shaped deformation. It is difficult to analyze this deformation. Hence an equivalent plane structure analysis is used, which will always be approximate. The bending moment and shear force due to concentrated loads on one-way slabs are analyzed by the equivalent width method:

- **METHODS BASED ON THEORY OF PLATES FOR CONCENTRATED LOADS ON TWO-WAY SLABS (PIGEAUD METHOD)**

The two well-known methods based on theory of elasticity for determination of bending moments in slabs due to concentrated loads, e.g. wheel loads, are **Pigeaud's** and **Westergard's** methods. Of these, the former method is more popular than the latter and is commonly used for design of bridge slabs.

However, it should be remembered that the standard design curves available for **Pigeaud's** method are for loads placed at the centre of two-way simply supported slabs as shown in Figure, and for other cases of loading and support conditions of values from the curves have to be suitably modified.



Position of concentrated load for Pigeaud's curves.

DESIGN OF BENDING MEMBERS FOR SERVICEABILITY REQUIREMENTS OF DEFLECTION AND CRACKING

INTRODUCTION

In addition to the two limit state condition (durability on exposure to the environments and ultimate strength at overloads), reinforced concrete structures must also satisfy the serviceability conditions under the action of the dead and live toads that act normally on the structure.

Two of the important serviceability conditions are:

- ✓ 1.The member should not undergo excessive deformation (i.e. limit state of deflection
- ✓ The crack width at the surface in the reinforced concrete member should not be mocetjttt that which is norma allowed by codes of practice (i.e. the limit state of cracking).

Even though other limit states like limit state of vibration can be specified and are applicable to special structures such as bridges, The two conditions above are generally accepted as very important conditions to be satisfied by every structure under service loads. Codes also specify the partial safety factors for load combinations under which these are to be checked.

According to IS 456, Table 18 the combinations of loads for serviceability conditions should be the largest of the following:

- 1.0DL+1.0LL
- l.0DL+1.0WL
- 1.0DL+0.8LL+0.8WL(EL)

For control of defleceion, two methods are usually described in codes of practices:

- The empirical method of keeping the span-effective depth ratios of the members not more than those specified in the codes.
- The theoretical method of calculating the actual deflection and checking it with the allowable deflection in codes of practice.

Similarly, for control of crack width two methods are recommended:

i. The empirical method of detailing the reinforcements accord to the provisions of the code regarding spacing of bars, minimum steel ratios, curtailment and anchorage of bars, lapping of bars, etc.

ii. The theoretical method of calculating the actual width of cracks and checking whether they satisfy the requirements in the codes for the given environmental conditions.

Greater attention to deflection and cracking of concrete structures has to be given with the aid of modern methods of R.C. design structures, as these methods allow higher stresses than the conventional method, both in concrete and steel.

During the past few decades, the maximum allowable stresses have nearly been doubled for steel and increased considerably for concrete. Thus, whereas most of the steel used in older R.C.C. members were only of grade Fe 250, in modem construction, steel of grades of Fe 415 and Fe 500 are very commonly used. This necessitates better control of deflection and cracking conditions.

DESIGN FOR LIMIT STATE OF DEFLECTION

Excessive deflection of beams and slabs is not only an eyesore in itself, but it can also cause cracking of partitions. As given in 18 456, Clause 23.2, the commonly accepted limits of allowable deflections are:

- ❖ A final deflection of **span/250** for the deflection of horizontal bending members like slabs and beams due to all loads so as not to be noticed by the eye and thus is not an eyesore.
- ❖ A deflection of **span/350 or 20 mm**, whichever is less, for these members, after the construction of the partitions and finishes etc., to prevent damage to finishes and partitions.

Even though methods for estimating deflection by calculation, are available the empirical method to limit deflection are enough for routine design of slabs and beams.

EMPIRICAL METHOD OF DEFLECTION CONTROL IN BEAMS

One can roughly express the allowable deflection/span ratio of beams in terms of length/depth ratio as can be shown by the following derivation.

Let the deflection of a simply supported beam under UDL be expressed by the formula

$$a = \frac{5}{384}\frac{wL^4}{EI} = \left(\frac{5}{48}\right)\left(\frac{wL^2}{8}\right)\left(\frac{L^2}{EI}\right) = \frac{5}{48}\left(\frac{M_{max}}{I}\right)\frac{L^2}{E}$$

Putting $M/I = f/y$ and $y = d/2$, we get

$$\frac{a}{L} = \frac{5}{48}\left(\frac{2f}{d}\right)\left(\frac{L}{E}\right) = \frac{5}{24}\left(\frac{f}{E}\right)\left(\frac{L}{d}\right)$$

Thus deflection/span is a function of L/d, taking f and E as constant values.

If one were to assume allowable values of

$$f = 5\ \text{N/mm}^2, \quad E = 10\ \text{kN/mm}^2, \quad \frac{a}{L} = \frac{1}{350}$$

hen the span/depth ratio can be obtained as

$$\frac{L}{d} = \frac{1\times 24\,(10\times 10^3)}{350\times 5\times 5} = 27$$

This means that assuming certain terms as constants, the allowable deflection/span ratio can be controlled by the span/depth ratio. This principle is used for specifying the span/depth ratio for control of deflection in beams and slabs.

PROCEDURE FOR CHECKING DEFLECTION

Step 1: Depending on condition of supports, choose the basic span/effective depth ratio from Table 9.1 if the span is 10 m or less. If it is greater than 10 m, reduce the values as indicated in figure.

Step 2: Determine modification factor F which depends on the type of steel used (corresponding to the service stress in steel) and the percentage of steel required in the beam at the point of maximum deflection.

The modification factor F is to be obtained from IS 456 . This figure gives F factor for different stress levels and percentages of tension steel. The value of the stress in steel at working stress (for which deflection is to be determined) is assumed to be 0.58 of the yield stress of steel.

Thus the steel stress which will depend on the actual area of steel provided at the section is to be calculated from the following formula:

$f = 0.58f_y$ [A_s (required) ÷ A_s (provided)]

This equation is very useful for determining the value of F more accurately than the one given in Fig. 4 of IS or when computer procedures are used for checking the deflection. The service stress L is obtained more accurately from the equation

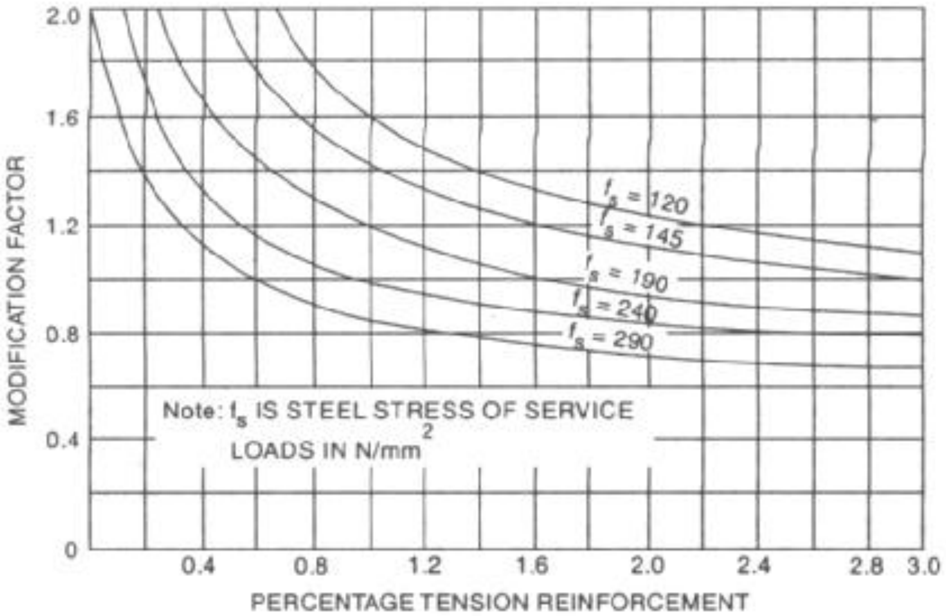
$$f_s = 0.58 f_y \frac{A_s\text{ (required)}}{A_s\text{ (provided)}} \frac{1}{\beta}$$

where

β = moment at section after redistribution

Note: The equation corresponding to BS 8110 (1985) is much simpler and is given by

$$F_1 = 0.55 + \frac{477 - f_s}{120\left(0.9 + \dfrac{M}{bd^2}\right)} \leq 2.0$$



$$f_s = 0.58\, f_y \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

Fig. 9.2 Modification factor for tension reinforcement (IS 456 Fig. 4)

$$F_1 = \frac{1}{(0.225 + 0.00322 f_s + 0.625 \log_{10} p_t)} \leq 2.0$$

where

$$p_t = \frac{100 A_s}{bd}$$

F is

s is

made by the special factor of T beams, viz. F as given in step 4.

It should be remembered that with higher grades of steel used (i.e. higher service stresses or with larger theoretical percentage of steel needed for the beam) the value of the multiplying factor F becomes smaller, i.e. the necessary depth for the same span increases.

Step 3: Determine the modification factor F corresponding to the percentage of compression reinforcement provided at the point of maximum moment. The larger the percentage of compression reinforcement, the larger will be the factor F For T beams the width to be considered is the effective flange width.

The compression steel can include all bars in the compression zone. It may be noted that increasing the percentage of compression steel is the best method to control deflection in critical cases as it can be done without decreasing the strain in tension steel required by limit state design.

The corresponding expression in 85 8110 (1985) is

$$F_2 = 1 + \left(\frac{p_c}{3 + p_c}\right) \leq 1.5$$

Step 4: As the factors F and F for flanged beams are calculated with the effective flange width (bf), a reduction factor F should be used to allow for the reduced area of concrete in the tension zone. In normal rectangular beams the concrete in the tension zone also contributes to the stiffness of the member.

The reduction factor depends on the ratio of web width (bw) to effective flange width (bf) as follows:

- For b_w/b_f = 0.3 and below, the value of F_3 = 0.8
- For b_f/b_w= 1.0, the value ofF_3 = 1.0
- For intermediate values, the value of F3 is obtained by linear interpolation

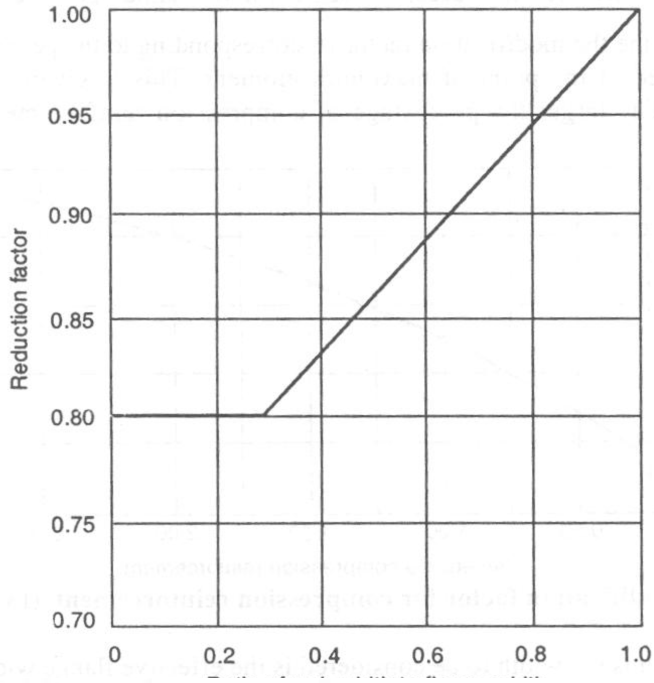


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Fig. 6 Fig. 6)

$$F_3 = 0.8 + \frac{2}{7}\left(\frac{b_w}{b_f} - 0.3\right) \not< 0.8$$

Step 5: The final span depth ratio allowed is

$$(\text{Basic ratio})\ (F_1)\ (F_2)\ (F_3).$$

Deflection Control in Slabs

At present, there are no accurate methods for estimating deflection of slabs. The empirical procedure recommended for control of deflection for slabs is the same as in beams, i.e. to limit the span/depth ratio as indicated. The same modifying factors as given above are used. In two- way slabs supported on all four sides, the shorter span of the two-way slab is taken for calculation with the amount of reinforcements in that direction at the centre of this span taken as the percentage of tension reinforcement.

THEORY OF SINGLY REINFORCED MEMBERS IN BENDING (LIMIT STATE OF COLLAPSE — FLEXURE)

INTRODUCTION

Beams and slabs carry loads by bending action. In the limit state method, these members are first designed for strength and durability, and their performance is then checked with regard to other limit states of serviceability, e.g. deflection and cracking. Many of these formulae are given in IS 456 Annexure .

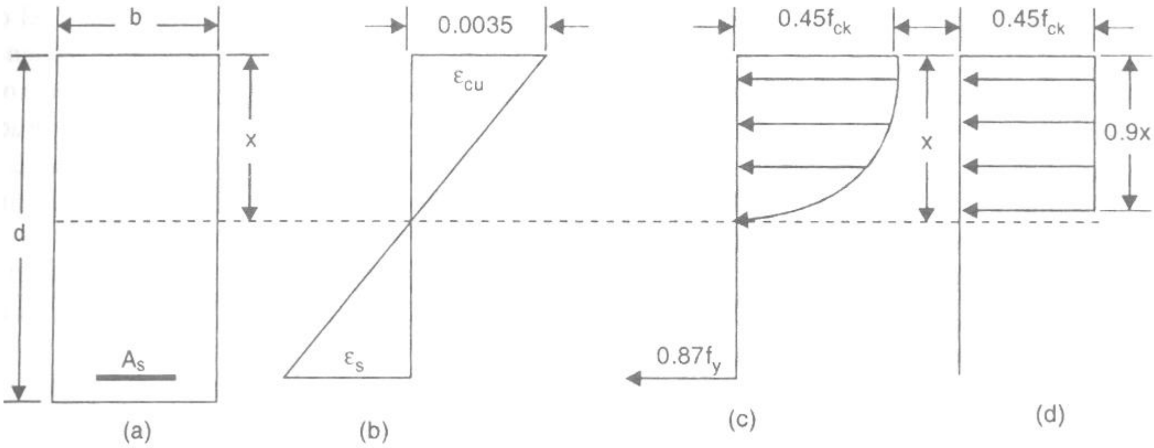
Even though it may not be necessary for a designer to know the derivation of all the formulae, it is advisable for the student who is studying the subject for the first time to become familiar with these derivations as it gives a better insight into the design process. For practical designs, however, one may make use of the tables and charts in the "Design Aids to IS 456 (1978)" published as SP16 by the Bureau of Indian Standards or other publications.

ULTIMATE STRENGTH OF R.C. BEAMS (UMIT STATE OF COLLAPSE BY FLEXURE)

The following assumptions are made for calculating the ultimate moment of resistance or the strength at limit state of flexural collapse of reinforced concrete beams (IS 456, Clause 38.1):

- Plane sections remain plane in bending up to the point of failure (i.e.) strains are proportional to distance from the neutral axis).
- Ultimate limit state of bending failure is deemed to have been reached when. the strain in concrete at the extreme bending compression fibre reaches 0.0035.
- The stress distribution across the compression face will correspond to the stress-strain diagram for concrete in compression. Any suitable shape like parabolic, rectangular or any combinations of shapes that

give results which are in substantial agreement with tests may be assumed for this compression block. For design purpose, the maximum compressive strength in the structure is assumed as 0.67 times the characteristic laboratory cube strength. With an additional partial safety factor of γ_m = 1.5 applied to concrete strength



4.1 Strain diagram and stress blocks: (a) Section; (b) Strain diagram (plane sections remain plane); (c) Stress block with partial safety factors; and (d) Simple rectangular stress block (BS).

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0. for all practical purposes. In Fig, it should be noted that γ_m = 1.5 is applied over the whole stress-strain curve to obtain the design stress-strain curve for concrete.

4. The tensile strength of concrete is neglected as the section is assumed to be cracked up to be neutral axis.

5. The stress in steel will correspond to the corresponding strain in the steel, and can be read off from the stress-strain diagram of the steel. For design purposes, a partial safety factor of 1.15 is used for strength of steel so that the maximum stress in steel is limited to fy/1.15= 0.87fy. It should be noted that the design stress-strain curve for cold worked steel is obtained by applying partial safety factor γ_m = 1.15 over the region starting from 0. &j of the actual stress- strain curve for steel .

6. In order to avoid sudden and brittle compression failure in singly reinforced beams, the limiting value of the depth of compression block is to

be obtained according to IS 456 by assuming the strain of tension steel at failure to be not less than the following:

$$\varepsilon_{su} = \frac{f_y}{1.15E_s} + 0.002 = \frac{0.87f_y}{E_s} + 0.002$$

where

ε_{su} = strain in steel at ultimate failure

f_y = characteristic strength of steel

E_s = modulus of elasticity of steel = 200×10^3 N/mm^2

BALANCED, UNDERREINFORCED AND OVERREINFORCED SECTIONS

- ❖ Reinforced concrete sections in bending are assumed to fail when the compression strain in concrete reaches the failure strain in bending compression equal to 0.0035.
- ❖ Sections, in which the tension steel also reaches yield strain simultaneously as the concrete reaches the failure strain in bending, are called balanced sections. Sections, in which tension steel reaches yield strain at loads lower than the load at which concrete reaches failure strain, are called under reinforced sections.
- ❖ It should be remembered that yielding of steel does not mean ultimate failure of the beam. When steel yields, there will be excessive deflection and consequent cracking but complete rupture of steel takes place at a much higher strain, of the order of 0.20 to 0.25 (i.e. 20 to 25 per cent elongation based on the original length) compared to the actual steel yield strain of 0.0038.
- ❖ The latter is only of the same order as failure strain of concrete. The ultimate failure of under reinforced beams in all practical cases is therefore finally due to the concrete reaching the ultimate failure strain of 0.0035.

- It is preferable that a beam be designed as an under reinforced beam, where 'failure' will take place after yielding of steel, with enough warning signals like excessive cracking and deflection taking place before ultimate failure.
- R.C. sections, in which the failure strain in concrete is reached earlier than the yield strain of steel is reached, are called over reinforced sections. Such beams, if loaded to full capacity, will again fail by compression failure of concrete but without warning. Such designs are not recommended in practice.

EQUIVALENT COMPRESSION BLOCK IN CONCRETE

If an idealized stress-strain curve of concrete is used as in the third assumption the magnitude of total compression which is given by the area of the stress block in the beam will be as shown in Fig. and can be expressed as $C = k_1 f_{ck} xb$

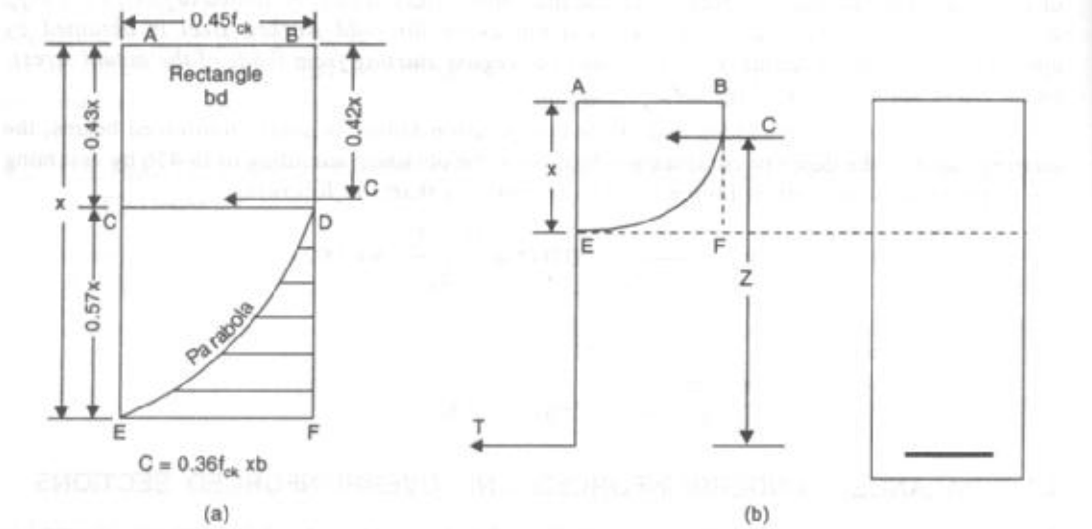


Fig. 4.2 **Calculation of resisting moment of a section: (a) Properties of I.S. stress block in compression; and (b) Forces acting on a section subjected to bending.**

Solving q and taking the lesser of the two values (as different from Procedure 1), we get

$$q = 0.50 - (0.25 - f_2)^{1/2}$$ (

The area of steel is obtained by substituting back for q in the expression

$$A_s = \frac{qbdf_{ck}}{f_y}$$

$$M_u = 0.87 f_y \left(\frac{p}{100}\right)\left(1 - \frac{1.005 f_y}{f_{ck}} \frac{p}{100}\right) bd^2$$

(USE OF DESIGN AID SP 16 WITHOUT CALCULATIONS)

The special publication No. 16 by Indian Standards Institution gives charts and tables for quick design of R.C. sections. Charts 1 to 18 and Tables Ito 4 are for singly reinforced beams. These charts and tables are derived from which is

Charts Ito 18 have been prepared by assigning different values of M per unit width and plotting d vs. p Tables 1—4 which cover a wider range gives the value of Pt for various values of M/bd^2

To determine the percentage of steel required for a given value of M, b, d, f_{ck}, fy from Tables 1 to 4 of SP 16, proceed as follows:

- Calculate M/bd^2
- Enter the table corresponding to the given value of fck and fy in SP 16.
- Read off the percentage of steel required.

Most designers use this handbook method as it is also the easiest method.

GUIDELINES FOR CHOOSING WIDTH, DEPTH AND REINFORCEMENT OF BEAMS

The following guidelines may be used to arrive at the dimensions of R.C. beams:

- ✓ The minimum percentage of tension steel used in beams should be around 0.3 per cent. Usually, the depth of singly reinforced beams is so arranged that the percentage of steel required is only around 75 per cent of the balanced steel.

- ✓ At least two bars should be used as tension steel, and not more than six bars should be used in one layer in a beam.
- ✓ The diameter of hanger bars should not be less than 10 mm and that of main tension bars
- ✓ 12mm. The usual diameters of bars chosen for beams are 10, 12, 16,20,22,25 and 32mm. When using different sized bars in one layer, place the largest diameter bars near the beam faces. The areas of steel should be symmetrical about the centre line of the beam.
- ✓ The width of the beam necessary for accommodating the required number of rods will depend on the specification for cover and minimum spacing. Table gives the required cover to main steel for beams. Assuming the nominal cover of 20 mm for mild steel environment and using 8 mm diameter stirrups the clear cover to steel works out to 28 mm IS 456, Clause 26.3.1 gives the minimum distance between bars as the diameter of bar or maximum size of aggregate plus 5 mm. The maximum size of aggregates normally used in India is 20 mm so that clear

DESIGN OF ONEWAY SLAB

INTRODUCTION

Reinforced concrete solid slabs are constructed in one of the following ways

- **One-way slabs**
- **Two-way slabs**
- **FIat slabs**
- **Flat plates.**
  - ❖ One-way slabs are those supported continuously on the two opposite sides so that the loads are carried along one direction

only. The direction in which the load is carried in one-way slabs is called the span. It may be in the long or short direction.

- One-way slabs are usually made to span in the shorter direction since the corresponding bending moments and shear forces are the least. The main reinforcements are provided in the span direction.
- Steel is also provided in the transverse direction, to distribute any unevenness that may occur in loading and for temperature and shrinkage effects in that direction. This steel is called distribution steel or secondary reinforcement. The main steel is calculated from the bending moment consideration and under no circumstances should it be less than the minimum specified by the code. The secondary reinforcement provided is usually the minimum specified by the code for such reinforcement.
- Two-way slabs are those slabs that are supported continuously on all four sides and are of such dimensions that the loads are carried to the supports along both directions.
- Flat slabs and flat plates are those multi span slabs which directly rest on columns without beams. Flat slabs differ from flat plates in that they have either drop panels (increased thickness of slab) or column capitals in the regions of the columns.
- Flat plates have uniform slab thickness, and the high shear resistance around the columns are obtained usually by the provision of special reinforcements called 'shear-head reinforcements' placed in the slab around the columns.

LIVE LOAD ON SLABS IN BUILDINGS

a. Dead load of slabs consists of its own weight and in addition, the weight of finishes, fixtures and partitions.

Live load or imposed load is specified as per IS code. This live load varies according to the use for which the building is to be put after construction.

b. It is important to note that for design of buildings these live loads are considered as either acting on the full span or assumed to be absent altogether in the span. In continuous slabs, they are to be so placed as to get the maximum bending moment and shear effect in the structure. In design of slabs for other structures like bridges, the effects of partial loading of the slab may have to be considered.

According to IS 875, the loading on slabs for buildings are calculated as follows:

- ❖ Self-weight at 25 kN/m for reinforced concrete

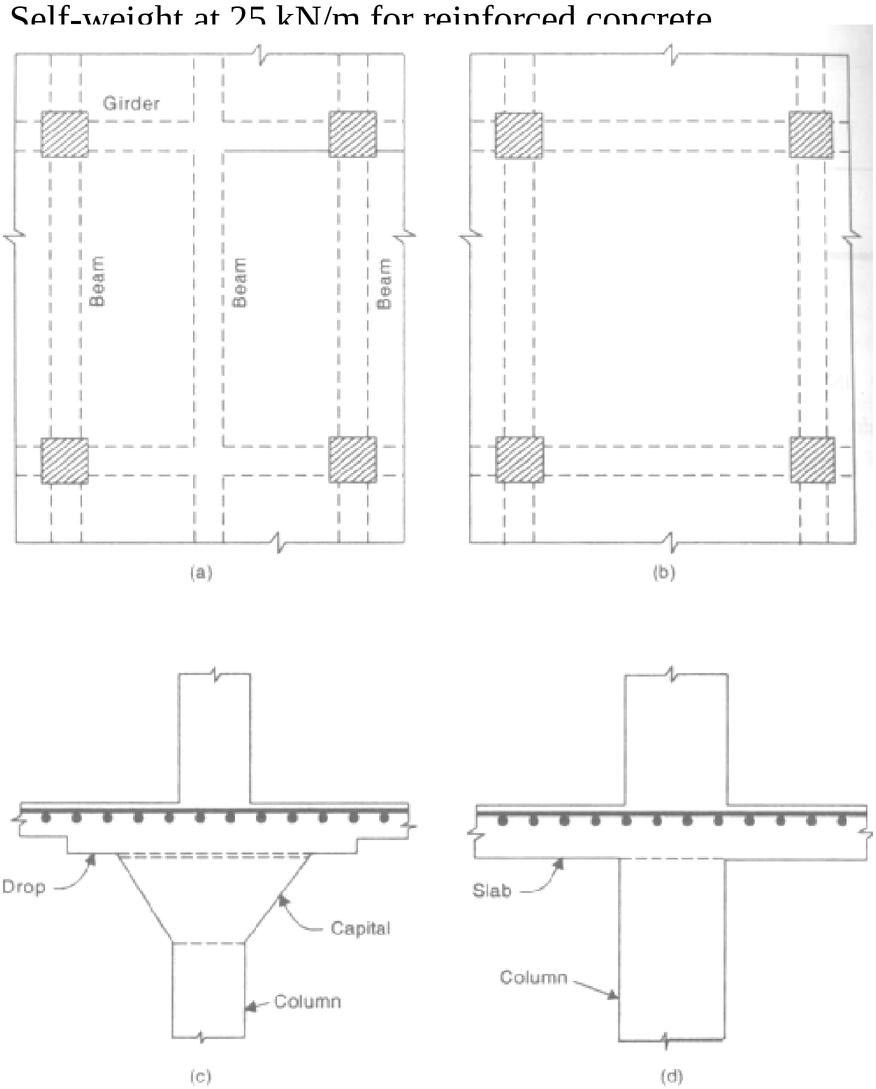


Fig. 11.1 Types of reinforced concrete slabs: (a) One-way slab; (b) Two-way slab; (c) Flat (d) Flat plate.

STRUCTURAL ANALYSIS OF ONE-WAY SLABS WITH UDL USING COEFFICIENTS

One. way slabs, because of their one-way action, are analyzed as beams of unit width. However, codes allow the use of simple coefficients for calculation of moments and forces in continuous beams and slabs of uniform loading with more or less equal spans and continuous on at least three spans.

Spans are considered as equal, if the differences in span are not more than 15 per cent of the larger of the spans. These coefficients for ultimate bending moments and shears are given in Tables 12 and 13 of IS 456 .It should be noted that when using this table, redistribution of moments between sections is not allowed.

TABLE 11.1 BENDING MOMENT AND SHEAR FORCE COEFFICIENTS FOR BEAMS AND ONE-WAY SLABS
(IS 456, Tables 12 and 13)

	End support	Near middle of end span	At first interior support	At middle of interior span	At all other interior supports
Moment					
DL		+ 1/12	− 1/10	+ 1/16	− 1/12
LL		+ 1/10	− 1/9	+ 1/12*	− 1/9
Shear					
DL	0.4		0.6 (outer) 0.55 (inner)		0.5
LL	0.45		0.6 (outer) 0.6 (inner)		0.6

Notes: (i) For bending moment the coefficient is multiplied by total design load and effective span.
(ii) According to IS 456, Clause 22.5.2, beams with partially restrained supports, develop restraining moment which may be taken as WL/24 and the corresponding increase in the shear coefficients as 0.05.
(iii) For SF the coefficient is multiplied by total design load.

DESIGN FOR SHEAR IN SLABS

Normally the thickness of slabs is so chosen that the shear can be resisted by concrete itself and the slab does not need extra shear reinforcements. It is only in extreme situations where the thickness tends to be very large that shear reinforcements are allowed to be used for slabs. Shear tests on solid slabs have shown that when these shallow members ate less than 300 mm thick, they have an increased shear resistance compared to members such as beams which are 300 mm or more in depth. This shear enhancement factor for shallow depths is given in IS 456, Clause 39.2 .

TABLE 11.2 SHEAR MULTIPLYING FACTOR FOR SLABS
($k\tau_c$ for slabs: IS 456, Clause 40.2.1.1)

Overall depth of slab (mm)	k
300 or more	1.00
275	1.05
250	1.10
225	1.15
200	1.20
175	1.25
150 or less	1.30

The enhanced shear is (kç), where k is called the shear enhancement factor for slabs. According to the above table, k varies from 1.30 for slabs of 150mm or less to 1.00 for slabs of 300mm or more. BS 8110 also allows the use of the shear enhancement factor and its value is given by the expression.

According to IS 456, the rules to be followed in the design of slabs for shear are:

- No shear reinforcement should be provided for slabs less than 200 mm thick. However, the increased value of shear resistance in slabs can be taken into account in design.

- It is preferable to design slabs without any extra shear reinforcements.
- However, if found necessary, shear reinforcements may be allowed to be provided in slabs which are 200 mm or more in thickness. The spacing of these reinforcements can be increased to "d" instead of "0.75d" as in beams.
- In no case, even with provision of shear reinforcements, according to IS 456, Clause 40.2.3.1, should the maximum shear stress allowed in slabs due to ultimate load exceed one-half that allowed in beams as given in Table 20 of IS 456.

CONSIDERATIONS FOR DESIGN OF SLABS

➢ Choosing Span Effective Depth Ratio for Slabs

For a given type of support condition, the same span/depth basic ratio as given for beams in IS 456 are applicable for slabs also. However, as the percentage of reinforcements in slabs is generally low, the effective span/depth ratios can be much larger than the basic ratios .

It should also be remembered that with heavy loadings when the percentage of steel in slabs increases, this ratio will tend to be the same as in beams. For the first trial, a convenient percentage of steel may be assumed for the stabs and the span/effective depth ratio calculated.

Thus, assuming 0.3 per cent of steel, the correction factor F from Fig. 4 of 1S 456 will be 1.4, and the span/effective depth ratio for a continuous slab will be of the order of 1.4 x 26 = 36. Because of these considerations. IS 456, Clause 24.1 recommends the following span-overall depth factor for two-way slabs using Fe 415 steel; up to 3.5 m span and loading class up to 3 kN/m^2.

Simply supported two-way slabs (35×0.8) = 28 (beams 20)
Continuous two-way slabs (40×0.8) = 32 (beams 26)
Cantilever slabs = 12 (beams 7)

For one-way slabs, a ratio of 25 and 30 may be more appropriate.

Concrete Cover

The nominal cover specified for slabs for different exposures is given in Chapter 3. Also, a minimum cement content and maximum water cement ratio are specified for different environment conditions. However, as increased cover raises the dead load, the slab has to carry, one should be judicious in the choice of cover. Strict maintenance of the chosen cover during construction and using a good grade of concrete for the construction will go a long way to ensure durability of slabs and reduce the dead load due to cover.

Calculation of Steel Area In Practice

It should be noted that the depth of slab chosen for deflection requirements will be usually greater than the depth required for balanced design. Hence the area of steel required wi/I be less than the balanced amount. The fundamental formula

$$M_u = (f_{st} A_{st}) \times (\text{lever arm})$$

Any one of the procedures explained in Section 4.13 for determining steel area for an under reinforced section may be used for the calculations. The three other rules for choosing steel for slabs are the following [26.5.2]

- The diameter of steel should not exceed 1/8 total thickness of slab
- The amount of steel area in either direction should not be less than 0.12 per cent of the total sectional area when using Fe 413 steel

- Spacing of main steel >3d or 300 mm; secondary steel 5d or 450 mm.

DESIGN PROCEDURE

The procedure of design of one-way slabs is to consider them as 'beams of one meter width in the short direction. The various steps in design are as follows:

Part 1: Assume depth to take care of deflection and design sleet

Step 1: Assuming a suitable overall thickness for the slab, calculate the factored loads (dead and live load) for design. This initial guess for thickness of slab may be made from empirical relations between depth and span. The allowable span/overall depth ratio of stabs may be taken as given in Section 11.5.1. The minimum depth for ease of construction is 100 mm. The factored load is (1.5 DL + 1.5 LL). A suitable cover depending on exposure condition should be assumed.

Step 2: Considering the slab as beam of one meter width and using effective span, determine the maximum bending moments M for the ultimate factored load. For continuous slabs, coefficients of Table 11.1 (IS 456 Table 7) may be used for this purpose. Otherwise, any established elastic analysis may be used. In the latter case redistribution of moments is also allowed.

Step 3: Using the formula $M = Kf_{CK}bd^2$ and b = 1000 mm, find the minimum effective depth required as in beams. Add cover and find the total depth of slab from strength considerations. Check the depth with the depth assumed in step 1. Generally, the depth from Step 1 will be more than that obtained from the strength formula.

Step 4: Check the depth used for shear. As the actual percentage of steel at supports is not known, the check is only approximate. A value of τ_c corresponding to the lowest percentage of steel in Table 13 of IS 456 may be used for this purpose. This value can be increased by a factor k. The depth used should be such that, in the final analysis, the slab is safe without any shear reinforcements.

Step 5: As the depth selected is usually greater than the minimum depth d, the tension steel required will be less than the balanced amount for the section. Determine the steel required by a suitable formula or by use of SP 16 charts and tables.

Part 2: Check for cracking by obeying rules for detailing.

Step 6: Check whether this steel is not less than the minimum percentage of the gross section specified for slabs, namely, 0.12 per cent with high yield steel and 0.15 per cent with mild steel bars. Provide at least the specified minimum. Table 11.3 may be used for this purpose. (IS 456, Clause 26.5.2.1)

TABLE 11.3 SPACING OF DISTRIBUTION STEEL FOR SLABS (cm)

Thickness of slab (cm)	Fe 250 (0.15%) Diameter of bar (mm)				Fe (0.12%) Diameter of bar (mm)			
	6	8	10	12	6	8	10	12
10	18	30			23			
11	17	30			21	35		
12	15	27	40		19	33		
13	14	25	40		18	30		
14	13	23	35		16	29		
15	12	22	30		15	27	40	
17.5	10	19	29		13	23	35	
20	9	16	26	35	11	20	30	
22.5	8	14	23	30	10	18	29	40
25	7	13	20	30	9	16	26	35

Step 7: Choose a suitable diameter for the main reinforcement and determine the spacing of steel. For crack control, this spacing should suit the bar spacing rules for slabs. In general, the spacing of main steel should not exceed three times effective depth or 300 mm whichever is smaller.

Step 8; Recheck for shear stresses, using the actual percentage of steel available.

Step 9: Check the adopted depth for deflection using the empirical method .

Step 10: Provide necessary distribution (secondary) reinforcement. This too should not be less than the specified per cent of the cross-sectional area of the slab, namely, 0.12 per cent for high yield bars and 0.15 per cent for rolled mild steel bars. This steel is usually placed over the main reinforcement bars to maximize the effective depth for the main reinforcements and facilitate the order of placing of steel. Check also the spacing of the secondary steel, so that it does not exceed five times the effective depth or 450 mm.

Step 11: For the slab forming the top flange of a T or L beam, the transverse reinforcement provided on the top surface should extend across the full effective width of the flange. According to IS 456, Clause 23.1.1, this transverse steel should not be less than 60 per cent of the main steel at mid-span of the slab and according to BS8110 the amount should not be less than 0.15 per cent of the longitudinal cross-sectional area of the flange (both for high yield steel and mild steel).

These steels are only for tying up the slabs with the beams, and not for absorbing any stresses in the compression zone of T beams. When the slab is spanning across T beams, the negative steel may be used for this purpose.

When the slab is spanning in the direction of the T beams, separate steel has to be employed for this transverse steel to make the beam act as a T beam.

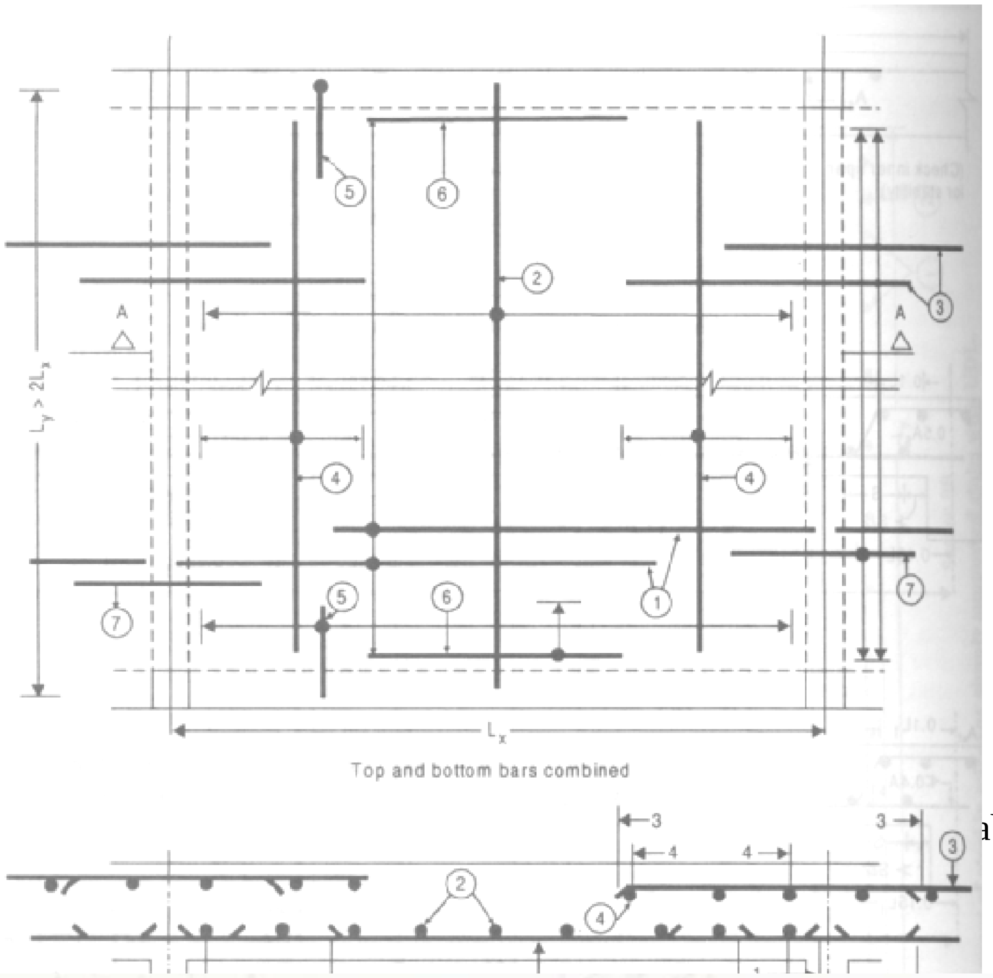
USE OF DESIGN AIDS SP 16

IS Publication SP 16 gives various tables and charts for rapid design of slabs. One may use either tables 1 to 4 of SP 16, which are usually used for design of singly reinforced beam sections by assuming b = 1000, or tables 5 to 44 giving the moment of resistance of slabs of specific thicknesses 10 to 25 cm for different values of f_{ck} and The tables are made for the standard 15 mm clear cover to slab reinforcement.

The minimum steel ratio of 0.12 per cent for Fe 415 and 0.15 per cent for Fe 250 with respect to gross area to be provided as distribution steel can also be read off for the various thicknesses of these slabs.

CONCENTRATED LOAD ON ONE-WAY SLABS

The bending effects due to concentrated loads on one-way slabs are usually analyzed by the effective width method .The corresponding effects on two- way slabs are analyzed by Pigeaud's method. It may be noted that even though one-way slabs can also be analyzed by Pigeaud's method, effective width method is more commonly used for such slabs.



Som ab

also,

$$a = x_1 + 2\text{ (wearing coat + effective depth)}$$

Fig. 11.3b Reinforcement drawings for one-way continuous slab using straight bars.

Similarly, let $e = y_1 + 2$ (wearing coat + effective depth). The first method is more conservative and generally used for highway slabs. The second method may be used in other safer situations. The effective width b is then calculated by the expression given in IS 456, Clause 243.2.1 .

$$b_e = kx\left(1 - \frac{x}{L_e}\right) + a$$

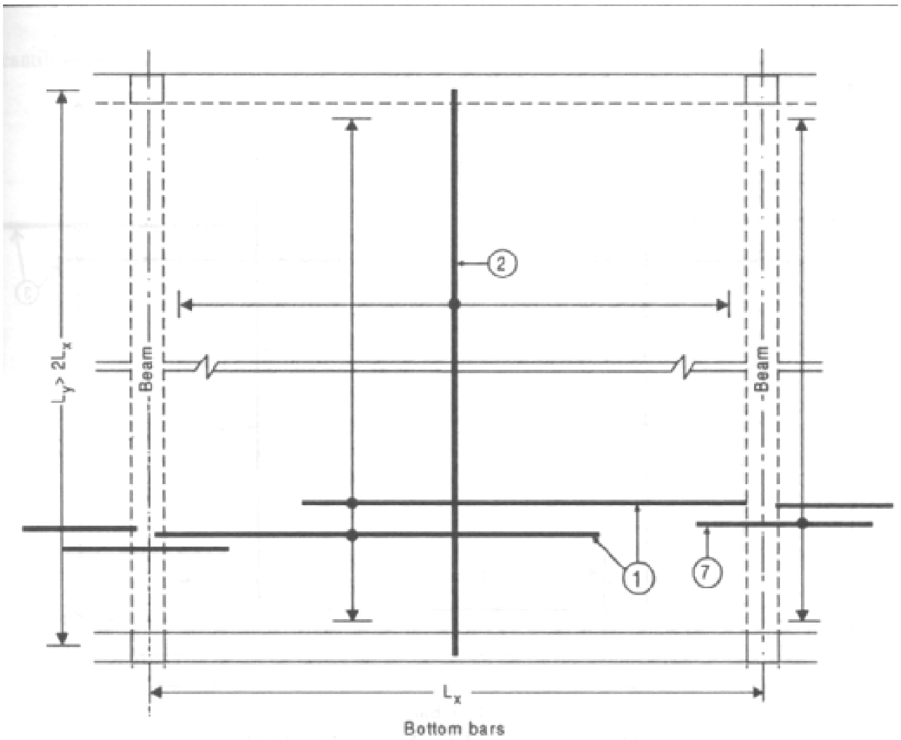


Fig. 11.3c Layout of bottom bars in Fig. 11.3b.

ratio of the width of the slab (B) to its effective span (L), and the nature of the slab whether it is simply supported or continuous. (A value of k = 1.2 on either side of toad width as shown in Fig may be assumed for alt one-way slabs.

x = distance of the centroid of load from the near support

L_e= effective span

a = dispersed width of the contact area of the concentrated load parallel to the supported edge. (Dispersion is taken at 45° through wearing coat only.)

Under no circumstances should the effective width exceed the actual width of the slab, and when the concentrated load is close to an unsupported edge of the slab, effective width should not exceed the above value or one-half the above value plus the distance of the load from the unsupported edge, whichever is less.

For determining the effective length of the load in the direction of the span, the dispersion of the load along the bridge through full effective depth of the slab is usually taken into account, thus assuming at 45° through both the wearing coat and the effective depth.

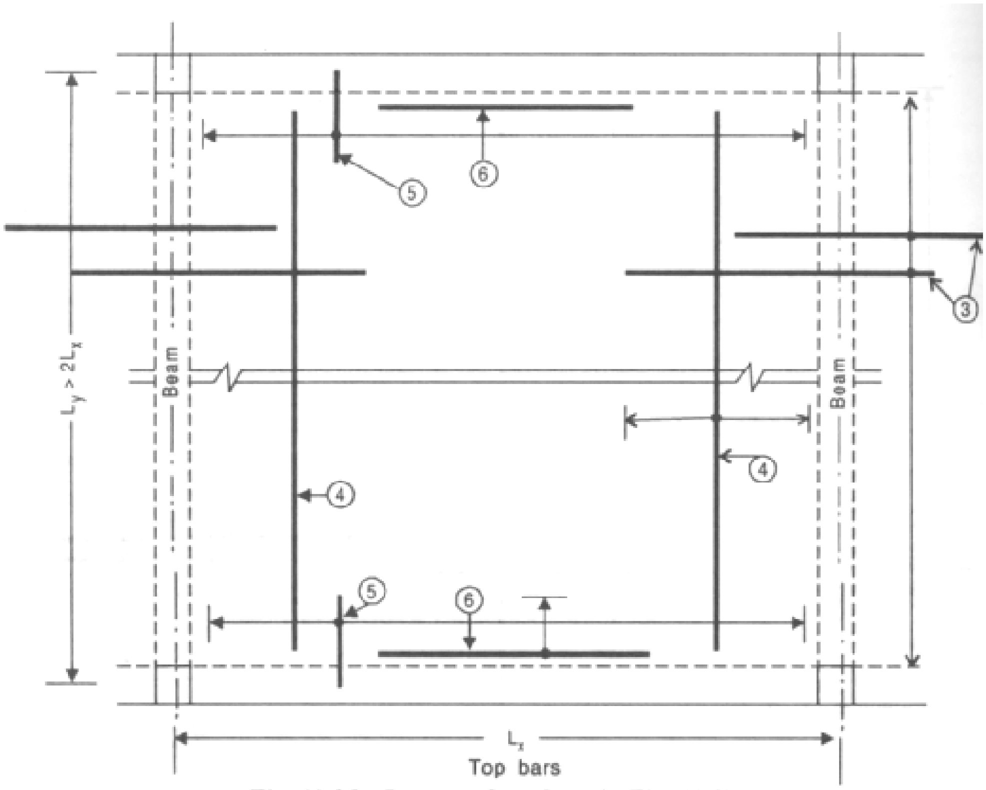


Fig. 11.3d Layout of top bars in Fig. 11.3b.

; are placed one behind the other on the slab along the direction of the span.

When the effective widths and the effective lengths of loads as calculated above overlap, for two or more loads, the effective widths for each load should be considered separately and marked. If the effects overlap, the slab should be designed for the combined effects of the two loads on the overlapped portion.

DESIGN OF TWO-WAY SLABS

Slabs which are supported on unyielding supports like walls on all four sides are called two-way slabs. The span in the larger direction is denoted by l_y and l_x that in the shorter direction by ly The distribution of loads in the l_y and l_x directions will depend on the ratio l_y/l_x When $l_y/l_x > 2$, it can be shown that most of the loads are transmitted along the shorter directions and the slab acts as a one-way slab.

Beam supports which are sufficiently stiff can be considered as unyielding and slabs on these beams also act as two-way slabs. Beam supports which deflect significantly under the loading from slabs, come under slabs on flexible beams, and cannot be strictly classified as conventional two-way slabs.

In these slabs the load distribution and bending moments produced are different from slabs on unyielding supports.

The boundaries of a two-way slab can be fully restrained (continuous), simply supported, or partially restrained at the edges as shown in Fig.

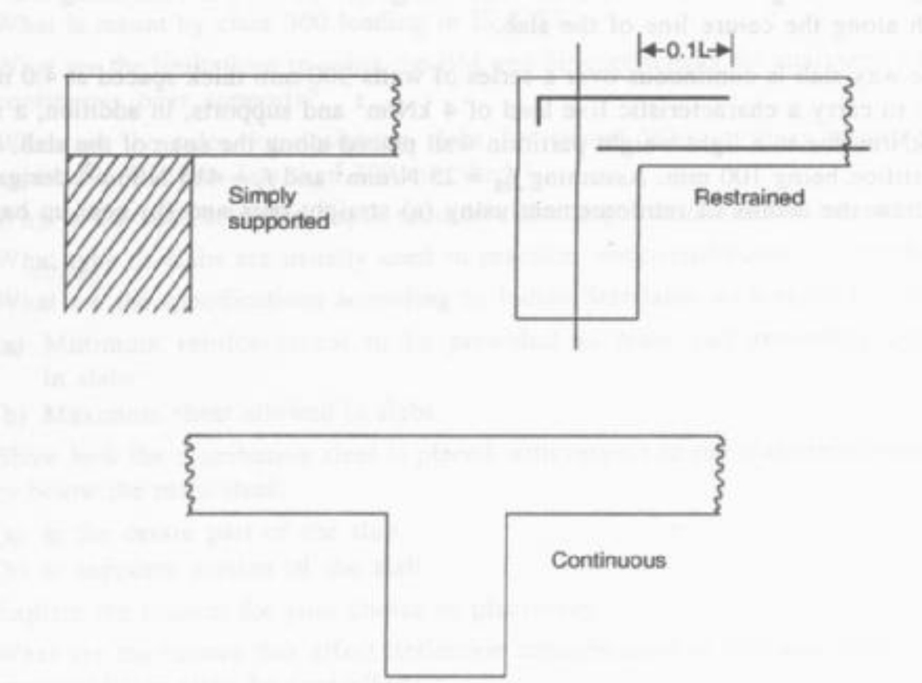


Fig. 12.1 Conditions of support of two-way slabs.

UNIT III

LIMIT STATE DESIGN FOR SHEAR TORSION BOND AND ANCHORAGE

DESIGN FOR TORSION

INTRODUCTION

Many types of loadings produce torsion in reinforced concrete members. The resultant torsion may be classified into two types

- **Primary or equilibrium torsion**
- **Secondary or compatibility torsion**

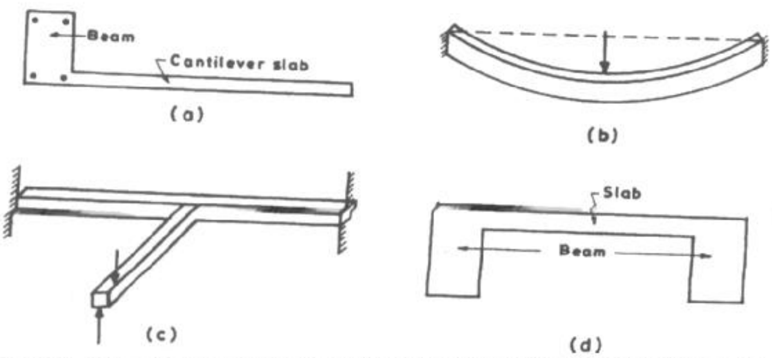


Fig. 18.1 Primary and secondary torsion: (a) Portico slab; (b) Bow girder; (c) Indeterminate frame; (d) Edge beams.

- The first type is that which is required to maintain basic static equilibrium, and the second is the one required to maintain only compatibility condition between members.
- In general, one may say that torsion in statically determinate structure is of the equilibrium type and torsion in statically indeterminate structures may be either of the equilibrium or the compatibility type.
- In statically indeterminate structures, there are more than one load path along which loads can be distributed and equilibrium maintained, so that the structure can be made safe without taking minor torsional effects into account. Such neglect, at most, will produce some cracking, but not failure.
- However, in structures in which a large part of the load is applied unsymmetrical, torsion will have to be considered carefully.
- Torsion is a major component if it is of the equilibrium type as also in situations where the torsional stiffness of the members has been taken into account in the structural analysis.

- In other cases of secondary torsion, provision of nominal shear reinforcements according to codes of practice may be assumed to take care of the incidental effects.
- Thus the small amount of unintentional torsion in most of the conventional beams and slabs can be ignored in design and supplied by proper detailing of reinforcements.

ANALYSIS FOR TORSIONAL MOMENT IN A MEMBER

- Moments provided in a structure on application of loads are taken by some members in bending and other members by torsion, depending on their disposition.
- Just as bending moments are distributed among the members sharing the moments in proportion to their bending stiffness, i.e. (EI/L) values, the factor that determines the transfer of torsional moment is the torsional stiffness (GCIL), where G is the elastic shear modulus and C the torsion constant. This principle is used to determine torsional moments carried by members in structural analysis .

18.2.1 Calculation of Torsion Constant *C*

The value C for a rectangle $b \times D$ (where b is the smaller dimension) is given by

$$C = KDb^3$$

where K is St. Venant's torsional constant which varies with the ratio of D/b. The value of K for various ratios of D/b is given in Table 18.1.

TABLE 18.1 VALUES OF K AND α IN TORSION OF RECTANGLES

$$\left(C = KDb^3, \quad \tau_{max} = \frac{T}{\alpha b^2 D} \right)$$

D/b	Value of		D/b	Value of	
	K	α		K	α
1.0	0.141	0.208	3.0	0.263	0.267
1.2	0.166	0.219	4.0	0.281	0.282
1.5	0.196	0.231	5.0	0.291	0.291
2.0	0.229	0.246	10.0	0.312	0.312
2.5	0.249	0.258	α	0.333	0.333

A more convenient expression for C, giving values close to those obtained from the above table, for value of $D/b < 10$, has been derived by Timoshenko as

(18.1)

:tions

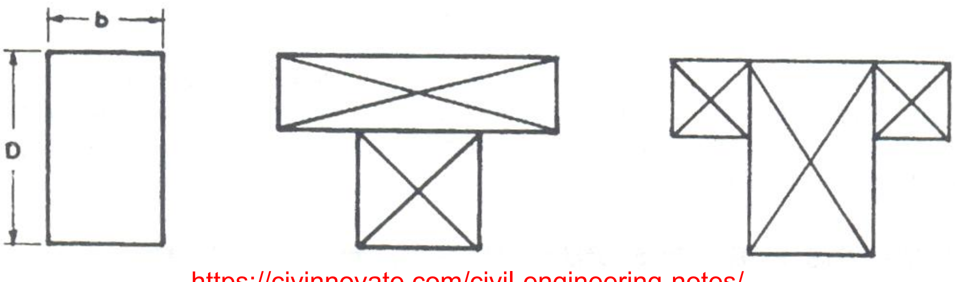


Fig. 18.2 Torsion of flanged sections.

- For analysis, T L or I sections are divided into component rectangles and the C value is the combined (added up) value of the component rectangles. The division should be such that the value of C obtained for the whole section, i.e. should be the largest possible value.

$$C = \Sigma\left(1 - 0.63\frac{b}{D}\right)\left(\frac{b^3 D}{3}\right)$$

- Hollow box sections can be treated as solid when the will thickness is more than 13/4 (one. fourth the depth). Otherwise, it can be divided into its component rectangle, and the value of the total torsional stiffness determined. It should, however, be remembered that sections of thickness less than / are not suitable for reinforced concrete in torsion due to its large flexibility.

Bending and Torsional Stiffness of R.C. Members

That the magnitude of distribution of moments as torsion to adjoining members is small can be seen from the following argument: First,

$$\text{Value of } G = \frac{E}{2(1+\mu)}$$

If $\mu = 0.15$, then

$$G = \frac{E}{2.3}, \quad \mu = 0, \quad G = \frac{E}{2}$$

which shows that G is very low as compared to E.

Secondly, for a rectangle $D = 2b$, the value of I is about three times that of C.

The ratio of stiffness in bending to stiffness in torsion for adjoining members can therefore be obtained as

$$\frac{EI}{L} : \frac{GC}{L} = 6.9 \quad \text{(approx. 7)}$$

Hence beams are several times more stiff in bending than in torsion. Thus very little of the moment is transferred as torsion to members while the major part is transferred as bending moments.

Torsional Rigidity of R.C. Members

- On the basis of laboratory tests, BS 8110 (part 2): Clause 2.4.3 states that for structural analysis or design, the torsional rigidity may be calculated by assuming G = 0.42 times the modulus of elasticity of concrete and C equal to one-half of the St. Venant value calculated for the plain concrete section.

Hence the maximum shear by plastic analysis is given by

$$\tau_t = \frac{2T}{b^2(D - b/3)}$$

For the hollow section the constant shear stress on the walls can be approximated as (see Fig. 18.4):

$$\tau_1 t_1 = \tau_2 t_2 \quad \text{for shear flow}$$

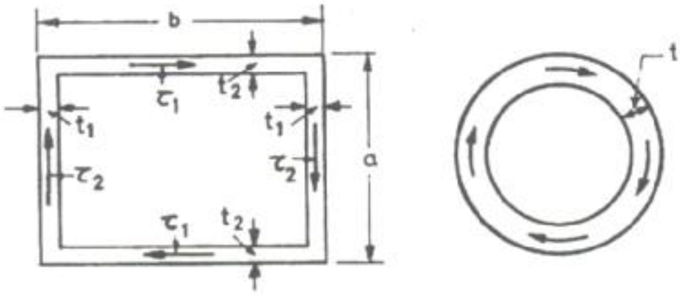


Fig. 18.4 Analysis of hollow sections in torsion.

Also,

$$T = 2\tau_1 t_1 \times (a - t_2)(b - t_1) = 2\tau_1 t_1 A$$

For a section of uniform width,

$$\tau_t = \frac{T}{2At}$$

where

A = area enclosed by the centre line of the walls

t = wall thickness

where

A = area enclosed by the centre line of the wails

T1= wall thickness

- It should be noted that the combined effect of bending shear and torsional shear is to increase the shear on one side and decrease it on the opposite side.

Torsional Strength of Concrete Beams (According to BS 8110)

- According to 88 8110, one has to assess the shear produced in bending shear and torsional shear separately.

$$\tau_v = V/bd$$

- The safe strength of concrete in bending shear without shear reinforcement is given by ç (Table 19, 15 456). Similarly, the shear produced by torsion can be evaluated by plastic theory by

$$\tau_t = \frac{2T}{b^2(D - b/3)}$$

18.4 TORSIONAL STRESS IN FLANGED SECTIONS

To determine maximum shear stress in sections *I*, *T* or *L*, and other rectangular sections, the section may be assumed to be divided into component rectangles with the largest possible rectangle as one of its components. The torsion can be assumed to be distributed to each rectangle in proportion to its torsional stiffness (KDb^3). Then by elastic analysis, we get

$$T_n = T \frac{K_n D_n b_n^3}{\Sigma K_n D_n b_n^3} \quad (18.4)$$

By plastic analysis the value is given by

$$T_n = T \frac{D_n b_n^3}{\Sigma D_n b_n^3} \quad (18.5)$$

The maximum stress in each rectangle is then found by Eq. (18.2) or (18.3) for elastic and plastic analysis, respectively.

REINFORCEMENTS FOR TORSION IN R.C. BEAMS

- Ultimate failure of a beam in torsion, according to Hsu's skew bending theory, is by rotation about a skew axis as shown in Fig. 18.5. (This can be easily demonstrated by taking a brittle material like a piece of chalk and giving it a twist at each of its ends by hand in the opposite directions.)
- Because of the skewness of the surface of failure (unlike in simple bending where it is planar), reinforcements for torsion should consist not only of transverse links (loop reinforcement) but also of longitudinal steel provided at all the four corners of the beam (Fig. 18.6).
- The requirement can be explained both by the skew bending theory as also by the simple space truss model (Fig.18.7) similar to the plane truss model proposed a early as in 1929, for bending shear.

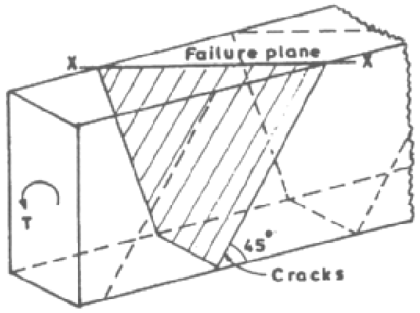
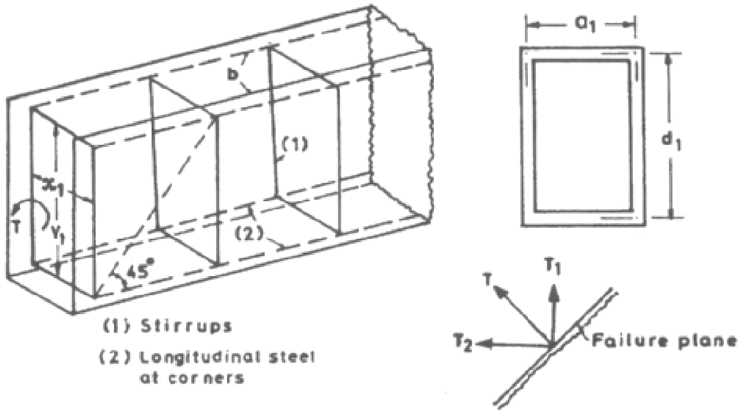


Fig. 18.5 Failure of beam section in torsion.



18.6.1 Interaction Curve for Concrete without Web Steel under *V* and *T*

If

V_{u0} = shear strength of beam without torsion

T_{u0} = torsional strength without shear

T_u, V_u = the design torsion and shear respectively

It has been found that for beams without web reinforcement the interaction curve can be represented by a quarter circle, as shown in Fig. (18.8a).

$$\left(\frac{T_u}{T_{u0}}\right)^2 + \left(\frac{V_u}{V_{u0}}\right)^2 = 1 \tag{18.6}$$

Equation (18.6) has been assumed by ACI in its design.

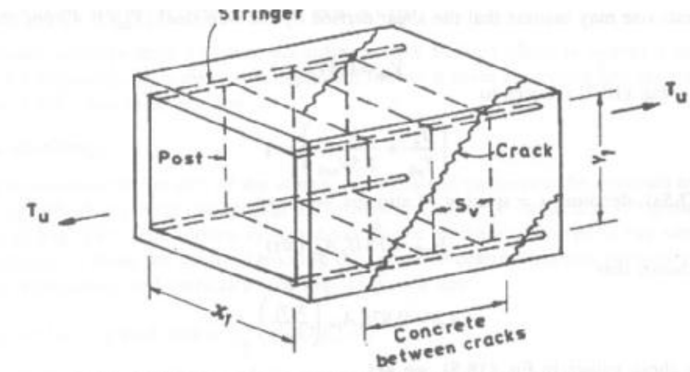


Fig. 18.7 Space-truss model for torsion in R.C.C. beams.

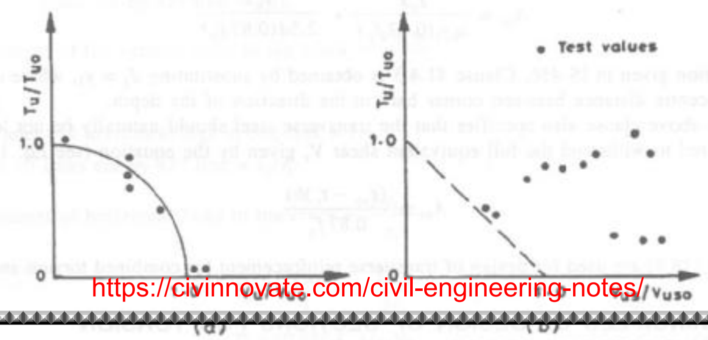


Fig. 18.8 Interaction curves for torsion and shear: (a) without web steel; (b) with web steel.

Based on tests one may assume that the shear carried by the web steel (V_{us}) is 40 per cent of V_u, i.e.

$$V_{us} = 0.4V_u$$

Accordingly, Eq. (18.7) reduces to

$$\left(\frac{T_u}{T_{u0}} + \frac{V_u}{2.5V_{us0}}\right) = 1 \qquad (18.8)$$

From Eq. (7.5a), denoting s = spacing of stirrups, we have

$$V_{us0} = 0.87 f_y A_{sv} (d/s)$$

It can be shown that

$$T_{u0} = 0.87 f_y A_{sv} \left(\frac{x_1 y_1}{s}\right)$$

Substituting these values in Eq. (18.8), we get

$$A_{sv} = \frac{T_u s}{x_1 y_1 (0.87 f_y)} + \frac{V_u s}{2.5d(0.87 f_y)} \qquad (18.8a)$$

The equation given in IS 456, Clause 41.4.3 is obtained by substituting $d_1 = y_1$, where d_1 is the centre-to-centre distance between corner bars in the direction of the depth.

The above clause also specifies that the transverse steel should naturally be not less than that required to withstand the full equivalent shear V_e given by the equation (see Eq. 18.12)

$$A_{sv} = \frac{(\tau_{ve} - \tau_c)bs}{0.87 f_y} \qquad (18.8b)$$

Equations (18.8) are used for design of transverse reinforcement for combined torsion and shear.

PRINCIPLES OF DESIGN OF SECTIONS FOR TORSION BY DIFFERENT CODES

- The design procedure to be adopted when torsion is present in R.C. members depends on the code to be used.
- The IS, BS and ACI propose different methods for torsion design, even though the resultant design is considered equally safe.
- When torsion is present along with ‘bending shear’, IS recommends the use of an equivalent shear for which the shear steels are calculated.
- Again in IS when torsion is present as combined with bending, an equivalent bending moment is calculated and reinforcement for this equivalent bending moment is provided as longitudinal steel.
- In the British practice, the section is separately analyzed for maximum torsional stresses, and depending on the magnitude of the resultant stress, the torsional reinforcements are calculated.

- Steel is also calculated separately for shear and bending moments. The values of reinforcements thus calculated individually are combined and provided as stirrups and longitudinal steel.
- The A procedure for design for torsion is to accommodate torsional shear in the same way as in flexural shear, i.e. part of the torsional moment may be considered as carried by concrete without web steel and the remainder by stirrups.

DESIGN FOR TORSION BY BS 8110

According to BS 8110, torsion is treated separately and provided for separately. This method is explained first as it gives an insight into the fundamentals of design of R.C. beams for torsion.

1. Area of stirrups

- A simple expression for the area of the stirrups to withstand torsion can be obtained by assuming the cracking pattern in torsion (which is in the form of a helix) to be inclined at 45° to the horizontal as shown in Fig. 18.5.
- The torsion is withstood by the moment of forces in the stirrups in the x and y directions about the centre.
- Denoting the centre-to-centre distances between the links as x and yi. horizontally and vertically (see Fig. 18.6) we get

Torsion force in each link $= \frac{A_{sv}}{2}(0.87 f_y) = F$

where A_{sv} is the area of both legs of the stirrups; moment of vertical legs in one link $= Fx_1$

No. of links cut by 45° line $= y_1/s_v$

Moment of the vertical links in the crack $= \frac{Fx_1 y_1}{s_v}$

Similarly,

Moment of horizontal legs in one link $= Fy_1$

No. of links cut by 45° line $= x_1/s_v$

Moment of horizontal links in the crack $= \frac{Fy_1 x_1}{s_v}$

$$T_u = \frac{2Fx_1 y_1}{s_v} = \frac{2A_{sv}}{2s_v}(0.87 f_y)\, x_1 y_1$$

$$T_u = (0.87 f_y A_{sv} x_1 y_1)/s_v \quad (18.9)$$

Using an efficiency factor 0.8, we get

$$T_u = \frac{0.8 A_{sv}(0.87 f_y) x_1 y_1}{s_v}$$

$$\frac{A_{sv}}{s_v} = \frac{T_u}{0.8 x_1 y_1 (0.87 f_y)}$$

or

$$\frac{T_u}{0.8 x_1 y_1} = \frac{A_{sv}(0.87 f_y)}{s_v} \quad (18.9a)$$

2. Area of additional longitudinal steel (Ast)

- At least four numbers of steel bars should be placed symmetrical inside the four corners of the links to be effective.
- According to this concept, the volume of the longitudinal steel required will be the same as the volume of the transverse hoops.
- Taking a distance equal to spacing of stirrup and equating the forces, we get the area of additional longitudinal steel as

$$A_{s1}\,(f_{y1})s_v = \frac{A_{sv}}{2}\,(f_y)\,2\,(x_1 + y_1)$$

$$A_{s1}\,f_{y1}s_v = A_{sv}f_y\,(x_1 + y_1)$$

where

A_{s1} = total area of the additional longitudinal steel

A_{sv} = area of the two legs of stirrups

f_{y1} = yield stress of the longitudinal steel

f_y = yield stress of links

x_1, y_1 = centre-to-centre distance of links

The formula for design is

$$A_{s1} = \frac{A_{sv}}{s_v}\,\frac{f_y}{f_{y1}}\,(x_1 + y_1) \qquad (18.10)$$

This area of steel has to be provided in addition to the bending steel to take care of torsion.

PRINCIPLES OF DESIGN FOR COMBINED BENDING, SHEAR AND TORSION BY IS 456

- Design procedure for torsion according to IS 456, it is not necessary to calculate the shear stresses produced by torsion separately as in BS 8110.
- The former gives the analysis for combined effects of torsion shear and bending shear. Bending shear and torsion are combined to an equivalent shear ye.
- Similarly, the bending moment and torsional moment are combined to an equivalent bending moment Me. The R.C. section is then designed for V and Me.

Calculation of Equivalent Shear and Design for Stirrups

An empirical relation for equivalent shear due to the combined effects of torsion and shear has been suggested in IS 456, Clause 41.3 as

The equivalent nominal shear stress will be given by the expression

$$\tau_{ve} = \frac{V_e}{bD} \tag{18.12}$$

Under no condition should the above value exceed τ_{max} given in Table 20 of IS 456 (Table 7.2 of the text). The area of shear steel should satisfy two conditions: First, the area of reinforcement A_{sv} should satisfy the same equation as used for bending shear. (IS code: Clause 41.4.3). Hence

$$\frac{A_{sv}}{s_v} = b\left(\frac{\tau_{ve} - \tau_c}{0.87 f_y}\right) \tag{18.13}$$

where s_v = spacing of stirrups.

Secondly, a linear interaction curve as shown by Eq. (18.7) is assumed, and the steel area should satisfy the condition given in IS 456, Clause 41.4.3, already derived as Eq. (18.8a).

$$\frac{T_u s_v}{A_{sv} b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y) A_{sv}} = 1$$

Rewriting this equation in the usual form for design of shear steel, it becomes

$$\frac{A_{sv}}{s_v}(0.87 f_y) = \frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1} \tag{18.13a}$$

The above two conditions should be satisfied by the shear steel.

18.9.2 Calculation of Equivalent Bending Moment and Design for Longitudinal Steel

In IS 456 the effect of bending moment and torsion is converted into an equivalent total bending (M_e) given in IS 456, Clause 41.4.2 by the equation

$$M_e = M_u \text{ (bending)} \pm M_t \text{ (equivalent torsion)}$$

The equivalent bending moment (M_t) due to torsion T_u is given by

$$M_t = \frac{T_u (1 + D/b)}{1.7} \tag{18.14}$$

so that the equivalent total bending moment is given by

$$M_e = M_u \pm \frac{T_u (1 + D/b)}{1.7}$$

where

T_u = design torsional moment

M_u = design bending moment

D = overall depth

b = breadth of beam

$$V_e = V_u + 1.6\ T_u/b$$

where

V_e = equivalent shear

V_u = design shear

T_u = design torsion

b = breadth of section

The above equation for bending moment is derived from the interaction curve between bending moment and torsion and the three possible modes of failure, the theory of which is beyond the

18.9.3 Design for Compression Steel

If $M_t > M_u$, then there can be reversal of moment, and longitudinal steel has to be p the flexural compression face also, so that the beam can withstand the equivalent m

$$M_{e2} = M_t - M_u$$

Additional steel is provided for this moment on the compression side of the beam.

DETAILING OF TORSION STEEL

- IS 456, Clause 25.5.1.7 gives the rules regarding detailing of torsion steel, which are to be read along with Clause 40.4.3. These rules can be summarised as follows:
    - The spacings of stirrups should not exceed x1
    - or 300 mm
- There should be at least one longitudinal bar placed at each corner of the stirrups.
- When the cross-sectional spacing exceeds 450 mm additional longitudinal bars should be provided to satisfy the minimum reinforcement and spacing rules regarding side face reinforcement.
- That is, there should be a minimum of 0.1 per cent longitudinal steel spaced at not more than 300 mm or thickness of web.IS 456, Clause 26.5.1.3.

18.11 DESIGN RULES ACCORDING TO IS 456

The following is the procedure for design of beam for torsion according to IS 456, Clause 40.

18.11.1 Design from Fundamentals

Step 1: Determine design moments shear and torsion M_u, V_u and T_u.

18.11.2 Procedure for Design with SP 16

The calculation in steps 5 and 6 for shear steel can be facilitated by use of SP 16, Table 62 (Table 7.3 of the text) which gives $\frac{A_{sv}(0.87 f_y)}{s_v}$ values for different sizes of steel and spacings. Steps 5 and 6 above can be modified respectively as

$$(\tau_{ve} - \tau_c)\, b = \frac{A_{sv}}{s_v}(0.87 f_y)$$

$$\frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1} = \frac{A_{sv}}{s_v}(0.87 f_y)$$

Calculate larger of the two values

$$[(\tau_{ve} - \tau_c)\, b] \text{ and } \left(\frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1}\right)$$

DESIGN PROCEDURE ACCORDING TO BS 8110

Rules for Design

- Let the section be subjected to bending moment M, shear V, and torsion T.
- It is necessary to design the transverse and longitudinal steel.
- First, the section is designed for longitudinal steel for the bending moment M.
- Then it is designed for the shear produced by the bending shear and torsion.

Checking of Shear Caused by V and T

$$\tau_v = \frac{V}{bd}$$

$$\tau_t = \frac{2T}{b^2(D - b/3)}$$

$$\tau_{tc} = 0.067\sqrt{f_{ck}} \not> 0.4 \text{ N/mm}^2$$

$$\tau_v + \tau_t \not> \tau_{c\,(max)}$$

$$\tau_{c\,(max)} = 0.8\sqrt{f_{ck}} \not> 5.0 \text{ N/mm}^2$$

The rules for design are given in Table 18.2.

Design Formula for Torsion

TABLE 18.2 DESIGN FOR SHEAR AND TORSION BS 8110 (1985)

Bending shear stress	Torsion shear stress	
	$\tau_t < \tau_{tc}$	$\tau_t > \tau_{tc}$
Less than safe in concrete ($\tau_v < \tau_c$)	Nominal shear steel, no torsion steel	Designed torsion steel
Greater than the safe value in concrete ($\tau_v > \tau_c$)	Designed shear steel, no torsion steel	Designed shear and torsion steel

- It has already been shown that for design of stirrups for torsion, one has to calculate the areas of links and the longitudinal steel.
- These are given by the following formulae:
  - The area of links is given by Eq. (18.9) as

$$\frac{A_{sv}}{s_v} = \frac{T_u}{0.8 x_1 y_1 (0.87 f_y)}$$

2. The area of longitudinal bars is given by Eq. (18.10) as

$$A_{s1} = \frac{A_{sv}}{s_v}\left(\frac{f_y}{f_{y1}}\right)(x_1 + y_1)$$

PROCEDURE FOR DESIGN FOR TORSION BY BS 8110

- Step 1: Find the area of tension steel for M.
- Step 2: Calculate V/bd.
- Step 3: Calculate ç due to torsion.
- Step 4: Design shear and torsion steel as per Table 18.2.
- Step 5: Calculate additional longitudinal steel

ARRANGEMENT OF LINKS FOR TORSION IN FLANGED BEAMS

- It is easy to arrange the stirrup and longitudinal steel for rectangular section. However, when treating flanged beams like T or I beams, it is not to be treated as one whole.

- It should be split into its constituent rectangles as detailed for torsion.
- Studies have shown that treating flanged beams with the largest rectangle, by taking the torsion gives conservative values, and this procedure is recommended in IS 456, Clause 40.1.1.
- In those cases where different rectangles are taken as resisting torsion, each rectangle must be suitably reinforced with the necessary links (Fig. 18.9).

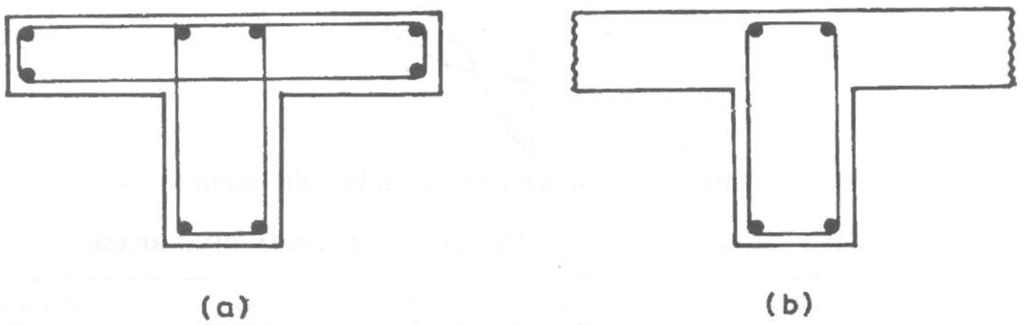


Fig. 18.9 Reinforcing flanged sections in torsion: (a) Torsion shared by two rectangles; (b) Torsion taken by one rectangle.

TORSION IN BEAMS CURVED IN PLAN

- Circular beams (e.g. the ring beam under a circular water tank supported on columns) are subjected to bending and torsion moments.
- Their distribution along the circumference depends on the number of column supports. The magnitude of these moments can be expressed as a function of the uniformly distributed load wand the radius of the ring beam r as follows: Let aand flbe taken as given in Fig. 18.10.
- The general formulae for torsion and bending moment at angle /3 are given by

1. $T_\beta = wr^2 (\beta - \alpha + \alpha \cos \beta - \alpha \sin \beta \cot \alpha)$
2. $M_\beta = wr^2 (\alpha \sin \beta - 1 + \alpha \cot \alpha \cos \beta)$
3. T_β is maximum when $M_\beta = 0$

Bond, Anchorage, Development Lengths

INTRODUCTION

- The term ‘bond’ in reinforced concrete design refers to the adhesion or the shear stress that occurs between concrete and steel in a loaded member.

- It is the bond between steel and concrete that enables the two materials to act together without slip.
- The assumption that in a R.C.C. beam plane sections remain plane even after bending will be valid only if there is perfect bond or no slip-between concrete and steel.
- The magnitude of this bond stress at a point is called local bond. It varies along a member depending on the variation of bending moment as shown in Fig. 10.1.
- Similarly, in order to develop the full tension in the steel placed at the mid-section of a beam, it should be properly anchored on both sides of the section so that the full tension capacity of the steel reinforcement is

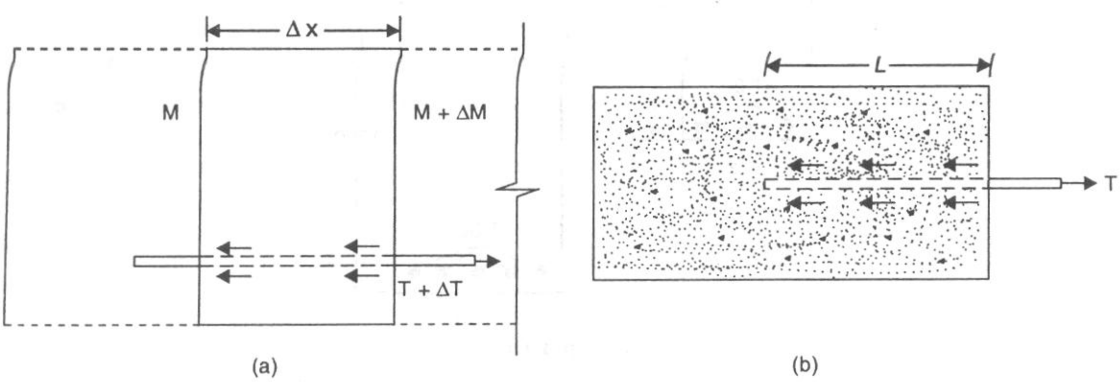


Fig. 10.1 Bond stresses: (a) Local (flexural); (b) Anchorage (average).

- The average stress that acts along this anchorage length is called the average anchorage bond.
- Even though local bond varies along the length of the anchorage, its 'average value' is taken for design.
- The length or extension that should be provided on either side of the point of maximum tension in the steel so that the average bond stress is not exceeded, is called the development length in tension.
- Development length should be ensured in compression steel also. This length for development of compressive stress in steel is called development length in compression.
- Till recently, since smooth mild steel bars were used as reinforcements, both local bond and development lengths were important and they had to be checked separately in routine designs.

- However, with the wide use of high bond bars (where the mechanics of bond is more complex and the action is not only adhesion of steel with concrete, but also mechanical locking by the projections on the steel bars as shown in Fig. 10.2), more emphasis is laid on development length requirements than on local bond.
- Thus, IS 456, Clause 26.2.2 deals with requirements for proper anchorage of reinforcements in terms of development length, Ld only.
- Fig. 10.2 Nature of bond in reinforcement bars: (a) Smooth bars; (b) Deformed bars.
- Hooks, bends, extensions etc. provided at the ends of bars are sometimes referred to as end anchorages, and their anchorage length is denoted by the symbol La.

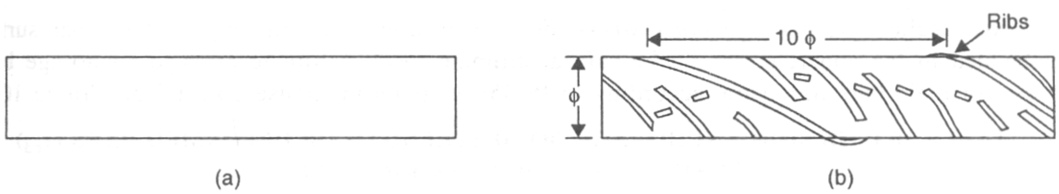


Fig. 10.2 Nature of bond in reinforcement bars: (a) Smooth bars; (b) Deformed bars.

Hooks, bends, extensions etc. provided at the ends of bars are sometimes referred to as *end horages,* and their anchorage length is denoted by the symbol L_a.

LOCAL (OR FLEXURAL) BOND

- Local bond (also known as flexural bond) at a point is 'fhe rate of change of tension in the steel at a given location in a R.C.C. member.
- With mild steel smooth bars where adhesion and friction are the main components of local bond, this is important, and an expression for the magnitude of flexural bond at a point can be derived as follows: In a distance d over the length of the beam, let the increase in tension be equal to T as given by Fig. 10.1. Then we get

$$T = dM/jd$$

Let u be the local bond stress and ΣO the perimeter of the steel area provided. Equating the forces, we obtain

$$u(\Sigma O)\,(dx) = T = jM/jd$$

$$u = \frac{dM}{dx}\left(\frac{1}{\Sigma Ojd}\right)$$

Therefore,

$$\text{Local bond stress} = \frac{V}{\sum Ojd}$$

- However, as already mentioned, the mechanism of bond between ribbed steel (which has projections along its length) and concrete is different from that in smooth bars and the above expression for local bond stress is not strictly valid for ribbed steel.
- The projections shown in Fig. 10.2 are so designed that bond failures will not normally occur, that hence checking of local bond stresses (which is obligatory for designs with mild steel smooth bars) is not required when using high bond bars.

AVERAGE (ANCHORAGE) BOND STRESS

- With modern high bond bars the mechanism of reinforcement anchorage is due to
    - Adhesion of concrete and steel,
    - Shear strength of concrete, and
    - Interlocking of ribs with concrete.
- Codes specify that, with high bond bars, the condition to be satisfied is that the average resistance called the average bond stress, developed along the full length of the bar surface embedded in the concrete, should be safe at ultimate loads.
- Ultimate average anchorage bond stress for plain bars in tension according to IS 456 is given in Clause 26.2.1.1 and Table 10.1.

TABLE 10.1 DESIGN ANCHORAGE BOND STRENGTH OF DEFORMED BARS (τ_{bd}
(Fe 415 Bars in Tension: IS Clause 26.2.1.1)

Concrete grade	15	20	25	30	35	40 and above
Bond strength N/mm^2	1.60	1.92	2.24	2.40	2.72	3.04

Notes: 1. Design ultimate average bond stress for

$$\text{Fe 250 steel} = \left(\frac{\tau_{bd} \text{ for Fe 415}}{1.6}\right)$$

2. BS 8110 uses the expression

$$\tau_{bd} = \beta\sqrt{f_{ck}}$$

In tension,

$\beta = 0.28$ for plain bars, 0.50 for deformed bars

For bars in compression, $\beta' = 1.25\ \beta$

It may be noted that the value of design ultimate anchorage bond stress in compression is larger by 25% because

- the compression tends to increase the diameter and tension tends to decrease the diameter of the bar,
- the end of compression bar also contributes to the transfer of load, and
- the adverse effects of flexural cracks are absent in the compression zone.

DEVELOPMENT LENGTH

- The length of bar necessary to develop the full strength of the bar is called the development length L see Fig. 10.3.
- The expression of Ld can be derived as taking design yield strength in tension as 0.874,

$$L_d\,(\pi\phi)\;\tau_{bd} = \frac{\pi\phi^2(0.87 f_y)}{4}$$

$$L_d = \frac{(0.87 f_y)\phi}{4\tau_{bd}} \quad \text{for tension}$$

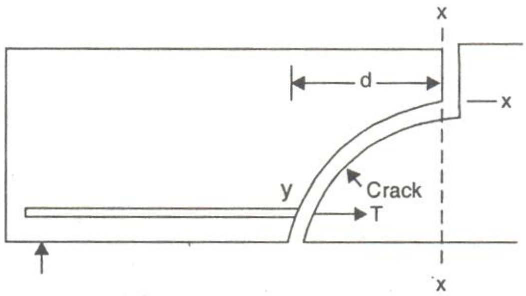


Fig. 10.3 Necessity of extending bar by *d* beyond theoretical cut-off point.

; design strength of steel in compression also as $0.87f_y$ for Fe 415 steel, we get

$$L_d = \frac{(0.87 f_y)\phi}{4\tau_{bd}(1.25)} \quad \text{for compression} \qquad (10.2a)$$

- Hence as a rough rule full anchorage of steel stressed to ultimate strength may be taken as 400 for tension and 300 in compression. The provisions in BS and ACT for development length are more complex than these simple rules in IS 456.

- It should also be noted that when the actual reinforcement provided is more than that theoretically required, so that the actual stress in steel will be less than the full stress.
- This principle is used in design of footings and other short bending members where bond is critical.
- By providing smaller sizes of bars or more steel than required by theory, the bond requirement can be satisfied.
- It should be remembered that Ld is calculated from the point of maximum stress.

END ANCHORAGE OF BARS

- It is the practice in detailing of steel to continue the reinforcement (both tension and compression steel) beyond the point where it is theoretically required for a distance equal to the effective depth of the beam or 12 times the diameter of the bar.
- This length is called end anchorage, La.
- The need for extension of reinforcement is evident from Fig. 10.3. If there is a diagonal crack, the force in steel will correspond to force at X and not Y (Refer SP 24 and IS 456, Clause 26.2.3.1).
- The condition to be satisfied is
- La = d or 120. whichever is greater

CHECKING DEVELOPMENT LENGTHS OF TENSION BARS

- The development length of bars which are in tension due to bending should be checked (in theory) at the following places.
- When detailing of bars is carried out, special care should be given at points where there is a change of direction of steel so that they do not tend to break away the concrete cover provided for the steel or produce very high compression in the concrete at the bend.
- This reinforcement should also be restrained by stirrup and other devices.
- For example, when column bars are bent at top of floors and spliced with steel from the next storey, laterals at close spacing should be provided for the horizontal component of the forces in these bent column bars.

ANCHORAGE OF A GROUP OF BARS (BUNDLED BARS)

- Anchorage of steel bars is accomplished by fixing of the tension or compression bars in concrete and providing the required development lengths.
- In conventional practice, placing of bars, one touching the other was not allowed.
- A space equal to the diameter of bar was to be left in between the bars to develop the bond stresses.
- In modern practice up to four bars can be bundled together to avoid congestion in heavily reinforced sections such as over the supports of continuous T beams.
- However, groups of bars, when used in compression, should be carefully examined for the additional provision of links for containment of the compression bars.
- According to IS 456, Clause 26.2.1.2, the development length required by each bar of the bundled bars is that of the individual bar increased by 10 per cent for the two bars, 20 per cent for three bars and 33 per cent for four bars which are in contact.
- More than four bars in contact are not allowed to be used.
- Care should also be taken not to stop all the bars of the group together.
- A spacing of at least 400 (which is equivalent to full development of a bar) should be maintained between the cutoff of each alternate bar at least till the bars are reduced to two.

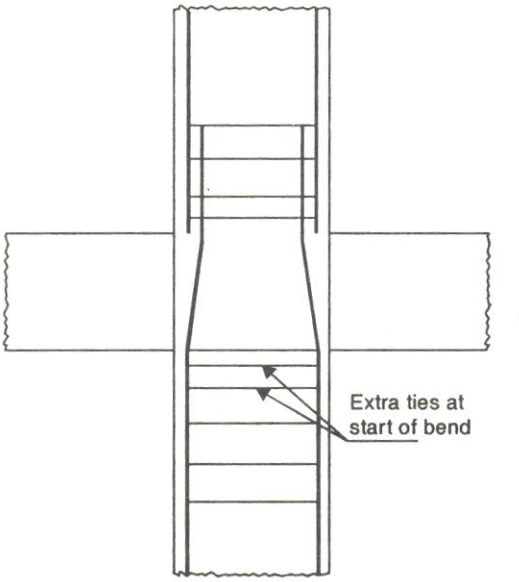
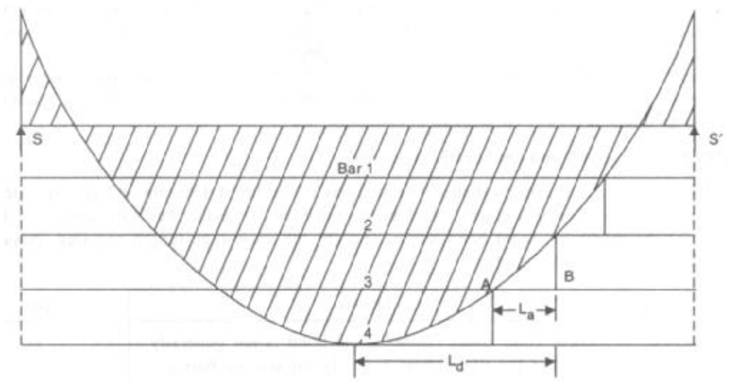


Fig. 10.7 Laterals at change in direction of bars.

CURTAILMENT OF BARS AND THEIR ANCHORAGE

- Bending moment at mid-spans of beams requires maximum area of steel. Towards the ends of the beam some of these steel may be stopped by curtailing them (Fig. 10.9).



. 10.9 Cut-off point of reinforcements: (A, theoretical cut-off point (TCP); B, physical cut-off point (PCP)).

- In practice, the theoretical cut-off point (TCP) and the actual or physical cut-off point (PCP) differ.
- The distance at which the PCP occurs should not be less than either the effective depth of the member or twelve times the bar size.
- In addition, the bar as a whole should satisfy the requirement of development length.
- Simplified empirical rules have been laid down for detailing of reinforcements for slabs and beams of nearly equal spans with UDL to comply with anchorage and other requirements.

USE OF SP 16 FOR CHECKING DEVELOPMENT LENGTH

- Tables 64 to 66, SF 16 give the tension and compression development lengths (I required for a design strength of 0.87f (the same in tension and compression).

$$L_d \text{ (in compression)} = \frac{L_d \text{ (in tension)}}{1.25}$$

- The length required (L for any other stress level that exists in the structure can be determined from these values by the expression

$$L'_d = \frac{f_s}{0.87 f_y} L_d$$

- Table 67 of SP 16 gives the anchorage value of hooks and bends for tension reinforcement.
- In tension anchorage, the effect of hooks, bends and straight lengths beyond bends, if provided, can be considered as development length. In compression bars, only the projected length of hooks, bends etc. are generally considered as effective towards development length.

IMPORTANCE OF LAPS AND ANCHORAGE LENGTH

- Laps in steel increase cost as well as difficulty in placing of concrete.
- When using large diameter bars in columns and foundations such as rafts, correct estimation of laps is required.
- Savings in lap length can considerably reduce cost and consumption of steel in the structure.
- Recent codes for footings and other foundation structures have relaxed the provision for anchorage length of starter bars for foundation structures .
- These can be utilized to effect economy of steel in construction.

UNIT IV

LIMIT STAYE DESIGN OFD COLUMNS

DESIGN OF AXIALLY LOADED SHORT COLUMNS

INTRODUCTION

- Members in compression are called columns and struts.
- The term 'column' is reserved for members which transfer loads to the ground and the term 'strut' is applied to a compression member in any direction, as those on a truss.
- Column members whose height is not more than three times its lateral dimension are called pedestals while the term 'wall' is used to compression members whose breadth is more than four times the thickness of the wall.
- It is well known from the theory of structures that the modes of failure of a column depend on its slenderness ratio.
- This ratio is expressed in IS and BS practice for reinforced concrete rectangular columns as the ratio of the effective length I. to its least lateral dimension (d), (L/d) ratio.
- In steel columns the slenderness ratio is generally expressed as the effective length to its least radius of gyration (L) ratio.
- This practice is continued for R.C.C. columns in ACT code. Effective length 1e of a column is different from its unsupported length L.
- Columns, when centrally loaded, fail in one of the three following modes, depending on the slenderness ratio (see Fig. 13.1).

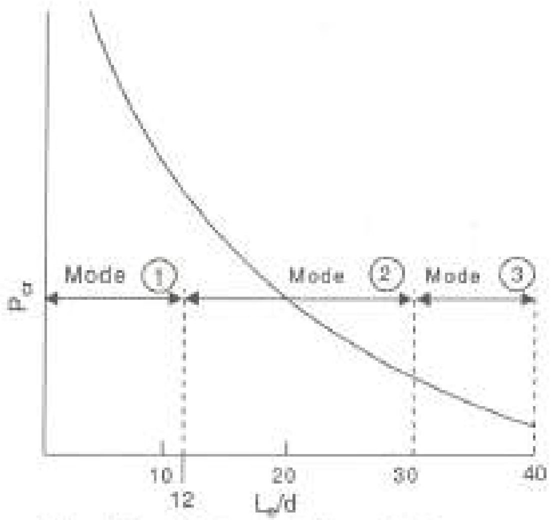


Fig. 13.1 Modes of failure of columns.

- **Mode 1: Pure compression failure**
    - The column fails under axial loads without undergoing any lateral deformation. Steel and concrete reach the yield stress values at failure.
    - The collapse of the column is due to material failure.
- **Mode 2: Combined compression and bending failure**
    - Short columns can be subjected to direct load (F) and moment (M). Slender columns even when loaded axially undergo deflection along their length as beam columns, and these deflections produce additional moments in the columns.
    - When material failure is reached under the combined action of these direct loads and bending moment, it is called combined compression and bending failure.
- **Mode 3: Failure by elastic instability**
    - Very long columns can become unstable even under small loads well before the material reaches yield stresses. Under such cases the member fails by lateral 'elastic buckling'
    - Failure by the third mode is unacceptable in practical construction (see Fig. 13.1).
    - R.C.C. members that may fail by this type of failure is prevented by the coda! provision that columns beyond a specified slenderness (30 for unbraced columns) should not be allowed in structures.

SHORT COLUMNS

- IS 456 classifies rectangular columns as short when the ratio of the effective length (Li) to the least dimension is less than 12.
- This ratio is called the slenderness ratio of the column. If the column is of dimension b x D, then there are two slenderness ratios namely,
    - **Slenderness ratio about major axis =Lex/D**
    - **Slenderness ratio about minor axis =Ley/b**

- If any of these two ratios is equal to or more than 12, it is called a slender column.
- If both ratios are less than 12, it is a short column.
- In BS the dividing ratio is taken as 15 for braced column and 10 for unbraced columns.
- In ACI the Dr ratio is used instead of the l/b ratio. The dividing line is taken as Dr = 34 for braced columns and Dr = 22 for unbraced columns.

BRACED AND UNBRACED COLUMNS

- columns can be planned in a structure so that they do not have to withstand any horizontal load like wind and earthquake loads.
- Thus, for example, when the column of water-tower are braced, the wind load is taken by the interaction of column bracings.
- In tall buildings, lateral supports like shear walls can be provided so that the lateral loads are taken by them.
- Such columns are called braced columns.
- Other columns, where the lateral loads have to be resisted in addition to vertical loads by the strength of the columns themselves, are considered as unbraced columns.
- Bracings can be in one direction or in more than one direction, depending on the likelihood of the direction

Design of Short Columns with Moments

- Columns, such as the external columns of framed buildings, or columns carrying crane toads through corbels of a column, are subjected not only to direct loads (F), but also to moments (Al) due to the eccentricity in application of the load (Fig. 14.1).
- In the above columns, the eccentricity is with respect to one axis only and these columns are said to be under uniaxial bending.

- On the other hand, a corner column of a building is subjected to eccentric load along both the X and Y axes.
- Such columns are said to be under biaxial bending. In Chapter 13 it was pointed out that the short column formula takes into account accidental eccentricity to a certain extent.
- (whichever is more) is greater than 0.05D already provided in the formula for short column, then the short column formula cannot be used for design of such columns.
- The theory of short columns subjected to axial load and moments should be used for their strength calculations.
- Slender columns, even when they are subjected to central load, bending moments are produced as they undergo deflection along its length as a beam column.
- Such columns also have to be designed as eccentrically loaded columns.

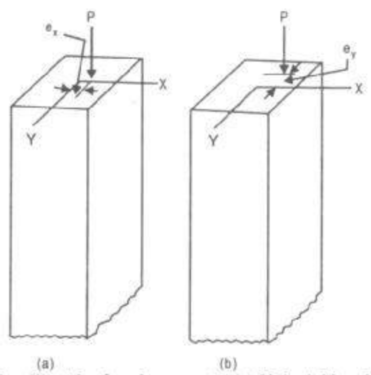


Fig. 14.1 Columns under direct load and moments: (a) Uniaxial bending; (b) Biaxial bending.

METHODS OF DESIGN

- The three methods that are commonly used to design these columns are:
  - Use of design charts (interaction diagrams)
  - Use of equilibrium equation to determine the minimum steel required
  - A simplified approximate method considering the section as a doubly reinforced beam.
- Of these, the interaction diagram is extensively used for design of rectangular or circular columns with symmetric arrangement of steel.

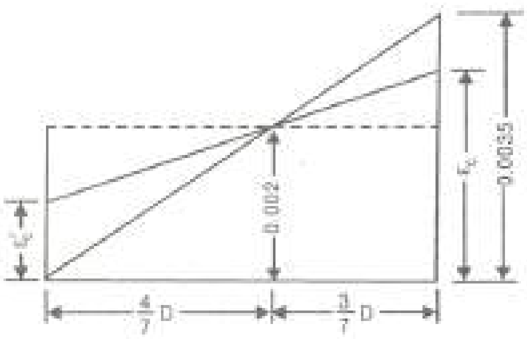
- The equilibrium method is based on fundamental concepts and is applicable to any cross-section and any arrangement of steel.
- The simplified method is found useful for columns with large eccentricities where the column acts more like a beam.

UNIAXIAL BENDING (DESIGN ASSUMPTIONS)

- In eccentrically loaded columns the strain distribution across the section will not be uniform as in the case of centrally loaded columns.
- As there is bending in addition to direct load, the strain distribution will vary linearly across the section as in the case of beams.
- The following assumptions as given in IS 456, Clause 39.1(b) are used to calculate the value of P and M of a given section:
- Plane sections remain plane even after bending.
- The strain at different points in the section will be different. The maximum compressive strain in concrete at failure is the governing criterion for ultimate failure. The magnitude of this failure strain is given by the expression (see also Fig. 14.2) $\varepsilon_c = 0.0035 - 0.75\varepsilon'_c$
- where e is the 'maximum' strain in compression in the section at failure and 4 is the 'minimum' strain in compression in the section at failure.
- The minimum compression strain in a beam 4 is equal to zero so that e = 0.
- In an axially loaded column, e = 4 so that e at failure = 0.002.
- It is interesting to note that the above expression is also equivalent to assuming that the strain distribution diagram rotates around a fulcrum at a distance 3D/7 from the highly compressed edge.
- This fulcrum point is the same point where the concrete strain will be 0.002 at failure with the neutral axis at the far edge of the section, as shown in Fig. 14.2.

- The design compression stress block under the varying strain is the same as assumed for beams. It is rectangular parabolic with the maximum stress value ° (approximately equal to 0). at failure.
- The design stress-strain curve for steel in compression is the same as in tension. The same design curve as for beams is assumed for columns also.

- The tensile strength of concrete is ignored.



Failure strain in concrete under direct load and moments.

STRESS-STRAIN CURVE FOR STEEL

- One should have a good idea of the stress-strain curves recommended by IS for the different types of steels to correctly assess the stresses corresponding to the strain in the steel.
- According to IS 456, mild steel (Fe 250) has a bilinear stress-strain curve, and failure strain on the design stress- strain curve is given by the expression

$$\varepsilon_s = \frac{0.87 \times 250}{200 \times 10^3} = 0.00109$$

- However, the design stress-strain curves of cold drawn bars, like Fe 415 steel, is not bilinear.
- There is a linear and a non-linear strain, for the stress levels beyond 0.801, so that the strains on the design stress-strain curve corresponding to the various design stress levels .

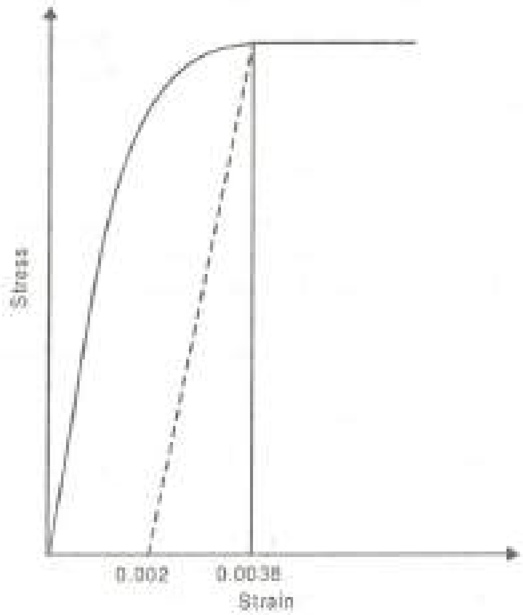


Fig. 14.3 Stress-strain curve for Fe 415 steel.

Points on the X-ordinate will be given by rearranging the terms as

$$\frac{M_u}{f_{ck}bD^2} = 0.36k\,(0.5 - 0.42k) \tag{14.4b}$$

Putting different values of k in Eqs. (14.4a) and (14.4b), the Y and X ordinates are obtained as

1. For $k = 1$, $Y = 0.36$ and $X = 0.029$
2. For $k = 0.5$, $Y = 0.18$ and $X = 0.12$
3. For $k = 0$, $Y = 0$ and $X = 0$

These are the values plotted in the interaction diagrams given in SP 16.

14.10 SHAPE OF INTERACTION CURVES

The shape of the interaction diagram will vary with the shape of the stress-strain diagram of steel, with kinks corresponding to the kinks in the stress-strain for steel. This can be noted from the differences in the shapes of the curves given in SP 16 for Fe 250, 415 and 500 steels.

14.11 ACCIDENTAL ECCENTRICITY IN COLUMNS

It was shown in Chapter 13 that the accidental eccentricity that should be assumed in a R.C. column according to IS 456 is

$$e_{min} = \frac{L}{500} + \frac{D}{30}, \quad \text{but not less than 20 mm (see Section 13.7)}$$

With small values of M, the consequent eccentricity $e = M/P$ may be small. In such cases one should note that the design moment—should be always larger than both of the following:

$M = M$ (given)

$M = Pe_{min}$ (due to accidental eccentricity)

14.12 USE OF INTERACTION DIAGRAMS FOR DESIGN AND ANALYSIS (METHOD 1)

Case 1: **Use of interaction curves to determine the area of steel required for a given column size for specified *P* and *M***

Step 1: Check whether the column is short or long. If it is long, proceed as in Chapter 15. If short, proceed as follows:

- Step 3: Determination of areas of steel from interaction curve is as follows: Choosing proper curve for grades of steel and d' find P/fck.
- Calculate A = p /(bD). Distribute this total area A as distributed in the sketch given in SP 16 for the interaction diagram.
- Case 2: Use of interaction curve to analyzed the safety of the given column for given P and M, with symmetrical distribution of steel
- **Step 1:** Find the following parameters:

$$\frac{a}{D}, \quad \frac{p}{f_{ck}}, \quad \frac{P_u}{f_{ck}bD}, \quad \frac{M_u}{f_{ck}bD^2}$$

Step 2: Determine safety of column as follows: Determine the point corresponding to the given

$$\frac{P_u}{f_{ck}bD}, \quad \frac{M_u}{f_{ck}bD^2}$$

on the corresponding interaction curve for p/f_{ck}. If the point is on or inside the interaction curve, the column is safe. If the point is outside the curve, it is unsafe.

DESIGN OF ECCENTRIC COLUMNS BY EQUILIBRIUM EQUATION (METHOD 2)

- Another method that can be used for design of eccentrically loaded column is to work from fundamentals by using equilibrium equation and to arrive at the necessary steel for a given section of breadth b, depth D, with given P and M, as follows:
- Step 1: Assume the arbitrary depth of neutral axis. Let the extreme fibre in concrete reach failure strain as explained in the assumptions in Section 14.3 above.
- Step 2: Determine the strains in the steels.
- Step 3: Determine the compression force in concrete by using Table 14.1 or by other means. Find also the stress f, in compression steel that will be provided near the compression face from the strain at the level of steel.
- Step 4: Determine the area of steel to be provided at the compression face (A by taking moments of all the forces about the position of the steel at the tension face. The moment equilibrium equation will be

$$P[e + D/2 - d'] = \begin{pmatrix} \text{Moment of compression} \\ \text{in concrete about tension} \\ \text{steel} + f_{sc} A_{s1}(d - d') \end{pmatrix}$$

Step 5: Determine A_{s2}, the steel required on the tension face, from the second equation of equilibrium of forces

$$P = C_c + f_{s1}A_{s1} \pm f_{s2}A_{s2}$$

Step 6: From these, determine the total area of steel

$$A_s = A_{s1} + A_{s2}$$

Plot the value of A_s against the value of the depth of the neutral axis assumed (Fig. 14.9).

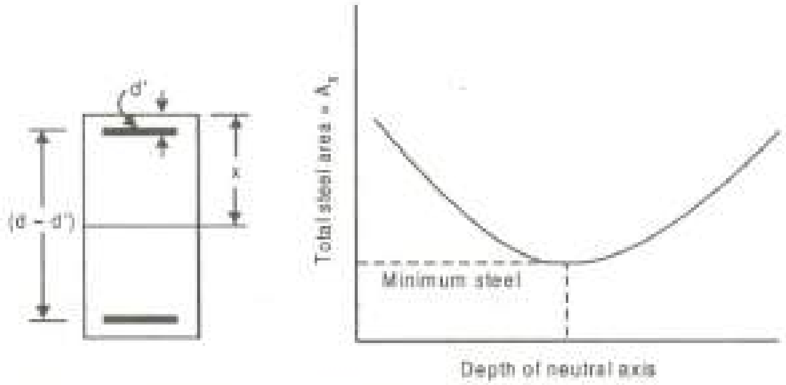


Fig. 14.9 Plot to determine minimum steel in section under direct load and moment.

- Step 7: Assume other suitable values of the depth of neutral axis and plot the values of A (total steel) needed, in the above diagram. The values of the neutral axis corresponding to minimum value of A can be taken as the optimum solution of the problem.
- Step 8: Provide the steel A and A as obtained in step 7 for optimum value of x. The procedure is similar.

SIMPLIFIED METHOD (METHOD 3)

- Columns under large eccentricity of load may be solved by considering them as equivalent to a doubly reinforced beam with a concentrated load acting on it (for equilibrium of forces) as shown in Fig. 14.10.
- The effect of P and M may be regarded as equivalent to a modified moment (M + Ma) and a force P applied along the steel on the tension side, where

$$M + M_a = M + P\left(\frac{D}{2} - d_2\right)$$

- In addition, P acts through the tension steel.
- As tables for doubly reinforced beams for these large eccentricities will not be available, calculation has to be made from the basic equations.
- The simplified stress block as uniform compression can be used for satisfying the equilibrium of forces.

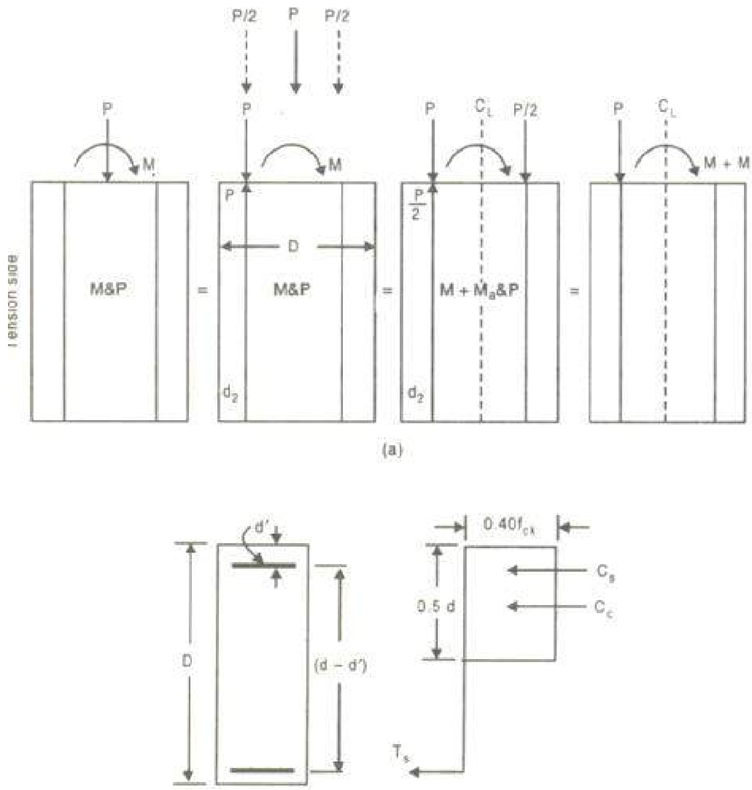
- Taking moments about the tension steel, we get

$$M_u = (0.4f_{ck}b)(0.5d)(0.75d) + 0.72 f_y A_{sc}(d - d')$$
$$= 0.15f_{ck}bd^2 + 0.72 f_y A_{sc}(d - d')$$

from which the value of A_{sc}, the area of compression steel, can be calculated.

For equilibrium of forces, the area of tension steel A_{st} is given by the equation

$$0.87 f_y A_{st} = (0.4f_{ck}b)(0.5d) + 0.72 f_y A_{sc}$$
$$= 0.2f_{ck}bd + 0.72 f_y A_{sc}$$



- However, a correction has to be made for the value of P acting as compression P on tension steel.
- This reduces the tension steel required and the area of the reduced tension steel is given $A_{st1} = A_{st} - \left(\frac{P}{0.87 f_y}\right)$
- It is evident from Eq. (14.3) that the equations are valid only when the beam theory as is valid for the situation, i.e. the eccentricity is larger than (D/2 -2).

MEMBER SUBJECTED TO BIAXIAL BENDING

- IS 456, Clause 38.6 deals with biaxial bending. The analysis may be used by oneof the following two methods:
    - By choosing the neutral axis which is in the XY-plane. Calculations are made from fundamentals to satisfy the equilibrium of load and moments about both the axes. (This method is quite tedious and is not generally recommended for routine design).
    - By the use of the formula recommended for use of IS 456. The above code recommends the use of the following relation:

$$\left(\frac{M_x}{M_{x1}}\right)^{\alpha_n} + \left(\frac{M_y}{M_{y1}}\right)^{\alpha_n} \leq 1.0 \qquad (14.9)$$

where M_x and M_y are the applied moments about the X and Y axes and M_{x1} and M_{y1} are the maximum uniaxial moments the column can take under the actual load P by bending along the XX- and YY-axis, respectively.

α_n is related to the value of P/P_z

where

P = design load on the column

$P_z = 0.45 f_{ck} + 0.75 f_y A_{sc}$ (i.e. value of P_u when $M = 0$)

α_n = exponent whose value is to be taken as follows:

P/P_z	α_n
0.2	1.0
0.8	2.0

The intermediate values are to be obtained by interpolation, as shown in Fig. 14.11. The value of α_n can be also determined by the equation

$$\alpha_n = \frac{2}{3}\left(1 + \frac{5}{2}\frac{P}{P_z}\right)$$

which should be within 1.0 and 2.0 as above.

- The IS code formula follows the "Bresler Load Contour Method". It is based on the concept of a failure surface which is the envelope of a number of interaction curves for different axes of bending of a column, as shown in Fig.14.12.
- Any point of the failure surface corresponds to failure condition in a column about a neutral axis and any point inside the surface can be considered safe.

- A horizontal section at each level defines a load contour corresponding to a particular PIPE value. The general form of the contour is defined by relation (14.9).
- It may be noted that the limiting value , = 1.0 for P/Pt = 0.2 represents the equation of a straight line and the value of a, = 2 for P/Ps = 0.8 represents a circle.
- These values are meant to represent the shape of the interaction diagrams at these points. Application of this formula is greatly facilitated by Chart 64 given in SP 16 (Chart 14.3 of the text).

SIMPLIFIED 85 8110 METHOD FOR BIAXIAL BENDING

- For the design of symmetrically reinforced rectangular columns under biaxial bending, results comparable with those obtained by the Bresler method can be obtained by the simplified design procedure recommended in BS 8110, Clause 3.8.4.5.
- The principle of the method is to transform the biaxial bending case to a uniaxial bending case which should withstand an increased moment about that axis according to the two conditions which are now given.
- Let the column be subjected to (P M and M,). Then it can be designed for uniaxial bending of (P. M) or (P. M). depending on the following conditions:

Condition 1: When $M_x/d \geq M_y/b'$, M_x controls the design and the column is to b
for P and M'_x, where

$$M'_x = M_x + \alpha \frac{d}{b'} M_y$$

Condition 2: When $M_x/d < M_y/b'$, M_y controls the design and the column is to b
for P and M'_y, where

$$M'_y = M_y + \alpha \frac{b'}{d} M_x$$

In the above expression,

d = effective depth with respect to major axis and total depth D

b' = effective depth with respect to minor axis and total depth b

$$\alpha = \text{coefficient} = \left(1 - \frac{7}{6}\frac{P}{f_{ck}bD^2}\right)$$

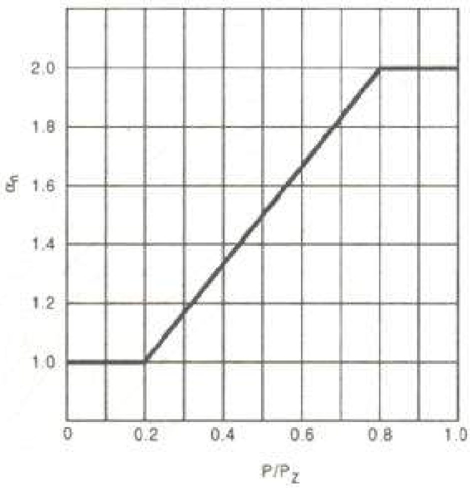


Fig. 14.11 Illustration of coefficient α_n for biaxial bending of columns.

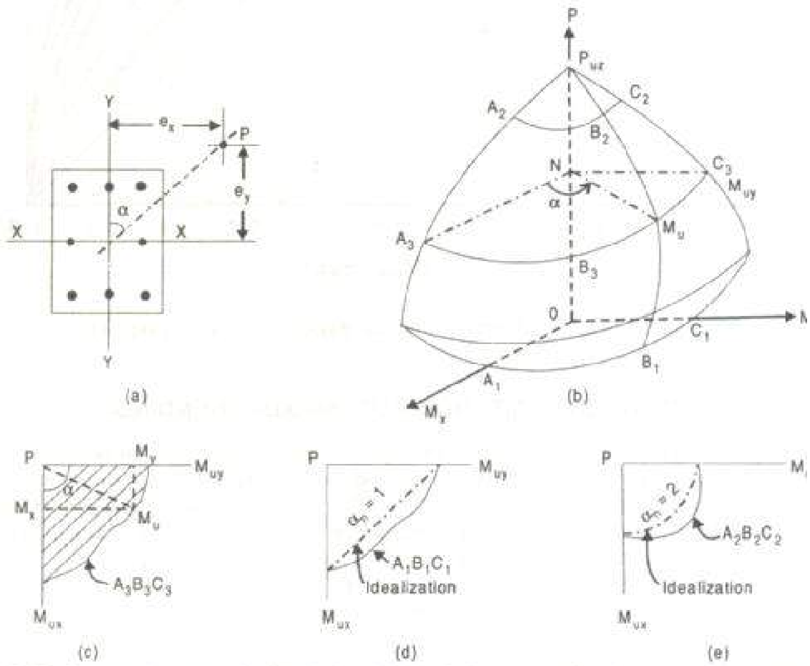


Fig. 14.12 Interaction curve for biaxial bending of columns: (a) Section; (b) Interaction s (c) Section at A_3C_3; (d) Section at A_1C_1; (e) Section at A_2C_2.

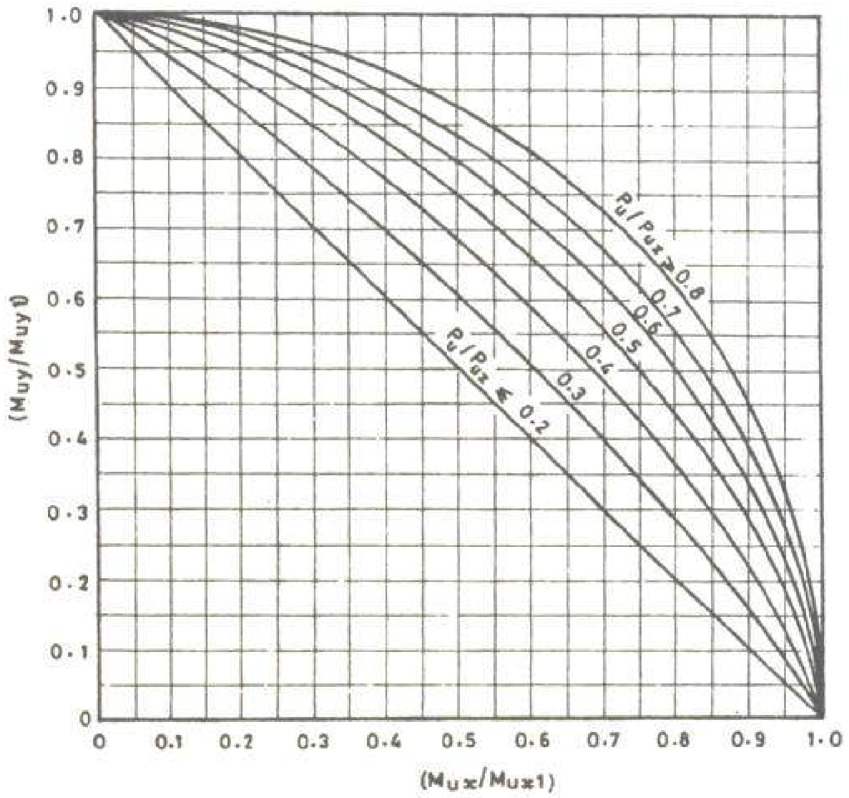
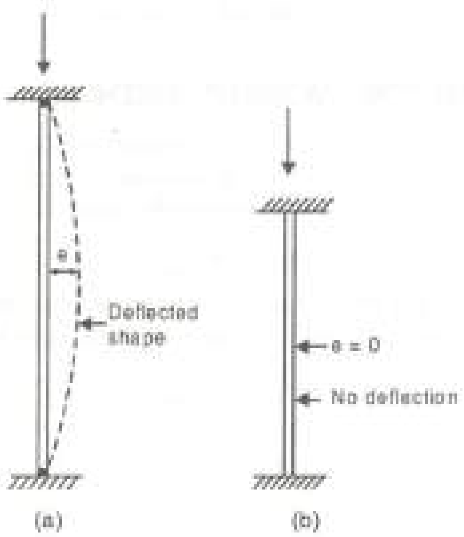


Chart 14.3 Check for Safety in Biaxial Bending [SP 16 Chart 64].

Design of R.C. Slender Columns

INTRODUCTION

- Rectangular columns when the ratio of the effective length of the columns to its lateral dimension exceeds 12, it is called a slender or long column according to IS code.
- In BS 8110, the limits for a short column are put as 15 for braced and 10 for unbraced columns.
- The difference between the behavior of short and slender columns is that, when slender columns are loaded even with axial loads, the lateral deflection (measured from their original centre lines along its length) becomes appreciable, whereas in short columns this lateral deflection is very small and can be neglected as shown in Fig.



1 **Behaviour of long and short columns: (a) Long column; (b) Short column.**

- In slender columns the moment produced by this deflection is large and has to be taken into account in design. Hence slender columns, even if centrally loaded, have to be designed for not secondary moment produced by the lateral deflection.
- There are three major methods that are used to take into account the slenderness effect of these columns. They are:
    - The reduction coefficient method
    - The additional moment method
    - The moment magnification method.
- The reduction coefficient method as given in IS 4 generally recommended for working stress design where designs are made for service loads (not factored loads) using allowable stresses in steel and concrete.
- The procedure is to reduce the allowable load by a reduction factor that depends on the slenderness of the column as given in IS 456, Clause 8.3.3 of Annex. B.
- In the limit state method, however, where one works with factored loads and ultimate strengths of steel and concrete, one of the other two methods is generally recommended.
- The British and the Indian codes specify the use of the additional moment method as given in IS 456, Clause 38.7.1, whereas ACT recommends the use of moment-magnification method.

MAXIMUM PERMITTED LENGTH OF COLUMNS

- In order to avoid buckling failures, IS 456, Clause 25.3.1 limits the unsupported length between restraints to 60 times the least lateral dimension. In practice, the unsupported length to breadth ratio of column is restricted to 60 in braced columns and 30 in unbraced columns.
- This restriction will ensure that the final failure will be due to material failure only and the classical buckling failure will be avoided.

BASIS OF ADDITIONAL MOMENT METHOD

- Slender columns, even when loaded axially, produce moments along their length due to lateral deflection.
- If is the maximum deflection, along the axis of the column, the moment produced by this deflection is given by the expression

$$M_{max} = Pe_{max}$$

This is the additional moment that has to be added to the external moments to obtain the design moment. This additional moment is usually designated as M_{add} or M_a so that

$$M_{add} = Pe_{add}$$

where

$$e_{add} = e_{max}$$

- The analysis for add, which is a second order analysis, is also called **P-J analysis.** In un braced frames such an analysis should also include the effect of sway deflections. However, this aspect is not dealt with in this text and is found in literature on structural analysis of frames.

EXPRESSION FOR LATERAL DEFLECTION

- In theory of structures the elastic deflection of a bending member is represented by th $\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R}$
- (is called the curvature, which may be defined as the change in slope over unit length of the member, assuming it to be constant along this length.
- Integrating the above expression twice, we get

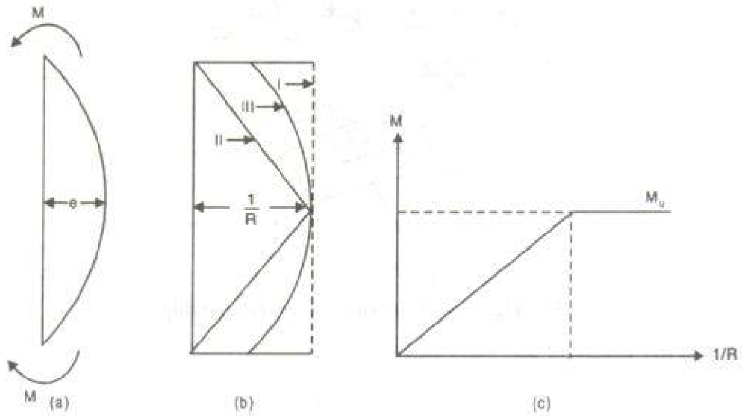
$$y = \iint\left(\frac{1}{R}dx\right)dx = \iint\left(\frac{M}{EI}dx\right)dx$$

- This means that the deflection depends on the distribution of curvature of MIE diagram, along the length of the member.
- Assuming that the moment curvature relationship is linear and is not influenced by axial load, one can express the maximum deflection in terms of l/r.
- Thus for a UDL on a simply supported beam with a parabolic distribution of bending moment, the maximum deflection can be expressed as

$$e_{max} = \frac{5}{384}\left(\frac{wL^4}{EI}\right) = \frac{(5)8}{384}\left(\frac{wL^2}{8}\right)\frac{L^2}{EI}$$

$$= \frac{(M_{max})}{EI}\frac{L^2}{9.6}$$

- Taking a very conservative estimate of a uniform rectangular distribution of the bending moment .
- Taking a very unconservative estimate of a triangular distribution of the bending moment diagram, we obtain



2 Curvature of bent columns: (a) Deflected shape at ultimate load; (b) Curvature (M/EI) diagrams showing rectangular, triangular, and assumed probable distribution; (c) Linear moment-curvature relationship.

$$e_{max} = \frac{L^2}{12}(1/R)_{max}$$

Hence, one may assume that a reasonable estimate of e_{max} can be obtained from the expressio

$$e_{max} = \frac{L^2}{11}(1/R)_{max}$$

Thus the eccentricity to be taken into account for calculation of the additional moment is equa to

$$e_{add} = \frac{L_e^2}{11}(1/R)_{max}$$

where L_e is the effective length of the column.

Assuming that the bending strains at failure (ultimate conditions) are as given in Fig. 16. (balanced failure), we have

$$(1/R)_{max} = \frac{\varepsilon_c + \varepsilon_s}{D} = \frac{0.0035 + 0.002}{D} = \frac{1}{182D}$$

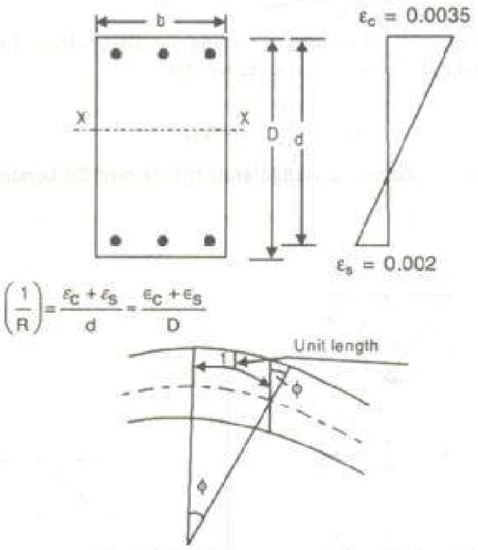


Fig. 16.3 Curvature-strain relationship.

$$e_{add} = \frac{D}{2000}\left(\frac{L_e}{D}\right)^2$$

Both IS 456 and BS 8110 use the reduced form of this equation, viz.

$$\frac{e_{add}}{D} = \frac{1}{2000}\left(\frac{L_e}{D}\right)^2$$

Putting the right-hand side expression as β, we have

$$e_{add} = \beta D$$

Therefore,

$$M_{add} = M_a = Pe_{add}$$

The value of β for (L_e/D) ratios can be read off from Table I of SP 16 or calculated from the above formula. If the value of additional moment on the XX-axis is designated by M_{ax} and that on YY-axis as M_{ay}, they can be calculated by the formulae

$$M_{ax} = \frac{PD}{2000}\left(\frac{L_{ex}}{D}\right)^2$$

$$M_{ay} = \frac{Pb}{2000}\left(\frac{L_{ey}}{b}\right)^2 \quad (16.1)$$

where

P = axial load on the member

L_{ex} = effective length in respect to major axis

L_{ey} = effective length in respect to minor axis

D = depth at right angles to the X-axis

b = breadth at right angles to the Y-axis

REDUCTION FACTOR FOR ADDITIONAL MOMENT

- It is quite obvious from physical considerations that the lateral deflection of a column must be less when a large portion of the column section is in compression.
- The expression for corn derived above was on the assumption that the curvature is the one corresponding to the balanced failure where the maximum strain in tension steel e = 0.002 and the maximum compression strain in concrete e = 0.0035 at failure.
- For any value of P for which the strain e is less than that of balanced failure, the deflection and hence the additional moment should also be less.
- Even though IS has made this modification optional, this reduction in many cases can be substantial, and should be taken into account from economy point of view.

$$k = \frac{P_z - P}{P_z - P_b} \leq 1 \quad \text{or} \quad k = \frac{1 - P/P_z}{1 - P_b/P_z} \leq 1$$

- The reduced deflection can be expressed by the reduction factor k given by IS

where

$$\frac{d_c}{d} = \frac{0.0035}{0.0035 + 0.002}$$

$$d_c = 0.636d$$

Assuming tension steel and compression steels are equal and are equally stressed so that $f_{s2} A_{s2} = f_{s1} A_{s1}$ for a section where equal steel is put on the two opposite sides, we have

$$P_b = (0.4 f_{ck} b)(0.636d) = 0.254 f_{ck} bd$$

The value of Pfr as obtained above is slightly different from that obtained by

16.5.2 Use of SP 16 for Determination of P_b

Table 60 on page 171 of SP 16 (Table 16.2 of the text) gives a more accurate method to calculate P_{bal}, especially when steel is distributed on all faces. It uses an expression in the following forms

1. *For rectangular section,*

$$\frac{P_b}{f_{ck}bD} = \left(k_1 + k_2 \frac{p}{f_{ck}}\right) \quad (16.4$$

2. *For circular section,*

$$\frac{P_b}{f_{ck}D^2} = \left(k_1 + k_2 \frac{p}{f_{ck}}\right) \quad (16.4a$$

k_1 and k_2 can be read off from the Table 16.1, where p is the percentage of the total steel in the column.

TABLE 16.1 VALUES OF k_1 AND k_2 FOR VALUES OF P_b
(SP 16, Table 60)

	Section	d'/D 0.05	0.10	0.15	0.20
Value of k_1	Rectangular	0.219	0.207	0.196	0.184
	Circular	0.172	0.160	0.149	0.138
	Rectangular				
	Equal steel on two sides	0.096	0.082	0.046	– 0.022
Value of k_2 for Fe 415 steel	Equal steel on four sides	0.424	0.328	0.203	0.028
	Circular	0.410	0.323	0.201	0.036

- It may however be noted that the value of k cannot be estimated until the area of the steel in the section is known. Hence, one has to initially assume a percentage of steel and determine the value of k and check for safety by successive approximation.
- Having obtained b as above, Chart 65 on page 150 of SP 16 (Chart 16.2 here) gives a quick method of obtaining k, the reduction factor from PIPE and P values.

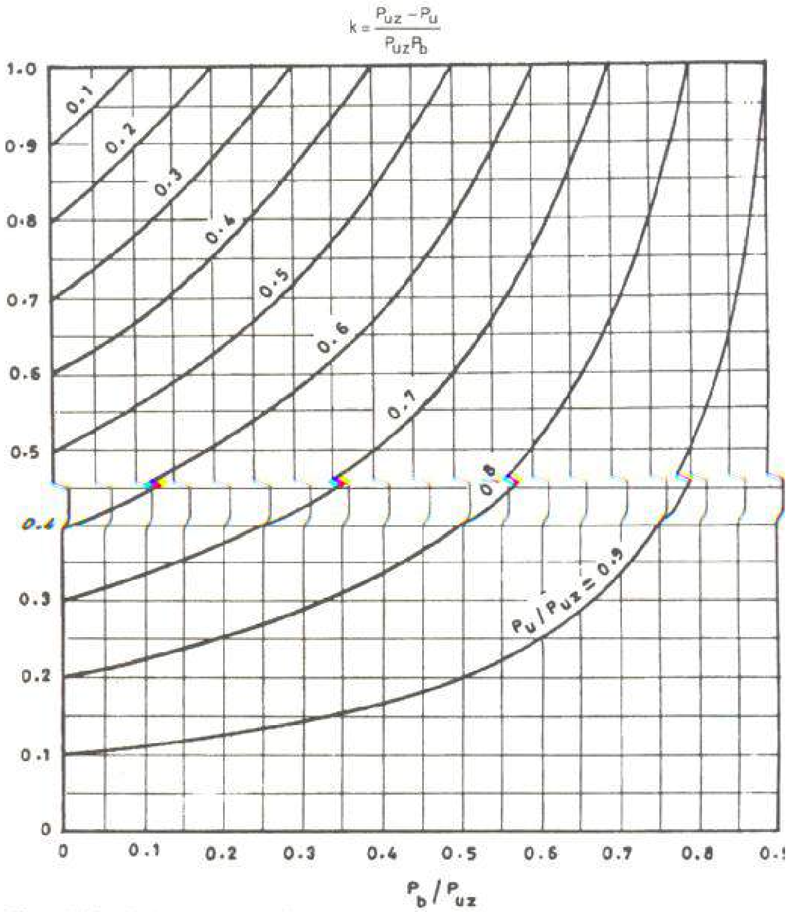


Chart 16.2 Reduction factor k for additional moment in columns (Chart 65 of SP 16)

FACTORS AFFECTING BEHAVIOUR OF SLENDER COLUMNS

- Columns are usually classified as one of the following types:
  - **Pin-ended columns**
  - **Braced columns**
  - **Unbraced columns.**
- They can also bend in single curvature or double curvature.
- The various types of bending that can occur in column is shown in Figs.
- Analysis of structures like building frames gives the value of the axial load and the moments at top and bottom of the columns.

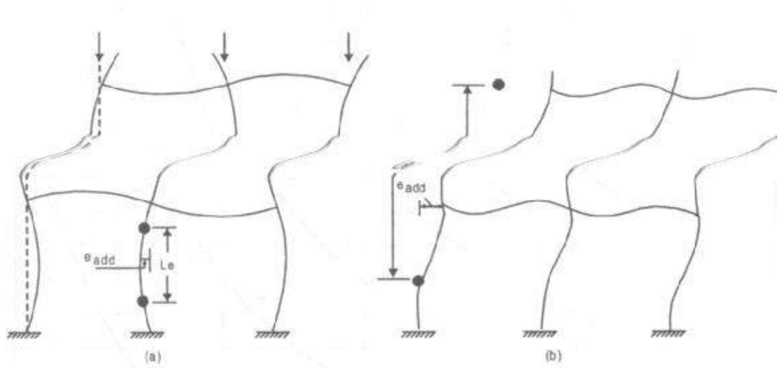


Fig. 16.5 Bending of columns in frames: (a) Braced; (b) Unbraced.

- The resulting bending moment diagram will indicate whether the column is bent in single or double curvature.
- If the moments on the two ends are opposite in sign, the column bends in single curvature.
- In designing slender columns to get the total moment M, for final design. one has to determine the combined effects of all the following three factors:
- The initial moment M caused by the end moments, M and M (The larger value is taken as positive and designated as M If the bending is in double curvature, M is taken as negative.)
- Moment due to accidental eccentricity usually designated by the term M or Mm.
- The magnitude of M will depend on whether the columns are in single or double curvature and whether they are braced or unbraced as explained in notes 1 and 2 of IS 456, Clause 39.7.1.
- The Explanatory Handbook on IS 456, SP 24, Section 38.7 may also be referred to for more details.